# Image Quality and Stability Criteria – Transmissive Systems

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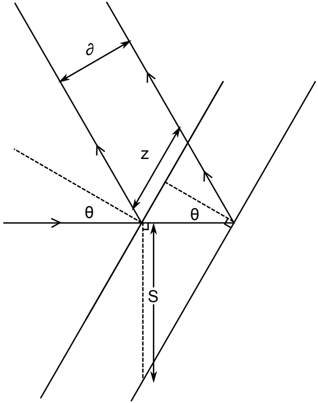
# Table Expansion

- If everything is fixed to the table, and the table expands uniformly, then everything moves radially away from the centre of expansion.
- The absolute change in position of any object depends on the centre of expansion chosen.
- However, the relative motion of any two objects remains radial, which can be seen by taking the centre of expansion as under one of the objects of interest.

So what happens when the light beam is not fixed to the table?

# Table Expansion and Pupil Shear

I somewhat regret ringing this bell. It should not be a problem after all. Let's look at the geometry.




$$z = 2 \times S \tan\theta \times \sin\theta$$

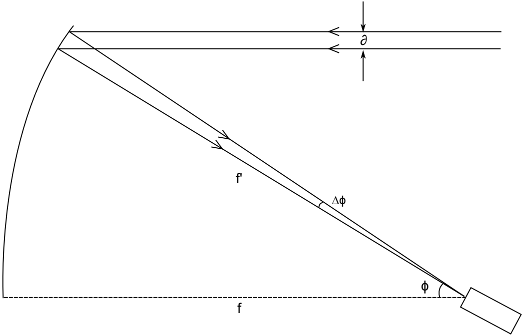
$$\delta = z \cos\theta$$

$$= 2 \times S \times \sin^2\theta$$

# Table Expansion and Pupil Shear

Now consider the effect of this displacement on the OAP.

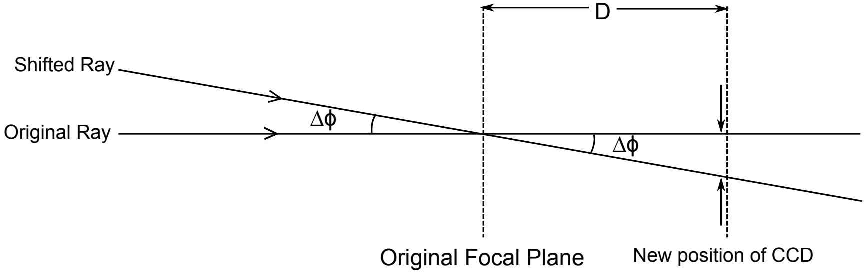
$$\Delta\phi \approx \frac{\delta}{f'}$$



# Table Expansion and Pupil Shear

So what effect does this have on the CCD? The image has moved by

$$\begin{aligned}\text{Observed Shift} &\approx D \times \Delta\phi \\ &= \alpha \Delta T \; f' \frac{\delta}{f'} \\ &= \alpha \Delta T \times S \times \sin^2 \theta\end{aligned}$$



# Table Expansion and Pupil Shear

So how big a shift does this correspond to?

$$\text{Observed Shift} = \alpha \Delta T \times 2 \times S \times \sin^2 \theta$$

$$\alpha = 18 \times 10^{-6} K^{-1}$$

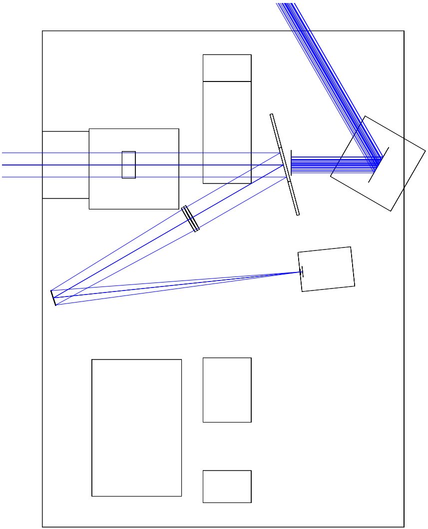
$$\theta = 15^\circ; \qquad \sin 15^\circ \approx \frac{1}{4}$$

$$\Delta T_{\max} = 5K$$

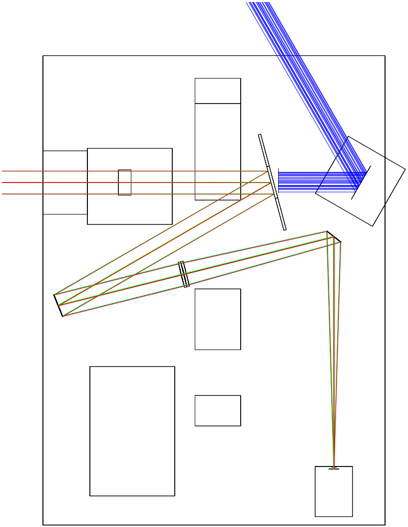
$$\text{Observed Shift} \approx 1.125 \times 10^{-5} \times S$$

So this is not going to be a problem after all. Sorry!

# Transmissive Layouts



# Transmissive Layouts



# Image Quality

Criterion was that the spot size should be on the order of the pixel scale across a field of view with side of 20 arcseconds. This field of view is the off-axis field of view we are required to guide within. For the common elements:

| Element | Degree of Freedom | Allowed movement |
|---|---|---|
| Dichroic | $\Delta\theta_{\mathrm{x}}$, $\Delta\theta_{\mathrm{y}}$ | $0.3^\circ$ |
| | $\Delta$z | Insensitive |
| | $\Delta$x, $\Delta$y | Unconstrained |
| | $\Delta\theta_{\mathrm{z}}$ | Unconstrained |
| Lens | $\Delta\theta_{\mathrm{x}}$, $\Delta\theta_{\mathrm{y}}$ | $0.7^\circ$ |
| | $\Delta$z | 200$\mu$m |
| | $\Delta$x, $\Delta$y | Unconstrained |
| | $\Delta\theta_{\mathrm{z}}$ | Unconstrained |

The CCD has the same defocus tolerance as the lens, and all other directions are insensitive or unconstrained.

# Image Quality

For the setup with one folding mirror:

| Element | Degree of Freedom | Allowed movement |
|---|---|---|
| FM1 | $\Delta\theta_{\mathrm{x}}$, $\Delta\theta_{\mathrm{y}}$ | $0.8^{\circ}$ |
| | $\Delta$z | 100$\mu$m |
| | $\Delta$x, $\Delta$y | Unconstrained |
| | $\Delta\theta_{\mathrm{z}}$ | Unconstrained |

These figures are reassuringly large.

# Image Quality

For the setup with two folding mirrors:

| Element | Degree of Freedom | Allowed movement |
| --- | --- | --- |
| FM1 | $\Delta\theta_{\mathrm{x}}$, $\Delta\theta_{\mathrm{y}}$ | $0.3^\circ$ |
| | $\Delta$z | Insensitive |
| | $\Delta$x, $\Delta$y | Unconstrained |
| | $\Delta\theta_{\mathrm{z}}$ | Unconstrained |
| FM2 | $\Delta\theta_{\mathrm{x}}$ | $0.7^\circ$ |
| | $\Delta\theta_{\mathrm{y}}$ | $1^\circ$ |
| | $\Delta$z | 200$\mu$m |
| | $\Delta$x, $\Delta$y | Unconstrained |
| | $\Delta\theta_{\mathrm{z}}$ | Unconstrained |

So again, not too bad. Like the OAP system, it's the stability that is forcing our hand.

# Image Stability

Criterion was image should not move by more than 0.015 arcseconds, or $\Delta = 1.63\mu$m for the focal length being considered here. Common elements:

| Element | Degree of Freedom | Allowed movement |
|---|---|---|
| Dichroic | $\Delta\theta_x$, $\Delta\theta_y$ | 0.10 arcsec |
| | $\Delta$z | Insensitive |
| | $\Delta$x, $\Delta$y, $\Delta\theta_z$ | Unconstrained |
| Lens | $\Delta$x, $\Delta$y | 1.63$\mu$m |
| | $\Delta\theta_x$, $\Delta\theta_y$ | 30 arcsec |
| | $\Delta$z | Unconstrained |
| | $\Delta\theta_z$ | Unconstrained |
| CCD | $\Delta$x, $\Delta$y | 1.63$\mu$m |
| | $\Delta\theta_x$, $\Delta\theta_y$ | Insensitive |
| | $\Delta$z | Unconstrained |
| | $\Delta\theta_z$ | Unconstrained |

# Image Stability

With one folding mirror:

| Element | Degree of Freedom | Allowed movement |
| --- | --- | --- |
| FM1 | $\Delta$z | $6\mu$m |
| | $\Delta\theta_{\mathrm{x}}$, $\Delta\theta_{\mathrm{y}}$ | 0.18 arcseconds |
| | $\Delta$x, $\Delta$y | Unconstrained |
| | $\Delta\theta_{\mathrm{z}}$ | Unconstrained |

# Image Stability

With two folding mirrors:

| Element | Degree of Freedom | Allowed movement |
|---|---|---|
| FM1 | $\Delta\theta_x$, $\Delta\theta_y$ | 0.10 arcseconds |
| | $\Delta z$ | Insensitive |
| | $\Delta x$, $\Delta y$ | Unconstrained |
| | $\Delta\theta_z$ | Unconstrained |
| FM2 | $\Delta z$ | 1.3$\mu$m |
| | $\Delta\theta_x$, $\Delta\theta_y$ | 0.17 arcseconds |
| | $\Delta x$, $\Delta y$ | Unconstrained |
| | $\Delta\theta_z$ | Unconstrained |

These are certainly more ominous figures than for the image quality criterion.

# OAP Mirror vs Lens Comparisons

- Dichroic in both cases must be stable to better than about 0.1 arcseconds. This comes from

$$\frac{0.015 \times 14}{2} = 0.105 \text{ arcseconds}$$

- The most stringent positional requirement in either system comes from holding the relative shear of the OAP/lens and the CCD to around $1\mu$m. High incident angle folding mirrors must also be held to this accuracy if placed after the lens.
- There is at least one extra element to hold to high accuracy using the lens and folding mirror arrangement.
- Is the OAP harder to hold accurately than the lens due to the inherent asymmetry?

# Sample Error Budget - OAP

What are the stability requirements on each individual component?

| Element | Degree of Freedom | Allowed movement | Limiting Requirement |
|---|---|---|---|
| Dichroic | $\Delta\theta_x$, $\Delta\theta_y$ | 0.11 arcsecond | Position of image |
| | $\Delta z$ | Insensitive | n/a |
| | $\Delta x$, $\Delta y$ | Unconstrained | n/a |
| OAP | $\Delta x$, $\Delta y$ | 1.2-2$\mu$m | Position of image |
| | $\Delta z$ | 100$\mu$m | Quality of image |
| | $\Delta\theta_x$, $\Delta\theta_y$ | 0.11 arcsecond | Position of image |
| | $\Delta\theta_z$ | 0.76 arcsecond | Position of image |
| CCD | $\Delta x$, $\Delta y$ | 1.2-2$\mu$m | Position of image |
| | $\Delta z$ | 100$\mu$m | Quality of image |
| | $\Delta\theta_x$, $\Delta\theta_y$ | Insensitive | n/a |
| | $\Delta\theta_z$ | Unconstrained | n/a |

# Sample Error Budget - OAP

Can divide up our sources of error quite nicely:

- Some constraints affect quality, some position. These can be readily divorced, as where one element affects both, the position requirement is far more stringent. The only two quality requirements are OAP and CCD $\Delta z$, so we can assign 70$\mu$m here to each, so the RMS despace is still only 100$\mu$m.
- The constraints that affect position are either angles or shears. Both seem hard, so assign budget equally between the two - each contribute $\frac{\Delta}{\sqrt{2}}$ of movement, where $\Delta$ is the movement on the CCD equivalent to 0.015 arcseconds on the sky.
- With each of angles and shears, the contribution to $x$ and $y$ movement of the image can be separated (to first order), so each of these can contribute $\frac{\Delta}{2}$.

# Sample Error Budget - OAP

We have argued our way down to a list of pairs of (approximately) independent and linear variables that each must contribute $\frac{\Delta}{2}$. So when added in quadrature, we expect each member of the pair to contribute $\frac{\Delta}{2\sqrt{2}}$. As they are linear, we can simply take the tolerances found earlier and divide by $2\sqrt{2}$.

| Criterion | Type of Degrees of Freedom | Degrees of Freedom |
|---|---|---|
| Quality of Image | Position | $\Delta$ z$_{\mathrm{OAP}}$, $\Delta$ z$_{\mathrm{CCD}}$ |
| Position of Image | Tilt | $\Delta\theta$ x$_{\mathrm{D}}$, $\Delta\theta$x$_{\mathrm{OAP}}$ |
| | | $\Delta\theta$ y$_{\mathrm{D}}$, $\Delta\theta$y$_{\mathrm{OAP}}$, $\Delta\theta$z$_{\mathrm{OAP}}$ |
| | Shear | $\Delta$x$_{\mathrm{OAP}}$, $\Delta$x$_{\mathrm{CCD}}$ |
| | | $\Delta$y$_{\mathrm{OAP}}$, $\Delta$y$_{\mathrm{CCD}}$ |

# Sample Error Budget - OAP

Doing this, we arrive at a (somewhat naïve) error budget:

| Element | Degree of Freedom | Allowed movement | Limiting Requirement |
|---|---|---|---|
| Dichroic | $\Delta\theta_\mathrm{x}$, $\Delta\theta_\mathrm{y}$ | 0.039 arcsecond | Position of image |
| OAP | $\Delta$x, $\Delta$y | 0.43-0.71$\mu$m | Position of image |
| | $\Delta$z | 70$\mu$m | Quality of image |
| | $\Delta\theta_\mathrm{x}$, $\Delta\theta_\mathrm{y}$ | 0.038 arcsecond | Position of image |
| | $\Delta\theta_\mathrm{z}$ | 0.17 arcsecond | Position of image |
| CCD | $\Delta$x, $\Delta$y | 0.43-0.71$\mu$m | Position of image |
| | $\Delta$z | 70$\mu$m | Quality of image |