Differential Image Motion Monitor, Which Is Transportable:

Tales of Twinkling Stars
and Long Nights in the Cold

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August 8, 2006

A dissertation
submitted for the degree of Doctor of Philosophy
in the University of Cambridge
Declaration

This dissertation is the result of work carried out in the Astrophysics Group of the Cavendish Laboratory, Cambridge, between October 2000 and September 2004. Except where explicit reference is made to the work of others, the work contained in this dissertation is my own, and is not the outcome of work done in collaboration. No part of this dissertation has been submitted for a degree, diploma or other qualification at this or any other university. The total length of this dissertation does not exceed sixty thousand words.

Bridget O’Donovan

August 8, 2006
Acknowledgements

First of all I would like to thank my supervisor Dr. John S. Young for his constant support and help over the past 5 years. I hope that he has not been put off supervising after me, his first student! I would also like to say a big thank you to Bodie Seneta for designing the readout modes for the Starlight Xpress camera and for supporting me during my campaign in New Mexico. I would also like to thank the other members of the COAST team for their input and companionship on those ‘long nights in the cold’, observing at the COAST telescope: Chris Haniff, David Buscher, Ali Bharmal, James Keen, Donald Wilson, Roger Boysen, John Baldwin, Pete Warner and Natalie Thureau. An especially big thank you goes out to the MRO team in New Mexico, especially Dan Klinglesmith and Craig Wallace-Keck.

I gratefully acknowledge financial support from the Isaac Newton Institute, the Astrophysics Group and the European Fund in the form of research studentships and scholarships. I would also like to acknowledge the financial support I received from Lucy Cavendish College.

I would also like to thank the various occupants of room 988 over the years and the RADA gang, especially Ed Campbell (computer guru and fellow TV addict), Ross Williamson (office entertainer, rowing coach and matchmaker) and Rachel Parker (for daring Ross to try an epilady and coaching LCCBC). A big hug for Gayoung Chon and Elsa Arcaute - I’ll always have fond memories of our weekly lunches, sharing our PhD woes. To all the members of the Lucy Cavendish College Boat Club and Hughes Hall Boat Club - we had our ups and downs on the river but still managed not to embarrass ourselves in the bumps! Thanks to fellow crew member, Fiona Lee, with whom I had so many fantastic nights out, especially in the Junction! Thanks to my ex-housemate Anu Deshpande for all the late night coffee chats and the encouragement to finish my write-up. And a special thank you to Deirdre Hickey and Elizabeth Forde, my oldest and best friends, who sympathised and cheered me up when the going got tough. I’ll never forget sleeping 3 to a bed girls!

Finally I would like to thank my long suffering boyfriend Jasper Holman, who has quietly and supportively put up with having his girlfriend tied to a computer every weekend for the past year, my brother Francis O’Donovan, who inspired me to do a PhD in astrophysics in the first place, and my parents, Tom and Vera O’Donovan, who taught me that I could achieve anything I put my mind to and without whose love and encouragement I would not be where I am today.
Differential Image Motion Monitor, Which Is Transportable:

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Characterising the fluctuations of refractive indices in the atmosphere by certain statistical parameters allows us to quantify their effect on ground-based astronomical instruments such as stellar interferometers. Differential Image Motion Monitors (DIMMs) have been developed to measure these parameters at various sites around the world.

The aim of this work was to design and build a portable DIMM, named Differential Image Motion Monitor, Which Is Transportable (DIMMWIT), capable of measuring the spatial and temporal seeing. DIMMWIT was developed and tested at the Cambridge Optical Aperture Synthesis Telescope (COAST) site at Lord’s Bridge, Cambridge before being transferred to the Magdalena Ridge Observatory Interferometer (MROI) site in New Mexico, USA for a short campaign of seeing measurements. The DIMMWIT is based on a Starlight Xpress CCD camera with custom readout modes, fed by a defocused amateur telescope (or COAST unit telescope) with an aperture mask.

In addition to recording the images, software was developed to analyse the differential image motion. Centroiding algorithms were tested to ensure the correct image positions were being used. Spatial statistical information could then be derived following the work of Sarazin and Roddier (1989). Two methods of deducing temporal information from differential image velocities and differential image motion spectra were investigated. The results from these spatial and temporal analyses were then compared with measurements made by the COAST interferometer to test the reliability of the DIMM system. The use of seeing measurements in correcting interferometric measurements for changes in seeing was also investigated.

The performance of the DIMMWIT system at the MROI site was promising and it is hoped that a longer campaign will allow the site to be characterised before observations begin at the optical interferometer.
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1 Twinkling Stars and Astronomy

“Twinkle, twinkle little star
How I wonder what you are
Up above the world so high
Like a diamond in the sky.”

*Traditional*

Man has been attempting to explain this fascinating effect of the atmosphere since the time of Aristotle [30]. Modern theories reason that light from a distant star is bent by different amounts due to refractive index variations in the Earth’s atmosphere and the resulting intensity variations can be detected by the human eye. Hence, the stars appear to ‘twinkle’. What is actually happening is that the wavefronts of incoming starlight are essentially ‘flat’ until these fluctuations introduce random phase delays in different parts of the wave, resulting in a distorted wave front, like a crumpled piece of paper [7]. So, regions of the wavefront are depleted of light to the profit of other regions and there are variations in the brightness of the star that are constantly changing - the star appears to twinkle [58]. The nature of these refractive index fluctuations is completely random so only statistics can be used to describe the behaviour of the atmosphere [48].

This dissertation is concerned with characterising these fluctuations by certain statistical parameters in order to estimate their effect on ground-based astronomical instruments such as interferometers [55] [14] [4] [68] and adaptive optics systems [6]. A seeing monitor (seeing is another name for these effects of atmospheric turbulence) was designed to measure these parameters so that a campaign of observations could be carried out at the COAST site in Lord’s Bridge, Cambridge and at the MROI site in New Mexico, USA.
Chapter 1: Twinkling Stars and Astronomy

Figure 1.1: The layers of the Earth's atmosphere.
1.1 Atmospheric Turbulence

Figure 1.1 shows how the Earth’s atmosphere is made up of several layers such as the Troposphere, Stratosphere, Mesosphere and the Thermosphere. The Troposphere is the layer from the surface of the Earth to 11-12km above. The Earth reflects the radiation that falls on it from the Sun and this warms the air near the ground. So, in the Troposphere, the temperature decreases with height. Rising air encounters lower atmospheric pressures allowing the air to expand and cool. Sinking air encounters greater air pressures and warms by compression. In this way, convection of warm and cold air, in addition to the Coriolis effect, establishes a circulation pattern and creates air flows in the atmosphere.

As convection usually means cloud cover, it is not entirely responsible for the air flows. In the calmer regions of the atmosphere the motion of the air cannot be seen because there is no humidity. Something causes an instability in the temperature profiles of a region of air and triggers mechanical turbulence. As the turbulence develops with time, the layer becomes well mixed and the vertical temperature gradient tends to zero. For this stable layer to exist, thin sublayers above and below are created that have steep temperature gradients. These steep gradients cause turbulent motion, so these thin turbulent layers contribute to the seeing due to the atmosphere.

1.1.1 Creation of Eddies in the Atmosphere

Both the presence of a circulation pattern and seeing laminae cause air flows in the atmosphere that become chaotic (or turbulent). By definition, a flow becomes turbulent when the dimensionless parameter, the Reynolds number, \( \text{Re} = \frac{V L}{\nu} \), where \( V \) is the fluid velocity, \( L \) is a characteristic length scale and \( \nu \) is the kinematic viscosity of the fluid, exceeds a critical value. Kinematic viscosity is defined as the dynamic viscosity divided by the density of the fluid and is measured instead of dynamic viscosity when the shear stress and shear rate is influenced by density, i.e. the more dense the fluid, the faster it will flow, as is the case with the atmosphere. For air, the atmospheric flows have \( \text{Re} \geq 10^6 \), due to a low kinematic viscosity, which corresponds to fully developed turbulence. The kinetic energy of large scale motions is transferred to smaller and smaller scale motions until the Reynolds number is small enough to stop the process. The kinetic energy is then dissipated into heat by viscous friction. A nice image of this energy cascade is shown in figure 1.2. The large scale motion transfers energy into large eddies (large parcels of moving air), which transfer energy.
to small scale eddies. The eddies become smaller and smaller until the energy is dissipated into heat at the high spatial frequencies where viscous dissipation is most severe [24]. The **outer scale** of turbulence, $L_0$, and the **inner scale** of turbulence, $l_0$, are the characteristic sizes of the large and small eddies respectively. $L_0$ is the order of the thickness of the turbulence layers so it ranges from 10 to several 100 metres. $l_0$ ranges between 1 to 10 millimetres [52]. Kolmogorov [52] deemed that viscous dissipation is not significant between $1/L_0$ and $1/l_0$, i.e. the rate of energy transfer is maintained. In this ‘inertial range’ the energy of the turbulence is related to the spatial frequency $\kappa$, which corresponds to an eddy size $L \propto 1/\kappa$ [52], by Kolmogorov’s Law that the turbulent energy is proportional to $\kappa^{-5/3}$. (This will be derived in the section 1.1.2)

Figure 1.2: Energy cascade - large scale motion becomes large eddies which become smaller and smaller until the energy is dissipated into heat.

### 1.1.2 Kolmogorov Turbulence Model

In order to estimate the contributions of different regions of the atmosphere to the seeing conditions, the variation of the refractive index fluctuations must be known as a function of height above the ground. Kolmogorov’s model of turbulence [36] is used to calculate the statistical behaviour of these refractive index fluctuations. Two important parameters in determining the spectrum of refractive index are the rate of energy generation per unit mass $\varepsilon$ (Js$^{-1}$kg$^{-1}$ or m$^2$s$^{-3}$) and the kinematic viscosity $\nu$ (m$^2$s$^{-1}$). In Kolomogorov’s model, turbulence is assumed to be isotropic and homogeneous: isotropic because the variance of each of the three components of velocity are equal.
1.1 Atmospheric Turbulence

Figure 1.3: Model of many thin turbulent atmospheric layers moving independently of each other [39].

and homogeneous because its structure is independent of position. The structure function of the turbulence velocity field can then be written as

\[ D_v(R_1, R_2) \equiv \langle |v(R_1) - v(R_2)|^2 \rangle. \] (1.1)

From dimensional analysis, the structure function must be independent of viscosity and this is only possible if the structure function is proportional to length to a two-thirds power,

\[ D_v(R_1, R_2) = C_v^2 \cdot |R_1 - R_2|^\frac{2}{3}, \] (1.2)

where \( C_v^2 \) is the velocity structure constant. However, seeing is not caused by these velocity fluctuations in the atmosphere. In order to apply this statistical analysis to the random temperature mixing in the atmosphere, Tatarski [69] introduced the concept of a ‘conserved passive additive’. This meant that similar expressions to Kolmogorov’s Law (equation 1.2) would hold for any passive additive variable such as temperature or refractive index [19]. Hence, the structure function would also be proportional to the distance between two points to the two-third power, i.e.

\[ D_T(R_1, R_2) \equiv \langle |T(R_1) - T(R_2)|^2 \rangle = C_T^2 \cdot |R_1 - R_2|^\frac{2}{3}, \] (1.3)

where \( C_T^2 \) is the temperature structure constant.\(^1\) From the ideal gas law, and \( N \equiv (n - 1) \propto \rho \) (\( n \) is the refractive index and \( \rho \) is the density), the structure function of the refractive index can be derived [50]:

\[ D_n(R_1, R_2) = C_N^2 \cdot |R_1 - R_2|^\frac{2}{3}, \] (1.4)
Chapter 1: Twinkling Stars and Astronomy

where $C_N^2$ is the refractive-index structure constant which is used to measure the strength of the atmospheric turbulence.

Atmospheric turbulence is a stationary process, so the structure function depends on $R = |R_1 - R_2|$ and can be written as

$$D_N(R) = \langle |N(0) - N(R)|^2 \rangle. \quad (1.5)$$

Remembering that the covariance function $B_N$ is defined as [47]

$$B_N(R) = \langle (N(0) - \langle N \rangle)(N(R) - \langle N \rangle) \rangle, \quad (1.6)$$

$$B_N(0) = \langle N^2 \rangle - \langle N \rangle^2, \quad (1.7)$$

$$\Rightarrow D_N(R) = 2(B_N(0) - B_N(R)). \quad (1.8)$$

Using equation 1.8 and the Wiener-Khinchin Theorem (that the autocorrelation is equal to the Fourier Transform of the power spectrum),

$$D_N(R) = 2 \int_{-\infty}^{\infty} d\kappa (1 - \exp(-2\pi i\kappa R)) \Phi(\kappa), \quad (1.9)$$

$$\Rightarrow C_N^2 \cdot R^2 = 2 \int_{-\infty}^{\infty} d\kappa (1 - \exp(-2\pi i\kappa R)) \Phi(\kappa), \quad (1.10)$$

where $\Phi(\kappa)$ is the power spectral density of the refractive index. With further analysis [50] it can be shown that the power spectrum of Kolmogorov turbulence follows a $\kappa^{-5/3}$ law in the inertial range:

$$\Phi(\kappa) = 0.0365 C_N^2 \kappa^{-5/3}, \quad (1.11)$$

1.1.3 Behaviour of Turbulent Wavefronts

In order to estimate the effect of these refractive index fluctuations, the statistical behaviour of the wavefronts that have been corrupted by the fluctuations must be understood. A wavefront $\psi(x) = \exp i\phi(x)$ passes through a turbulence layer of thickness $\delta h$ at height $h$. The phase shift produced by refractive index fluctuations is

$$\phi(x) = k \int_{h}^{h+\delta h} dz n(x,z), \quad (1.12)$$

\footnote{There is no significant correlation between the magnitude of $C_N^2$ and $C_T^2$. The magnitude of $C_T^2$ is proportional to the local vertical temperature gradient, and thus $C_T^2$ can be almost zero despite the fact that the local wind velocity turbulence is quite strong and $C_T^2$ can be large in conditions where the wind velocity turbulence is weak [24].}
where \( k = 2\pi/\lambda \). The thickness of the layer is large compared to the size of the turbulent cells so Gaussian statistics apply. The coherence function \( B_h(r) \) of the wavefront after passing through the layer is expressed in terms of the phase structure function:

\[
B_h(r) = \langle \psi(x)\psi^*(x + r) \rangle,
\]

\[
\Rightarrow B_h(r) = \exp(-\frac{1}{2}D_\phi(r))
\]  

(1.13)

Next the covariance \( B_\phi(r) \) must be computed in order to calculate \( D_\phi(r) \)

\[
B_\phi(r) = \langle \phi(x)\phi(x + r) \rangle,
\]

\[
\Rightarrow B_\phi(r) = k^2\delta h \int_{-\infty}^{\infty} dz B_N(r, z),
\]  

(1.14)

by changing variables and introducing the covariance \( B_N(r, z) \) of the refractive index variations. It has been assumed that \( \delta h \) is much larger than the correlation scale of the fluctuations so the integral can be extended over \(-\infty\) to \(\infty\).

In this range, and assuming a near field approximation, the phase structure function (the mean squared differential phase difference between two points separated by a distance \( r \) in the aperture plane of a telescope) is related to the refractive index structure function. This relationship can be derived by applying equation 1.8 to the structure functions and using equation 1.14:

\[
D_\phi(r) = 2(B_\phi(0) - B_\phi(r)),
\]

\[
= 2k^2\delta h \int_{-\infty}^{\infty} dz [B_N(0, z) - B_N(r, z)],
\]

\[
= k^2\delta h \int_{-\infty}^{\infty} dz [D_N(r, z) - D_N(0, z)].
\]  

(1.15)

Inserting equation 1.4, which describes the structure function of the refractive index fluctuations, gives

\[
D_\phi(r) = 2.914k^2\delta hC_N^{2}\frac{r^5}{3},
\]  

(1.16)

which is the structure function of the phase fluctuations due to Kolmogorov turbulence in a layer of thickness \( \delta h \). Knowing the structure function, equation 1.16 can be inserted into equation 1.13 which is then integrated over the entire atmosphere to take the zenith angle into account giving

\[
B(r) = \exp \left[ -\frac{1}{2} \left( 2.914k^2C_N^2\delta h r^\frac{5}{3} \right) \right].
\]  

(1.17)

\(^2\)Note that the variables are defined as one-dimensional. If three-dimensional variables are used instead, then the result is a power spectrum \( \Phi(\kappa) \propto \kappa^{-\frac{11}{3}} \), where \( \kappa \) is a vector.
Defining the **Fried parameter** $r_0$ as

$$r_0 \equiv \left[ 0.423 k^2 (\sec z) \int \delta h C_N^2(h) \right]^{-\frac{2}{3}}. \quad (1.18)$$

simplifies the expressions for the phase-coherence function and the phase structure function:

$$B(r) = \exp \left[ -3.44 \left( \frac{r}{r_0} \right)^{\frac{5}{3}} \right], \quad D(r) = 6.88 \left( \frac{r}{r_0} \right)^{\frac{5}{3}}. \quad (1.19)$$

### 1.1.4 Parameters of Atmospheric Turbulence

The **Fried parameter** is a crucial quantity from which the phase structure function can be determined. It is one of two important parameters that describe the quality of a wave that has propagated through atmospheric turbulence. In order to interpret $r_0$ the resolving power, $R$, of an optical system is first investigated. $R$ is defined as the integral over the optical transfer function. It can be shown [50] that for long exposures the optical transfer function is the product of the telescope transfer function and the atmospheric transfer function, which is equal to the phase coherence function $B$. Hence, for the atmosphere/telescope system the resolving power is

$$R \equiv \int df S(f) = \int df B(f) T(f). \quad (1.20)$$

As shown in figure 1.4, in the absence of turbulence, $B(f) \equiv 1$ and the resolving power of the telescope is limited by diffraction only. If the diameter of the telescope is $D$ and the aperture is circular then

$$R_{tel} = \frac{\pi}{4} \left( \frac{D}{\lambda} \right)^2. \quad (1.21)$$

![Diffraction limited OTF and Seeing limited OTF](image-url)
1.1 Atmospheric Turbulence

For strong turbulence and large telescope diameters, several times the size of typical \( r_0 \) values, \( T=1 \) in the region where \( B \) is non-zero and the resolving power is limited by seeing. Hence,

\[
R_{atm} = \frac{\pi}{4} \left( \frac{r_0 \lambda}{\lambda} \right)^2.
\]  
(1.22)

Comparing equation 1.21 and equation 1.22 shows that the resolution of seeing limited images affected by atmospheric turbulence characterised by \( r_0 \) is the same as the resolution of diffraction limited images taken with a telescope of diameter \( r_0 \). So, the Fried parameter \( r_0 \) can be defined as the critical pupil diameter under which the resolving power of a telescope is diffraction limited [39]. Observations made with telescopes much larger than \( r_0 \) are seeing limited. The second interpretation of \( r_0 \) is to define it as the region of the wave front over which phase perturbations are small. This means that the mean-square phase variation over an aperture of diameter \( r_0 \) is about 1 rad\(^2\). This definition leads to a simplified picture of an atmosphere with \( r_0 \)-sized patches of random phase.

To understand the temporal changes of the turbulence pattern, use is made of Taylor’s hypothesis [50]. According to the Taylor hypothesis of frozen turbulence, the variations of the turbulence caused by a single layer can be modelled by a frozen pattern that is blown across the aperture of the telescope by the wind in that layer. If there are multiple layers of turbulence, the analysis is more complicated. However, the temporal behaviour of the turbulence can still be characterised by the coherence time, or correlation time, \( \tau_0 \) which is defined as the delay after which the mean square phase excursion is one radian. Applying Taylor’s hypothesis [58] yields

\[
D_\phi(\tau_0) = 6.88 \left( \frac{\tau_0 v}{r_0} \right)^{\frac{3}{5}} = 1,
\]
i.e. \( \tau_0 \equiv 0.314 \frac{r_0}{v} \)  
(1.23)

where \( v \) is the wind speed in the dominant turbulence layer. Hence, the temporal phase structure function becomes \( D_\phi(\tau_0) = (\tau/\tau_0)^{\frac{3}{5}} \). Typical values for \( r_0 \) and \( v \) are 10-20 cm and 10-20 m/s, so that the coherence time is around 3 ms at good astronomical sites.
Chapter 1: Twinkling Stars and Astronomy

1.1.5 Effect of Turbulence on Astronomy

Each region of the crumpled wavefront has a random phase slope. The starlight appears to come from a direction perpendicular to that slope. So, every part of a large telescope aperture sees the star in a different constantly changing place. For a long exposure image all the star positions blur together into one blob, the size of which is called the seeing disk. This effect means that large telescopes cannot see in any more detail than a smaller telescope, although they can detect fainter sources. One way of achieving diffraction limited, rather than seeing limited, performance is to operate at longer wavelengths where $r_0$ is larger because it scales as $\lambda^{5/6}$. However, diffraction limited resolution falls with increasing $\lambda$ so a compromise is required.

Fried’s parameter is important for interferometers because wavefronts from individual telescopes must be coherent (i.e. the phase variations must be around 1 rad$^2$ or smaller). So, the maximum useful aperture size of an interferometer depends on the value of the Fried parameter at the wavelength of their detectors. The constant variation of the atmospheric piston over time affects interferometers even more as the fringe pattern is smeared out after few 10ms (depending on the observation wavelength). Interference fringes will only be observed if the optical distance from the star to the detector is the same for both arms of the interferometer. For example, if the instantaneous optical path difference offset offset $\delta$ is 2.25 $\mu$m, assuming a wavelength of 1.0 $\mu$m and that $\Delta \lambda/\lambda = 0.1$, the resulting visibility amplitude is reduced by 10% (i.e. sinc function equals 0.9 in equation 6.8).

1.2 Outline of Thesis

The aim of this work was to design and build a portable image motion monitor capable of measuring the spatial and temporal seeing at various sites of astronomical interest. The monitor would then be tested at COAST before being moved to another site available in New Mexico. Of secondary concern was the characterisation of the seeing at both of these sites.

In the next chapter I will present the technique used to measure the two parameters, $r_0$ and $\tau_0$, that characterise the turbulent layers in the atmosphere. Previous versions of the equipment are reviewed, before outlining the prototype that has been developed.
for use at the COAST site in Lord’s Bridge, Cambridge. Also, the common features of
the data analysis are introduced so that they can be referred to by later chapters.

Chapters 3 and 4 outline the progress that has been made in developing equipment
that could measure both $r_0$ and $\tau_0$. The ability to derive these two parameters from
the data not only depends on the equipment, but on the way the data is analysed.
The various sources of error are outlined and results of measurements taken at Lord’s
Bridge are presented.

In chapter 5, the portable aspect of the equipment is emphasised. Results of a cam-
paign carried out at the Magdalena Ridge Observatory in New Mexico are presented
to demonstrate the ability of the equipment to measure the seeing at another astro-
nomical site. The possibility of calibrating interferometric data with measurements of
Fried’s parameter is then investigated in chapter 6.
Chapter 1: Twinkling Stars and Astronomy
2 Differential Image Motion Monitor, Which Is Transportable

Differential Image Motion Monitors, DIMMs, were developed to evaluate turbulent atmospheric conditions quantitatively. The principle of differential image motion measurements is to allow starlight into an imaging system through two small apertures. The images from the two apertures are separated and their differential position over time is recorded. This method has two advantages. The system is insensitive to tracking errors [62], as it is the relative motion of the images that is measured, and the instruments are compact because the distance between the apertures need only be a few times their diameter for the motion of the two images due to the atmosphere to be uncorrelated. Thus, DIMMs give an unbiased estimate of image degradation due to the atmosphere alone.

The aim of the DIMMWIT design was to avail ourselves of the high standard of amateur astronomical equipment. This meant that not only would the DIMM system be inexpensive but anyone could set up their own seeing monitor. With such a straightforward, cheap design the monitor could be easily transported to a site of interest. The same monitor could be used to characterise the seeing at several sites for comparison. Alternatively, the conditions at a new site could be tested. Extensive knowledge of the behaviour of the atmosphere at the site of a proposed interferometer, for example, would provide information about key design parameters such as operating wavelength, aperture size and coherent integration time.

2.1 Review of Differential Image Motion Monitors

Knowledge of seeing conditions at any site is important for ground-based optical astronomy especially in the fields of interferometry and adaptive optics. New sites with good seeing need to be found for the next generation of interferometers and image
quality at present sites needs to be understood to improve the performance of the instruments. The DIMM has become the conventional seeing monitor used to carry out seeing campaigns. Hence, several differential image motion monitors exist at sites around the world. In order to explain the facets of the DIMMWIT design, it is useful to outline the models that inspired its creation. These models show the progress that has been made in developing DIMMs since 1986, when the first monitor was introduced. DIMMs have become more compact and more straightforward, with reliable measurements of spatial seeing proved by comparison with large telescope measurements. Their ability to observe temporal changes is generally limited by the performance of the detectors that record the image positions. These detectors have previously been unable to record the positions at a rate fast enough to measure wavefront changes. The reader is referred to section 2.2 for the explanation of any technical terms used to describe the detectors if they are not included in this section.

2.1.1 ESO Differential Image Motion Monitors

The most important and influential DIMM was designed by ESO in 1986 [62]. This DIMM consists of a 35cm diameter telescope (which is about 13” for comparison with DIMMWIT) placed on a 5m high tower so that the instrument was at the same height as a nearby dome-enclosed telescope. Thus, seeing measurements made by both the DIMM and the telescope could be compared accurately as they are observing the same amount of atmosphere, in particular the ground layer. The DIMM detector and camera are installed at the focus of the telescope by re-imaging the entrance pupil behind the Cassegrain focus with a collimator as shown in figure 2.1. In order to form two star images, a roof prism in the pupil plane splits the starlight into two beams. The beams then pass through a pupil mask, glued onto the prism’s plane edge, that has two holes of diameter D (4cm) and separation d (20cm).
The chosen camera is a microchannel plate image inverter intensifier tube on a frame transfer CCD (see section 2.2.1) because it can detect targets as faint as magnitude $V=3-4$ with a large signal to noise ratio, allowing year-round operation of the DIMM. To measure the seeing conditions, 10ms exposures are taken with the electromagnetic shutter and a subframe around each star image is read out, digitised and transferred to the computer for later analysis. Any tracking errors between exposures are corrected by an autoguiding system that sends the centroids of each image back to the telescope.
The centroid positions of the two star images in each frame are calculated having first discarded frames with saturated pixels or frames with low overall intensity, subtracted a dark frame from each image frame and applied a threshold to the pixels. **Thresholding** is a technique used to increase the accuracy of the centroiding algorithm used to calculate the image positions. The threshold is defined as a fraction of the maximum pixel value for each image and all pixels in the region around the image used to calculate the centroid that are under that value are set to 0.

For one measurement of the seeing, the variance of 200 image centroids in the directions parallel and perpendicular to the mean direction of separation of the images is calculated. The variance is converted into arc-seconds-squared knowing that one detector pixel is 0.88 arc-seconds from observations of the separation of a binary star. The corresponding seeing value can then be calculated from expressions derived by Sarazin and Roddier that relate the variance of the motion to Fried’s parameter, $r_0$. The detector acquires images at a rate of 5 per second, which includes the time taken to expose the CCD and then download the image. So, including time for analysis, the ESO DIMM can produce a spatial seeing measurement every 2 minutes.

Traditionally, the image quality at a site has been characterised by the full width at half-maximum of a star at the focus of a large telescope. Fried’s $r_0$ is related to the full width at half maximum of long exposure images by [75]

$$\text{FWHM} = 0.98\lambda/r_0.$$  \hspace{1cm} (2.1)

The performance of the ESO DIMM could thus be tested by comparing $r_0$ measurements made by the DIMM with the FWHM of long exposure images observed by the large telescope available. The seeing monitor was moved inside the nearby dome and attached beside the primary mirror of the large 2.2m telescope such that both instruments could observe the same stars. 50s exposures were acquired on the large telescope for comparison with seeing measurements made simultaneously with the DIMM. There was an obvious agreement between the two instruments which proved the reliability of the measurements made with a DIMM system.

The setup had to be changed in 1993 (see figure 2.2) because the previous detector was incapable of measuring image positions at a high enough rate to be able to measure temporal as well as spatial seeing. A tilt mirror was added after the roof prism to move the images across the detector after each exposure in a direction perpendicular to the aperture separation. So, several pairs of images appear in each CCD frame. The exposure time of each pair is set by a chopping wheel blocking the light while the mirror moves the stars from one position to another.
2.1 Review of Differential Image Motion Monitors

Figure 2.2: Optical adaptor for the DIMM: a tilt mirror drives the two star images across the CCD detector [39].

For this setup, the exposure time of each pair is about 0.7ms and the sampling rate is about 500Hz. The additional change in design was to increase the aperture diameters to allow more light into the system during such short exposure times. With this new setup, it was possible to acquire images at a rate fast enough to measure temporal seeing. Any comparison of the measurements made with this new ‘coherence monitor’ and another instrument that measures seeing was not presented.

2.1.2 DA/IAC DIMM

The DA/IAC DIMM is a simpler version of the ESO DIMM with some parts available ‘over the counter’ [75]. A commercially available CCD is slotted into the eye piece of an 8” telescope on a stiff equatorial mount capable of automatic tracking. A mask, with a specially made wedge covering one of the apertures, is then attached to the telescope to form the two star images. Note that the mechanical pieces to attach the camera and eye piece to the telescope were custom made. The addition of an eyepiece increased the focal length of the telescope to match the pixel scale of the ESO DIMM.
Chapter 2: Differential Image Motion Monitor, Which Is Transportable

Figure 2.3: General scheme of the DA/IAC DIMM [75].

The detector is an intensified CCD camera with an electronic shutter as opposed to a less reliable mechanical shutter (this intensified CCD camera is similar to the detector used in the ESO DIMM). Shutter speeds of 1 to 10ms allow measurements of spatial seeing during high winds when the turbulence is changing on very short timescales. In order to avoid remnant images (light from the previous frame) due to the decay time of the phosphor screen in the intensifier, a frame is recorded every 40ms. The cycle time, which includes exposure time, frame rate and download time to the computer, is then further reduced by reading out a subframe around each image.

Only frames where the star images are a minimum distance from the edge of the subframe window are accepted. The centroids of the last five windows are stored and used to relocate the centroid window, keeping the images in the centre of the window and hence reducing the number of discarded frames. 200 acceptable frames are used in each calculation of the spatial seeing.

In order to test the reliability of the DIMM, a seeing campaign was carried out at the site of the Very Large Telescope where the DA/IAC DIMM and the ESO DIMM were run simultaneously. The two DIMMs were in good agreement once the same centroiding process was used for both sets of data. Vernin et al [75] discovered that using too high a threshold led to overestimating the seeing conditions, especially in more turbulent conditions. The reliability of the instrument was further proven because the ratio of longitudinal and transverse seeing values was always around unity (see chapter 3).
2.1.3 Hartmann Differential Image Motion Monitor

Bally et al [5] wanted to modify the designs suggested by Sarazin and Roddier [62] and Vernin and Munoz-Tunon [75]. The components of the H-DIMM simply attach to an existing telescope without needing custom made parts. The detector is inserted into the eyepiece of the telescope and the images produced by the apertures in the mask are separated by defocussing the telescope rather than placing a prism over one of the holes in the mask. Bally et al found that these defocused images were in nearly perfect focus if the imaging CCD was displaced from best focus by an amount less than the depth of field $\Delta Z$, where $2 \Delta Z \approx 1.22\lambda(f/D)^2$, and $D$ is the diameter of the apertures. Also, the images were well-separated in the image plane if the detector was at a distance from the focal plane greater than $\Delta z = 1.22\lambda(f/d)^2$, where $d$ is the separation of the apertures. As long as the detector is placed at a distance, $Z$, from the focal plane of the imaging telescope, where $\Delta z < Z < \Delta Z$, then the images will be well-separated and in nearly perfect focus. The optimum ratio of image separation to diameter is achieved by placing the detector at $Z \approx \Delta Z$. Hence, using a defocus rather than a prism to separate the images enables the use of a Hartmann mask with multiple apertures instead of a two aperture mask in order to measure differential image motion. These holes produce many simultaneous baseline pairs on which to measure the differential image motion. Choosing bright stars as targets mean a simple direct imaging CCD can be used instead of an image intensified one. The H-DIMM CCD is a commercially available CCD capable of exposure times as short as 10ms and frame rates of 1 every 10s. The cycle time is too long to allow temporal as well as spatial seeing measurements. The variation in the variance on redundant baselines and comparison between the results and the predictions of spatial seeing by Sarazin and Roddier [62] then lead to an estimate of the error due to various factors.

In 1995 at the South Pole, microthermal balloon flights were carried out at the same time as H-DIMM measurements. A 48 aperture mask was attached to a 60cm telescope mounted on a 12m tower. The tower was needed to minimise build up of surface snow on equipment, to isolate the system from nearby buildings and to eliminate as much surface seeing as possible. The comparisons between 5 balloon flights and DIMM measurements are presented in Marks et al [40]. The correlation between the sets of data is good for three flights but not for two that were affected by turbulence created by heaters used to keep optical surfaces ice-free. These heating problems were later resolved during a second campaign [41] that used another H-DIMM consisting of a 12 aperture mask on a 28cm telescope mounted on the same tower as the first and it was found that there was very good agreement between the results of the two systems.
2.1.4 Grating Scale Monitor or Generalized Seeing Monitor

The creators of the ESO DIMM produced the Grating Scale Monitor [43, 1, 44, 85, 42] in 1994 [43], which is capable of measuring spatial and temporal seeing. Two 10cm telescopes on a common equatorial mount work as a DIMM and an additional two telescopes on separate mounts are placed nearby to form a L-shaped configuration that is used to make further seeing measurements to determine other seeing parameters.

The L-shaped configuration is used to make additional measurements but it is only the DIMM operation of the GSM that is of interest here. In this mode, the GSM can either use two telescopes as two apertures or one telescope with a two sub-aperture pupil mask and any tracking difficulties are avoided by using the pole star as the only target.
2.1 Review of Differential Image Motion Monitors

Figure 2.6: The optical scheme of the GSM. Part (a) shows how input beams coming from the stellar source are directed into the telescope T by two mirrors M₁ and M₂. Part (b) shows the acquisition and detection module.

In order to measure temporal seeing [2, 18, 72, 86], the position of the differential images would have to be recorded every 5ms. However, they did not have a detector capable of such rates. The compromise was to measure the position in one direction only by rapidly scanning the stellar image over a Ronchi grating with a tilting plane mirror. Spatial and temporal seeing can be estimated from these one-dimensional image positions. The former is calculated from the variance of the positions in the same manner as other DIMMs. The latter is derived from power-spectra of the data. These power-spectra are regularly used with interferometric data to derive the wind speeds of the turbulent layers. The break frequency in the power spectrum is related to the wind velocity which in turn is related to Fried’s parameter and the temporal scale of the turbulent atmosphere (see chapter 4).

2.1.5 Outline of ideal DIMM system

This small sample of DIMMs demonstrates the advances that have been made in measuring astronomical seeing. The constant trend has been to find a simpler and cheaper way of designing a seeing monitor. This can be accomplished by using an amateur telescope and inserting a detector at the eyepiece, using a simple defocus rather than attaching prisms to the aperture masks and choosing a detector carefully. The DIMMs are designed to be insensitive to tracking problems. All of the present DIMMs have been reported to compare favourably with other seeing instruments. So, a DIMM with the same basic design should be able to measure Fried’s parameter reliably.
The main difficulty seems to have been finding a detector with cycle times short enough to measure temporal changes. The GSM found a way round this by settling for one-dimensional information in order to get very short cycle times. Is it possible to operate a CCD at a high rate and get the two-dimensional information needed to find the wind velocity and not just speed of the turbulent layers? The GSM determined wind speeds calculated from the power spectra of single image positions. Can any information on wind speeds be gleaned from differential motion? If the subframe read out was small enough, could a camera operate at short cycle times? And with such a small subframe, what kind of tracking capabilities would be needed? These questions would have to be answered in order to develop a cheap, transportable DIMM capable of both spatial and temporal seeing measurements.

2.2 Prototype DIMMWIT

As outlined in the previous section, the basic components of the DIMMWIT have to be an aperture mask, telescope and CCD. The ability to measure temporal seeing relies on the choice of detector. Seeing timescales can be as short as 2ms and the resultant image motion is too fast for existing detectors to measure. The challenge for the DIMMWIT system was to find a camera capable of cycle times as short as 3 or 4ms at least, which it was hoped would be short enough to measure the seeing at the site in Cambridge. We wanted to adapt an ‘off-the-shelf’ inexpensive charge coupled device to be able to perform at these high acquisition rates. Initially, only an SBIG camera was available for DIMMWIT measurements. However, the software that ran the camera could not be adjusted in order to achieve cycle times short enough for our purposes. Hence, we moved onto a selection of cameras from Starlight Xpress with freely distributed Linux software. The development of this software led to two new unique ways of operating the CCD at cycle times of 2 and 3 ms. To keep star images accurately within a selected region of the readout area of the CCD in order to achieve these cycle times, a separate autoguiding system has been added to the DIMMWIT. To explain how these new readout modes work, the following section is an introduction to CCDs and the terminology associated with them.
2.2 Prototype DIMMWIT

2.2.1 Charge Coupled Devices

A CCD is a rectangular array of imaging elements called pixels that are made from a crystal of semiconductor silicon. When an incoming photon of light falls on a pixel, the energy releases an electron within the semiconductor. The electrons are stored in the array of pixels until they can be moved. The array is made up of columns of pixels called vertical registers. The row of pixels at the bottom of the array is called the horizontal register. All the charge in these pixels is read out or transferred along these registers by raising or lowering the voltages on the pixel electrodes, which is called clocking. To read out an image:

1. The entire image is shifted vertically by one pixel so that the second last row of charge now lies in the horizontal register.
2. The charge in this row is then shifted horizontally one pixel at a time to an output amplifier where it is converted to an output voltage and digitised for transfer to a computer.
3. Another vertical shift of charge fills the output register.
4. Another horizontal shift of charge.
5. Etc...

This process continues until the entire image has been read out.

In a frame transfer CCD, the detector for the ESO DIMM, a duplicate imaging area lies next to the first and is covered by an opaque screen. The charge in the first area is shifted rapidly in a vertical direction down into the covered region, where it is read out in the usual manner at a slower rate.

Intensifier tubes (such as those used in some DIMM intensified cameras but not in the DIMMWIT camera) are used to convert incoming photons into electrons that are then accelerated to form high energy electrons that are converted back to light. In this way, the incoming flux is amplified several times. This amplification factor can be further increased by using microchannel plates that are very thin plates of conductive glass containing many small holes. More electrons are released in these holes which increases the amount of photons finally released.
Chapter 2: Differential Image Motion Monitor, Which Is Transportable

The Starlight Xpress camera is an interline transfer CCD consisting of vertical strips which are alternately opaque and light sensitive. The opaque strips conceal charge transfer registers. The charges are moved sideways at high speed from a light sensitive pixel to an opaque one. The charge is then transferred down these shielded registers to the horizontal register of the CCD. Interline cameras use microlenses to compensate for the fact that every second column in the array is opaque and this leaves small gaps between the pixels. The Starlight Xpress camera also contains a microcontroller, which is in charge of all the operations of a CCD: how long the exposures are, clearing the CCD of charge before an exposure and transferring the image into the storage area after each exposure. It also regulates the temperature of the CCD.

Other important terms to describe the performance of a CCD are:

- **Exposure Time**: the length of the time the CCD is exposed to light.
- **Binning**: the charge in multiple pixels is combined to create the effect of a single larger pixel. By having less pixels to read out in an image, the digitization and download times are reduced.
- **Dark Current**: electrons are also thermally generated in the CCD which means there are electrons present in each pixel even in the absence of light. CCDs are cooled from room temperature to reduce this affect.
- **Read Out Noise**: errors introduced by the CCD’s amplifier and the electronics that detect the charge.
- **Dark Frames**: the source of light is blocked and the CCD is given time to cool to normal operating temperature. A number of frames are recorded with short exposure times to ensure no stray light falls on the camera. The result is an image of the dark current and read out noise.
- **Cycle time**: refers to the time between successive images. To calculate the cycle time, you add the exposure time and the time taken to download the image to the computer. This assumes that another exposure is not started until the previous image has been transferred.
- **Pixel scale**: refers to the size of the CCD pixels in arc-seconds on the sky. This depends on the focal length of the optical system to which the CCD is attached.
2.2 Prototype DIMMWIT

2.2.2 Read Out Modes for Seeing Measurements

There are three read out modes of the Starlight Xpress cameras (developed by Seneta [65]) and these are used to measure the seeing conditions. The conventional subframe readout mode will hence be referred to as slow2d. The two new fast readout modes that have been specially developed for temporal seeing measurements are referred to as fast1d and fast2d. This nomenclature combines the speed (i.e. the cycle times which are either fast or slow) and image dimensions (2d refers to two-dimensional images and 1d to one-dimensional images) of these modes.

Figure 2.7: Control window for Starlight Xpress camera software

Figure 2.5 shows the control window of the ‘finder’ software that controls the CCD. The procedure for recording images in the conventional slow2d mode is to select a subframe of the CCD to be read out and choose the binning and the exposure time. The name of the file and number of frames recorded in each file are then saved in a parameter file and the image acquisition can begin. To record each image, the CCD is exposed and the subframe is then moved to the readout register and read out for transfer to the observer’s computer. Multiple frames are stored along the third axis of a three-dimensional Flexible Image Transfer System or FITS file. A FITS file usually contains a two-dimensional image with a ‘header’ attached. The header is a string array containing important information the user wishes to record, like the binning and exposure time of the subframes. By storing the 2D images all in one file with only one header attached, rather than in separate files with their own header, it is possible to reduce the space needed to store data files.
Chapter 2: Differential Image Motion Monitor, Which Is Transportable

With exposure times of 1 or 3ms, slow2d mode can be used to make reliable estimates of Fried’s parameter. However, cycle times of 4ms or less, similar to the timescales of atmospheric fluctuations, are needed to measure temporal seeing. It is not possible to read out a CCD array in the conventional way at these rates due to the limits of the Starlight Xpress camera’s USB microcontroller and the hardware that digitises the data before it is transmitted via a USB cable to a computer. The digital (A/D) converter is 16-bit which means slower digitisation and readout. However, it becomes possible to read out a small subframe of the CCD at high speed if [65]:

- The charge on the CCD is not cleared before an exposure.
- The subframe is at the CCD corner closest to the readout register.
- No clocking is performed after the subframe has been read out.
- The readout is triggered by a hardware interrupt (the USB 1ms frame signal) making the timing independent of the operating software.

The trick is that not only is the subframe closest to the corner read out but the clocking signals move charge elsewhere in the array towards the readout register. So, an image generated anywhere on the CCD will eventually be read out after a delay depending on its position. The two star images from a DIMM can be read out at high rates no matter where they fall on the CCD. Unfortunately, this technique relies on three things.

1. there are no other images on the CCD other than the two DIMM images.
2. the telescope and autoguiding system can keep the two images inside the small region on the CCD.
3. there are few processes running on the computer that captures the CCD image, so that nothing slows down the transfer of the data stream through the USB interface.

Hence, fast read out rates of about 500Hz require: checking the CCD images for ‘ghosts’ or unexpected images anywhere in CCD array, good telescope tracking and no use of the laptop at all during file acquisition - not even the mouse! This restricts the possibility of analysing the data files while new data is being acquired. Hence a three computer network is connected via a switched Ethernet hub. A single-board controls the camera, while a second ‘observer’ computer displays the control software locally or remotely. The third computer then accesses the observer’s computer and copies over
files for analysis without affecting the network activity between the observer’s computer and the single board connected to the camera because the single board doesn’t see the traffic between the other two computers in the network.

Fast1d was the first fast mode developed. A subframe of 64x64 pixels is binned vertically into a single line of 64 pixels and then read out. These lines are then stored as a 2d image in one FITS file as shown in figure 2.8. Each line shows only the horizontal position of the images so all vertical information is lost. The exposure and cycle time of this mode is 2ms. The second fast mode is called fast2d. A 64x32 region is binned into a 16x8 region and then read out. This mode has a cycle time of 3ms with lower resolution than the fast1d mode because the pixels are larger. The 2D images are then saved in a three-dimensional FITS file in the same manner as the slow2d mode (see figure 2.9). Each frame can be exposed for the entire cycle time without the need for a mechanical shutter because the CCD is capable of reading out a frame while exposing a new one.

Figure 2.8: Example of a fast1d file - each frame is binned into a single line of charge every 2ms and successive frames are stacked left to right.

Figure 2.9: Example of a single frame from a fast2d FITS file.
2.2.3 Control System, Telescopes, Autoguiding, Masks and Targets

There are two versions of the DIMMWIT. The first utilises a COAST unit telescope instead of an amateur telescope. Starlight Xpress cameras had been installed on all of the COAST unit telescopes for use as acquisition cameras, so it became possible to use one of these telescopes as part of the DIMMWIT system. The mask is a simple lens cap that attaches to the acquisition camera and the COAST autoguiding system can be used to track the targets. The second version of the DIMMWIT utilises a 12 or 14 inch amateur telescope with a separate autoguiding system and a mask attached to the front of the telescope. From the description of the fast modes in section 2.2.1, we can see that the size of the region in which the star images must stay is 64x64 (fast1d) or 64x32 (fast2d) pixels. This means that the telescope tracking must keep one star image in a region smaller than 32x32 CCD pixels, which is 62.4 arcseconds on the sky. Hence, an autoguiding system is needed as part of a closed loop tracking system to lock the images into place at the centre of these subframes on the CCD.

Controlling either version of the DIMMWIT system requires four separate computers - one for the telescope, one for the autoguiding system and two for the camera used to record the images. The reason for connecting the camera to a two computer network is to be able to operate it remotely and to overcome timing delays in the new CCD modes. It has been discovered that network activity or display updates on the observer computer causes delays in the readout rate of the CCD. By connecting the camera to a single board computer and running the ‘Finder’ software on this (i.e. the data acquisition software), a single file can be recorded without any interruptions and then transferred to the observer’s computer via Ethernet. Finder, software that runs on Linux operating systems, is displayed on the observer’s computer by an X Windows display protocol. The observer’s computer also acts as a file server as both Finder and the DIMMWIT data are stored on it. The single board computer has a limited...
amount of memory and this restricts the size of a file that can be recorded before being transferred to the observer’s computer. The longest a file can be recorded for is 3 to 4 minutes.

Expressions derived by Sarazin et al [62] for the variance of differential motion assume that the ratio of the separation of the apertures to the diameter of an aperture is at least 2. The parameters of the aperture mask (i.e. d and D) are a compromise between this assumption and practical considerations such as the size of the apertures and the brightness of possible targets. For the COAST DIMM setup, two apertures of diameter 4mm, with an accuracy of 0.1mm, were drilled into the cap on the acquisition camera. The separation between the centre of the apertures is 12mm. The apparent diameter and separation after taking the magnification of the optical system at COAST into account is 6.4cm and 19.2cm. For the MRO DIMM setup, two apertures of diameter 6.985cm, with an accuracy of 0.001cm, were drilled into the mask that attached to the front of the telescope. The separation between the centre of the apertures is 27.3cm.

Only targets with a magnitude less than 2 can be used when the exposure time of the camera is only 2 or 3ms. There must be enough light falling on the camera in this short time so that the signal to noise of the brightest pixel is at least 4 or 5. This is to ensure that there is enough signal to find the centre of the image positions accurately.
Chapter 2: Differential Image Motion Monitor, Which Is Transportable

2.3 Calibrating the DIMMWIT instrument

The detector of the DIMMWIT may be capable of exposure times short enough to freeze the turbulence, and cycle times short enough to record the temporal changes in the turbulence, but how are the atmospheric parameters derived from the observed image positions? The analysis will be dealt with in later chapters describing how seeing parameters are related to the differential image positions recorded by our DIMMWIT. All of the existing documentation on DIMM systems emphasises the importance of calibrating the instrument. Only then can the results be compared to other atmospheric instruments to prove its reliability. The files may be recorded in three different CCD readout modes that are then analysed in slightly different ways but each must be calibrated in the same manner. There are various aspects of the instrument that need to be calibrated and others whose effect on the centroids must be debiased in order to reduce the error in calculating the image positions:

1. **Dark Frames**
2. **Flatfielding**
3. **Pixel Scale**
4. **Sky Background**

2.3.1 Dark Frames

The chosen technique to calculate the centroid of the image position requires the dark field to have a zero mean. That means the average of the signal in the pixels where no light falls should be zero. If the average background count for each pixel could be calculated, then subtracting this from the original image would give a zero mean background. In addition, the standard deviation, $\sigma$, of each pixel, and the readout noise, $R$, of the CCD, which is the average standard deviation of a frame are calculated from analysis of these dark files. As mentioned in section 2.2.3, only frames where the signal to noise ratio of the brightest pixel (i.e. the ratio of counts in the brightest pixel to $\sigma$) is greater than 4 or 5 are acceptable, to ensure centroid accuracy. Finally, the readout noise parameter, $R$, is used to calculate the error in the variance of the centroids of an image due to readout noise (see chapter 3). The quoted dark current of the HX516 is less than 0.1 electrons/pixel/second and is the mean thermally generated signal in the absence of light falling on the CCD. This signal is due to electrons that are thermally
2.3 Calibrating the DIMMWIT instrument

generated in the CCD pixels. Hence the signal will have shot noise on it, with variance equal to the mean dark current signal. In addition there is a DC offset introduced by the characteristics of the CCD output amplifier and of the electronics of the video signal processor. There is no associated shot noise with this offset as it does not arise from a small number of discrete events. The DC offset also differs from the dark current signal in that it is independent of exposure time. Finally the readout noise is superimposed on the total signal (dark current and DC offset). Note that it is not necessary to use two parameters (\( \sigma \) and \( R \)) in the software. \( \sigma \) does not vary by more than 1% across the array which shows that the readout noise is fairly uniform across the CCD. Hence, \( R \) could be used to ensure that there is enough signal to noise in the image instead of \( \sigma \).

A series of dark frames (described in section 2.2.1) is acquired in all three readout modes at the end of each night of observing (as a matter of convenience). For a dark slow2d file, the exposure time is set to the shortest possible which is 1ms to avoid recording any stray light and the binning is set to 1x1. Note that it doesn’t actually matter what the binning mode is for a short exposure dark frame as the response should be the same irrespective. This also applies to using an arbitrary subframe to record dark files. The dark current (less than 0.1 electrons/pixel/second) is negligible for short exposures compared to the DC offset (about 10 electrons per pixel regardless of exposure time). Hence, analysis of the dark frames gives the DC offset introduced as part of the readout process, after binning, with the readout noise superimposed. The readout mechanism acts the same whether the pixels have been binned or not. Hence, the DC offset and readout noise level should be exactly the same for e.g. a 4x4 binned pixel as for an unbinned pixel. Standard practise is to use 1x1 binning anyway. A typical dark slow2d file contains 50 or more such subframes, so that the readout noise is averaged out. For the fast modes, the cycle or frame time is very short (2/3ms) so 1000 frames are saved as a dark file in each mode.

These three important parameters, average background count, \( \sigma \) and \( R \), are saved in a file in the same directory as the data. It is also important to record dark files on every night of observations because the ADC converter setting could change over time and recording dark files on each night allows the observer to check this.

2.3.2 Flatfielding

Before the mean background can be calculated from a set of dark frames, the response of the entire CCD array has to be tested for non-uniformity. Non-uniformities occur
when the pixels do not record the same signal level when they have the same illumination. In order to test the response of the CCD, the array is uniformly illuminated at a high enough level so that the mean signal in each pixel can be measured to less than 1%. The mean signal in each pixel then shows the relative QE of each pixel. This information is then input into a simulation of the centroiding algorithm using recorded data (one set of 8000 frames was taken from a recorded fast2d file and used as image data in the simulation) in order to test the impact of a non-uniform response on the centroids. Depending on the position of the ‘hot pixels’ relative to the centre of the star image, the presence of pixels with a 6% difference in response would result in a maximum 0.012 pixel change of the centroid from its true position (i.e. position if the CCD had a uniform response). This error is 0.3% of the rms differential image motion for the most heavily binned mode, fast2d. In addition, successive flatfield frames were averaged in order to check that the non-uniformities were not significantly larger at low signal levels. Hence, it was concluded that any non-uniformities in the CCD array did not have a significant impact on the accuracy with which the centroids of the star images could be determined. However, there was a stage during the development of the CCD software when the response of the CCD array was not uniform and hence flatfielding was required.

Flatfielding is the process where a light frame is divided by a normalised flat frame in order to correct the sensitivity difference among the pixels. Dark files are recorded with the light source off and flat files are recorded when a light source of uniform brightness falls on the CCD. The level of light allowed to fall on the CCD depends on the type of readout mode. The software allows the observer to ensure that intensity of light falling on the CCD is 50% of the pixel saturation intensity. The software also records the X and Y offsets of a subframe in the FITS header in units of unbinned pixels from the origin. This allows the observer to divide an observed file by the right subframe of a flatfield file, i.e. match up the pixels correctly. Regardless of what binning mode the observed file is recorded in, a flatfield file must be recorded in 1x1 binning to be able to use these X and Y offsets This is because the binned pixels are slightly offset from (0,0) co-ordinate of the unbinned full frame.

2.3.3 Pixel Scale

Calculating the pixel scale of the optical system is an important part of calibrating any DIMM system. It is not enough to trust the quoted focal length of the amateur telescope you have purchased. Determining the focal length already leads to a 3% or more
2.3 Calibrating the DIMMWIT instrument

error to the determination of Fried’s parameter. Incorrectly calculating the focal length will result in either under or overestimating this parameter and the DIMM will not appear to perform dependably when compared with another seeing instrument. The simplest method of determining the focal length of any optical system is to measure the separation of a (non variable) binary or multiple star whose separation is accurately known. The focal length is then calculated by comparing the separation of the stars in CCD pixels and in arcseconds on the sky and the error estimate is calculated from the variance of the 100 measurements of the separation. The known separation of the Mizar binary is $15.3771 \pm 0.0004$ arcseconds. Combining this uncertainty with the uncertainty of the mean of 100 measurements of the apparent separation (about 2.5%) leads to an error of 2.5% in the pixel scale and this contributes to a 3% error in the estimation of $r_0$.

The preferred separation of the stars should be between 10 and 30 arcseconds so that the two or more images are not too close to accurately centroid and to prevent any images falling outside the the CCD array. The procedure is to point a well-focused telescope at the chosen multiple star target and capture 100 or so frames of the CCD images. The focal length, and hence the pixel scale, of the DIMMWIT system can then be calculated from these measurements of the binary separation. The pixel scale parameter is then saved in the parameter file mentioned before. (Note: The best way to focus an amateur telescope is to focus the telescope with the aperture mask still attached so that the two separated star images become one again). It is well known that the focal length of a small telescope can change over the course of the night due to cooling effects. To ensure an accurate focal length measurement, the entire procedure of observing a binary target should be repeated at least three times: at the start, middle and end of the night. The change in focal length due to temperature can then be studied and the pixel scale values calculated from the focal length are reliable.

2.3.4 Sky Background

The contribution of the sky background to the recorded images should be checked at every new site as well, although it is unlikely to be significant in short exposures. The telescope is simply pointed at a dark patch of sky and 50 1s-exposure subframes are saved in a 1x1 binning slow2d file. If there is an insignificant amount of charge above the normal background level when the 50 frames have been added together, then the contribution of the sky to the background in an image is negligible.
2.3.5 Parameter Files

There are so many parameters that change from night to night of observing that a record of them is saved in a file called ‘settings’ in the directory of the nights data. In addition, the processing time of the dark files is quite long because of the time it takes to read in these large full frame files. Thus, they are processed separately from the DIMMWIT data at the start of every night and the results are then stored in the ‘settings’ file to be called on by other programs. Next the focal length of the system on the night of observations is calculated as in section 2.3.3 and the results are written out to the same ‘settings’ file. The ‘settings’ file also records what observatory the files were recorded at (either MRO or COAST), the latitude and longitude of this observatory and the RA and DEC of the targets so that the results can be corrected for zenith.

```
#Settings for analysing seeing files for : East_Telescope at COAST
#Calculated background information.
Slow2d file: Sigma 19.1995 Ave background 1071.65 RMS 19.3383
Fast2d file: Sigma 32.5275 Ave background 1356.76 RMS 38.4094
Fast1d file: Sigma 35.6815 RMS 39.4662
Ave fast1d background (average of one line because of artefact)
  1544.86  1528.45  1532.61  1521.27  1519.55  1516.43
  1514.77  1516.82  1515.97  1512.06  1517.11  1510.47
  1509.98  1510.81  1509.50  1505.23  1589.22  1603.77
  1600.57  1609.23  1593.98  1601.40  1598.08  1592.57
  1503.20  1500.16  1501.34  1590.09  1579.08  1599.41
  1591.64  1598.23  1599.78  1610.31  1585.02  1596.31
  1593.02  1611.35  1514.70  1526.44  1523.71  1524.65
  1521.24  1521.08  1525.45  1516.07  1516.20  1521.76
  1506.57  1522.88  1522.43  1525.82  1513.33  1515.12
  1515.08  1510.57  1504.51  1512.05  1506.01  1516.45
  1508.19  1516.03  1515.16  1511.81

#Other Important Settings
Focal Length of Chosen Telescope in cm: 312.588  3.95834
Observatory Settings (0 coast 1 mro): 0
#Zenith Correction Info
Latitude and Longitude: 52.16666666  0.1156666666
Targets alpboo
RA  14.261027777
DEC  19.1825
```

Figure 2.11: Typical parameter file called ‘settings’ written for each night of observing.
2.4 Conclusion

The DIMMWIT design is based on the basic components of similar DIMM systems. The main difference is the choice of amateur CCD camera to record the image positions with. Two new readout modes have been developed by Seneta [65] in order to get cycle times of 2 or 3 ms. The compromise for 2ms cycle times is one-dimensional information, while 3ms is achieved with lower resolution of the star images. The basic analysis of recorded data images is introduced here, while Chapter 3 and 4 will give more in-depth detail on how the spatial and temporal parameters are deduced from the recorded image motion. Hence, we can measure these turbulent parameters with a cheap, transportable DIMM.
Chapter 2: Differential Image Motion Monitor, Which Is Transportable
3 Spatial Structure of Turbulence

The performance of large aperture telescopes and stellar interferometers is limited by the effects of the atmosphere. These effects can be corrected for to some extent if they have been quantified by statistical analysis of the amplitude and phase fluctuations of the stellar wavefronts. Fried’s parameter, $r_0$, that is introduced in chapter 1 is a commonly used parameter to describe all seeing effects due to atmospheric turbulence. $r_0$ describes the spatial behaviour of the turbulent air flows in the atmosphere that limits the resolution of large telescopes.

Here I outline how Fried’s parameter is calculated from data recorded on the differential image motion monitors described in chapter 2. Sarazin and Roddier first developed the approach in 1989 by estimating Fried’s parameter from the variance of the differential image motion in two small apertures. This technique was then further developed by Tokovinin in 2002 by refining the DIMM measurements.

The aim of this chapter is to show that the DIMMWIT is a well calibrated instrument capable of measuring Fried’s parameter accurately. Only then can it be trusted to carry out seeing campaigns at other sites around the world. Once the three readout modes have been tested, the DIMMWIT results are compared with results from an alternate method of measuring $r_0$ using the COAST autoguider and with the FWHM of long exposure images.
Chapter 3: Spatial Structure of Turbulence

3.1 Fried’s parameter $r_0$

Turbulence causes small differential movements of the two DIMM images [62], so the phase perturbations of the wavefronts, and hence Fried’s parameter, can be derived from variations in the separation of the images, which corresponds to the difference in wavefront tilt across the two sub-apertures.

3.1.1 Sarazin and Roddier’s Equations for $r_0$

The deviation $z(x, y)$ of the wavefront surface from the average plane is proportional to the wavefront phase $\phi(x, y)$:

$$z(x, y) = \frac{\lambda}{2\pi} \phi(x, y). \quad (3.1)$$

Since light rays are normal to the wavefront surface, the component of the angle-of-arrival fluctuations in the x direction is given by:

$$\alpha(x, y) = -\frac{\partial}{\partial x} z(x, y) = -\frac{\lambda}{2\pi} \frac{\partial}{\partial x} \phi(x, y). \quad (3.2)$$

The covariance of the angle-of-arrival fluctuations defined by

$$B_\alpha(\mu, \eta) = \langle \alpha(x, y) \alpha(x + \mu, y + \eta) \rangle, \quad (3.3)$$

is obtained by taking the Fourier transform of their power spectrum. Therefore, it is related to the covariance $B_\phi(\mu, \eta)$ of the phase fluctuation by

$$B_\alpha(\mu, \eta) = \frac{\lambda^2}{4\pi^2} \frac{\partial^2}{\partial \mu^2} B_\phi(\mu, \eta). \quad (3.4)$$

Chapter 1 introduced the structure function which is related to the phase covariance by

$$D_\phi(\mu, \eta) = 2[B_\phi(0, 0) - B_\phi(\mu, \eta)]. \quad (3.5)$$

Substituting this into equation 3.4 gives:

$$B_\alpha(\mu, \eta) = \frac{\lambda^2}{8\pi^2} \frac{\partial^2}{\partial \mu^2} D_\phi(\mu, \eta). \quad (3.6)$$
3.1 Fried’s parameter \( r_0 \)

From chapter 1.1.3 we know that \( D_\phi(\mu, \eta) \) is related to Fried’s parameter \( r_0 \) by:

\[
D_\phi(\mu, \eta) = 6.88 \left( \frac{r}{r_0} \right)^{\frac{5}{3}} = 6.88 \left( \frac{\sqrt{\mu^2 + \eta^2}}{r_0} \right)^{\frac{5}{3}}, \tag{3.7}
\]

\[
\Rightarrow \frac{\partial^2}{\partial \mu^2} D_\phi(\mu, \eta) = \frac{5}{3} 6.88 r_0^{-\frac{5}{3}} \left[ (\mu^2 + \eta^2)^{-\frac{1}{6}} - \frac{1}{3} \mu^2 (\mu^2 + \eta^2)^{-\frac{7}{6}} \right]. \tag{3.8}
\]

\( \eta = 0 \) gives the longitudinal covariance in the direction of the tilt as a function of the separation \( \mu = d \) (\( d = \) separation), and \( \mu = 0 \) gives the transverse covariance in a direction perpendicular to the tilt as a function of separation \( \eta = d \):

\[
B_l(d) = B_\alpha(d, 0) = 0.097 \left( \frac{\lambda}{r_0} \right)^{\frac{5}{3}} \left( \frac{\lambda}{d} \right)^{\frac{1}{3}}, \tag{3.9}
\]

\[
B_t(d) = B_\alpha(0, d) = 0.145 \left( \frac{\lambda}{r_0} \right)^{\frac{5}{3}} \left( \frac{\lambda}{d} \right)^{\frac{1}{3}}. \tag{3.10}
\]

Note that the covariances in two orthogonal directions are dissimilar, although Kolmogorov’s theory assumes that every quantity is isotropic. The anisotropy is introduced by the gradient operator in equation 3.2. The covariance of the wavefront gradient is not independent of the orientation of the DIMM baseline with respect to the direction in which the gradient is measured. Hence, the differential tilt is an inherently anisotropic quantity, even for an atmosphere in which the phase variations are isotropic. Equations 3.9 and 3.10 are only valid in the inertial range and cannot be used to estimate the covariance at the origin. Instead, because the value at the origin is limited by aperture averaging, the expression for the variance of image motion derived by Fried is used:

\[
B_\alpha(0, 0) = 0.179 \left( \frac{\lambda}{r_0} \right)^{\frac{5}{3}} \left( \frac{\lambda}{D} \right)^{\frac{1}{3}}, \tag{3.11}
\]

where \( D \) is the diameter of the apertures in the DIMM mask. The variance \( \sigma^2(d) \) of the differential image motion observed over a distance \( d \) is given by[62]:

\[
\sigma^2(d) = < (\alpha(0) - \alpha(d))^2 >, \\
= < \alpha(0)^2 + \alpha(d)^2 + 2\alpha(0)\alpha(d) >, \\
= B(0) + B(0) - 2B(d), \\
\Rightarrow \sigma^2(d) = 2[B(0) - B(d)]. \tag{3.12}
\]

Putting equation 3.10 and equation 3.11 into equation 3.12 gives an approximate equation for the variance of the differential longitudinal motion, and putting equation 3.9 and equation 3.11 into equation 3.12 gives an approximate equation for the variance of the differential transverse motion as a function of \( r_0 \), assuming that \( d > D \), from which
Chapter 3: Spatial Structure of Turbulence

we can calculate $r_0$:

$$r_0 = \left[ \lambda^2 \sigma_i^{-2} D^{-\frac{1}{3}} \left[ 0.358 \left( 1 - 0.541 b^{-\frac{1}{3}} \right) \right] \right]^\frac{3}{5},$$  \hspace{1cm} (3.13)$$

$$r_0 = \left[ \lambda^2 \sigma_i^{-2} D^{-\frac{1}{3}} \left[ 0.358 \left( 1 - 0.810 b^{-\frac{1}{3}} \right) \right] \right]^\frac{3}{5}.$$  \hspace{1cm} (3.14)

where $b = d/D$, $d$ being the aperture separation and $D$ their diameter.

3.1.2 Tokovinin’s equations for $r_0$

Determining the image positions is the greatest source of error in determining Fried’s parameter from the variance of the differential motion. Sarazin and Roddier[62] calculated the image positions with the barycentre centroiding algorithm:

$$x_c = \frac{\int_a^b xI(x)dx}{\int_a^b I(x)dx}.$$  \hspace{1cm} (3.15)

It can be shown that the centroid is related to something called a G-tilt, which is defined as the average wave-front gradient across the aperture and so Sarazin and Roddier assumed that a DIMM measures G-tilts (also called angle-of-arrival fluctuations). However, G-tilts can be corrupted by atmospheric coma aberrations because the centroid heavily weights image wings and even slight asymmetry of the diffraction rings can influence the results. Hence, Tokovinin [71] introduced the concept that a DIMM could instead measure Z-tilts that are unaffected by these aberrations. A Z-tilt or Zernike-tilt is the slope of the least-squares fit plane to the wavefront over the aperture and hence, the calculated centre is the location of the maximum intensity in the focal plane or the location of minimum residual phase variance. Thus, Z-tilts are unaffected by distortions in the diffraction rings due to coma. A DIMM measures Z-tilts if the radius of the window used in the barycentre algorithm is that of the first dark ring in an Airy disk, or $1.22\lambda/D$ radians. The theoretical formulae relating the variance of the differential image positions to Fried’s parameter must now be modified to take this new interpretation into account. With $b = d/D$, $D$ is the diameter and $d$ is the separation of the centres of the apertures:

$$\sigma_1^2 = \lambda^2 r_0^{-\frac{5}{2}} D^{-\frac{1}{3}} \left[ 0.364 \left( 1 - 0.532 b^{-\frac{1}{3}} - 0.024 b^{-\frac{7}{3}} \right) \right],$$  \hspace{1cm} (3.16)$$

$$\sigma_1^2 = \lambda^2 r_0^{-\frac{5}{2}} D^{-\frac{1}{3}} \left[ 0.364 \left( 1 - 0.798 b^{-\frac{1}{3}} + 0.018 b^{-\frac{7}{3}} \right) \right].$$  \hspace{1cm} (3.17)
3.2 Analysis

One cannot always assume that a DIMM is capable of measuring Z-tilts. Depending on the resolution of the CCD, it may not be possible to choose a centroiding window that falls between the Airy rings. The centroiding window either cuts out critical light leading to inaccurate centroids or includes light past the first dark Airy ring and the DIMM is no longer measuring Z-tilts. Only if the starlight is spread over many CCD pixels is it possible to choose a window that is likely to contain only light from the central peak of the diffraction pattern so the DIMM is measuring Z-tilts. If this is not the case then it must be assumed that the DIMM is only measuring centroids or G-tilts.

Hence, the formulae in equations 3.18 and 3.19 are used to calculate Fried’s parameter. These equations are more accurate than equations 3.13 and 3.14 because of the crude approximation made by Sarazin and Roddier that \( d \gg D \).

\[
\sigma_i^2 = \lambda^2 r_0^{-\frac{5}{3}} D^{-\frac{1}{3}} \left[ 0.340 \left( 1 - 0.570 b^{-\frac{1}{3}} - 0.040 b^{-\frac{7}{3}} \right) \right], \quad (3.18)
\]

\[
\sigma_i^2 = \lambda^2 r_0^{-\frac{5}{3}} D^{-\frac{1}{3}} \left[ 0.340 \left( 1 - 0.855 b^{-\frac{1}{3}} + 0.030 b^{-\frac{7}{3}} \right) \right]. \quad (3.19)
\]

3.2 Analysis

Chapter 2 described how the DIMMWIT data is calibrated. Now we continue to describe how the three types of FITS files are analysed to calculate Fried’s parameter.

3.2.1 Finding stars

Analysing DIMMWIT FITS files to get the two image positions entails finding the two brightest pixels in each frame (or line in the case of the fast1d mode) and applying a centroiding algorithm around each bright pixel.

1. Before the files are analysed, a dark frame is recorded in order to estimate the background level in every pixel in a frame.

2. The positions of the two brightest pixels are found by searching for pixels that are brighter than all the pixels in a certain region around them.
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a) Analysis begins with the first frame containing exactly two locally-bright pixels, all previous frames in the file being discarded.

b) For frames after the first frame found in a), when more than two brightest pixels are found in an image, this set of brightest pixels is subjected to a few tests in order to reduce the number to two. These two pixels will hopefully correspond to the two star images. These tests are outlined in steps 3 to 5.

3. First, the intensity in each bright pixel must be more than 4 times the RMS of the background level for there to be enough starlight to calculate an image position accurately. Hence, any bright pixels with an intensity below this cut-off are discarded.

4. Then, the positions of the acceptable bright pixels are compared with those in the previous frame to find two sets of bright pixels around the positions of the two star images. The brightest pixel of each set is then chosen as the new star position.

5. To ensure that these two pixel positions relate to two separate star images, they undergo several tests. If at any stage during this process only one pixel is left then the frame or line is discarded as only one star has been identified and the differential position cannot thus be calculated. If the separation means they actually belong to the same star, the frame or line is discarded.

6. The image centroids are computed using centroid boxes centred on the chosen brightest pixels.

The centroiding algorithm uses a box of pixels around each of the bright pixels so if either pixel is too close to the edge of the frame or line the box falls off the array and the centroid is inaccurate because there are pixels missing in the calculation. (Note: the centroiding algorithm will be explained in the next section). To prevent this, if the pixel is deemed to be too close to the edge then the frame or line is discarded. Finally, if the frame or line has passed all these conditions, the two star positions are rearranged so that they are in the same order as they appear on the display so that an array of the first or second star’s positions always refers to the same star.

The number of frames discarded for the various reasons outlined above is recorded and displayed on the screen so that the user can judge the usefulness of this section of the file in calculating atmospheric parameters. To summarise, frames are discarded if: not enough starlight in each bright pixel, pixels too near the edge of the frame or line, and two bright pixels actually belong to the same star.
3.2 Analysis

3.2.2 Centroiding Algorithms

There are various centroiding algorithms that can be used to determine the image positions [67]. The most popular is based on the barycentre algorithm [27], which requires the background to have zero mean. For a chosen window, if \( I(x) \) is defined as the intensity of pixel \( x \) once the background level has been subtracted, the position of the centroid, \( x_c \), along the x axis is

\[
x_c = \frac{\int_a^b x I(x)dx}{\int_a^b I(x)dx}.
\]  

(3.20)

where \( a \) and \( b \) are the edges of the window. For a pixelised CCD image, the integral is replaced by the sum over all the pixels. Centroids, also called G-tilts ([74]), calculated by the barycentre method are related to the averages of the wave-front gradients over the sub-apertures.

The shape of the images in a DIMM are generally assumed to be close to that of an Airy disk [71]. However, the diameter of the apertures is usually greater than Fried’s parameter, so the shape of the image is affected by low-order atmospheric distortions such as astigmatism, defocus and coma. Coma, in particular, can shift the image centres. Tokovinin [71] researched how windowing could be used to overcome this effect. Windowing is the technique where only the pixels within a certain radius of the image centre are used in the calculation of the centroid.

\[
x_W = \frac{\sum_{\text{window}} x_{ij}I_{ij}}{\sum_{\text{window}} I_{ij}}.
\]  

(3.21)

If the radius of the window is that of the first dark ring in an Airy disk, or \( 1.22\lambda/D \) rad, we are now measuring the centre of the images which is related to the best fit of the wave-front gradients across the apertures or Zernike tilts. Z-tilts are not affected by coma aberrations so measuring the centre of the images is a more reliable way of calculating the image positions. In order to use a centroiding window of radius \( 1.22\lambda/D \), the readout mode must have good resolution, i.e. the light from an image must be spread over enough pixels to be able to cut off at a good approximation of the first dark Airy ring. The fast1d mode has this resolution but the other two modes bin the starlight and have lower resolution so the position of the first dark Airy ring could be anywhere within a pixel. Hence, the DIMMWIT measures Z-tilts in fast1d mode and G-tilts in the other two modes. (Note that the DIMMWIT only measures G-tilts in slow2d mode because 2x2 binning has been chosen to improve the cycle time)
Finally a word on other centroiding techniques. Also popular is the gradient approach where the local maximum of the intensity is calculated, i.e. the position where the derivatives on the vertical (Y) or horizontal (X) axes go to zero. Many believe that this is a more robust method but it is more complicated to quantitively the error introduced by the detector. Windowing, as described by Tokovinin, has a simple noise model so the centroid can be easily corrected for noise introduced by the detector.

### 3.2.3 Variance of the Differential Motion

Section 3.1 showed how Fried’s parameter is related to the variance of the differential image positions. In order to get a 24 second average of the seeing, the variance of the image positions in 12000 lines in a fast1d FITS file or 8000 frames in a fast2d FITS file are used to calculate $r_0$ (see figure 3.1). In the slow2d mode there is a delay of about 200ms between each exposure so recording a comparable number of frames would take too long. So, 1000 frames are used for each estimate of $r_0$ from a slow2d file as a compromise. 1000 frames takes about 10 minutes to record. The slow2d mode is only used as a means of testing the reliability of the two faster modes. Once the observer is sure that all three types of files give comparable answers, there is no longer any need to record in slow2d mode, unless the situation calls for 1ms exposures (see section 3.2.5).

![Sample of a fast1d file](a) sample of a fast1d file ![Frame from a fast2d file](b) frame from a fast2d file

**Figure 3.1:** Examples of a fast1d and fast2d file. 12000 of the lines in each fast1d file and 8000 frames, like the one shown, from a fast2d file are used.

In order to calculate the longitudinal and transverse value of the Fried parameter, the two components of the variance, one parallel and one perpendicular to the median separation of the images must be calculated. Hence, the original image positions on axes parallel and perpendicular to the bottom of the CCD array must be transformed to axes relating to the median separation of the images. If the median of the first star’s position on the x and y axis is $\tilde{x}_0$ and $\tilde{y}_0$, and the median of the second star’s position is $\tilde{x}_1$ and $\tilde{y}_1$, then the rotation angle $\theta_R$ to transform to the new axes can be calculated from equation 3.22:

$$\theta_R = \arctan \left( \frac{\tilde{y}_1 - \tilde{y}_0}{\tilde{x}_1 - \tilde{x}_0} \right).$$

(3.22)
3.2 Analysis

Once the median of each image positions, \(x_0, y_0, x_1\) and \(y_1\), has been subtracted from each image position, so that we are rotating about the median star '0' position, the co-ordinates can be transformed to the new axes \(l\) and \(t\), for longitudinal and transverse, by

\[
\begin{align*}
\text{long}_* &= \cos \theta R x_* + \sin \theta R y_* \\
\text{trans}_* &= -\sin \theta R x_* + \cos \theta R y_*
\end{align*}
\] (3.23)

(3.24)

where * can be 0 or 1 denoting the first or second star positions. The variances of the differential positions on these new axes, e.g. the variance of \(\text{trans}_1 - \text{trans}_0\), are then calculated and used in equations 3.16 and 3.17 (or the equivalent G-tilt equations) to calculate the Fried parameter in the longitudinal or transverse directions. As well as calculating Fried’s parameter from the variance, it is important to check that the image positions have a sensible distribution. A histogram of the differential image positions in the longitudinal or transverse directions should be a normal or Gaussian distribution as shown in figure 3.2. By fitting the histogram to a Gaussian model, the goodness of fit of the data can be determined. If the data is a poor fit to a normal model then it should be discarded and not used to calculate the Fried parameter.

![Histogram](image)

Figure 3.2: Histogram of image positions from a fast1d file on 24 May 2004 with the Gaussian distribution fit. The chi-squared value is 1.04 which is well within the empirically determined acceptable range of up to 1.3 for fast1d data. The chi-squared value of good and bad sets of data were compared in order to determine this range of acceptable fits. ‘Good’ and ‘bad’ sets were judged visually from the shape of the histogram. ‘Bad’ data sets had skewed histograms that poorly fit the gaussian.
3.2.4 Error Analysis

There are three sources of random error in determining the Fried parameter: the error in measuring the pixel scale (i.e. the size of a CCD pixel in arc-seconds on the sky), statistical errors from calculating a variance from a number of samples and the centroids or centres of the images [29].

Measuring the pixel scale was described in chapter 2 - observations of a binary star allow the focal length of the optical system to be determined. The pixel scale allows the units of the differential variance to be converted from (CCD pixels)$^2$ into (arc-seconds)$^2$ before the variance can be used to calculate Fried’s parameter. The pixel scale is a function of the focal length of the optical system and so the error in measuring the pixel scale is dependent on the error in the focal length measurement. Calculating the standard deviation of all the binary separations measured leads to an estimate of the error in the focal length and hence the pixel scale. The uncertainty in the pixel scale gives rise to an error that is common to all of the measurements of Fried’s parameter on a particular night. As mentioned in chapter 2, the error in the pixel scale leads to a 3% error in the estimation of $r_0$.

If the N samples (i.e. the short exposures) are considered statistically independent and the measurement time (i.e. the length of file analysed) is short enough that the statistical properties of the image motion remain constant, then, according to Frieden [25], the error in calculating a variance from a number of samples for this application is given by

$$\frac{\partial \sigma^2}{\sigma^2} = \sqrt{\frac{2}{N-1}},$$

(3.25)

where $\sigma^2$ is the variance of the differential motion in the longitudinal or transverse direction. 8000 and 12000 fast2d and fast1d frames/lines are used in each estimation of $r_0$. The rms change in $r_0$ over 3 minutes was calculated on several nights and was found to be consistent with zero. So, $r_0$ is unlikely to change (i.e. the mean of the underlying distribution of variances does not change) over timespans of length 60,000 frames. Here we are using a minimum of 8,00 frames. Hence, the error contributed to the estimation of $r_0$ by the number of frames used in the calculation will always be around 1% (i.e. $\partial \sigma^2 / \sigma^2 = 1.3\%$ hence, $\partial r_0 / r_0 = 0.8\%$).

The centroid can be affected by the noise in the CCD image due to the detector. Read-out noise, Poisson noise and non-uniformity of the CCD response all affect the recorded intensity in each pixel. Looking again at the barycentre algorithm in equation 3.15, you can see that points located away from the image centre are weighted by the square of
their distance from it when calculating a variance. If the background level does not actually have a zero mean, perhaps because the CCD pixels aren’t uniform in their response, then the centring process is badly degraded. Readout noise and Poisson noise then add additional centroid errors because their effect on the intensities is random and cannot be corrected for by simply subtracting a background level. Eliminating the background counts by either thresholding or windowing makes the centring algorithm much less sensitive to noise in the background. Windowing has already been described and **thresholding** is the process of subtracting a threshold from the image and then computing the centroid from the nonnegative pixels. Both processes reduce the weight of pixels in the image wings for an optimal centroid.

The barycentre algorithm may be more robust when windowing or thresholding is also applied but detector noise still contributes some error to the centroid determination, adding a noise variance to the atmospheric image motion variance. The contribution of readout noise to this noise variance is easily modelled for window-limited centroids, as shown by Tokovinin [71]. If $R$ is the RMS readout noise and $I_{tot}$ is the total flux in the chosen window, we can derive the readout noise variance from equation 3.21 to be

$$
\sigma^2_R = \frac{R^2}{I_{tot}^2} \sum_{\text{window}} (x_{ij} - \bar{x})^2.
$$

(3.26)

Here $\sigma^2_R$ has units of $(\text{CCD pixels})^2$ and relates to one spot only. $2\sigma^2_R$ must be subtracted from the variance of the differential image motion to remove the bias in the centroid due to readout noise. The noise variance due to the readout noise of the CCD adds to the atmospheric image motion variance and makes the apparently measured seeing worse. Subtracting $2\sigma^2_R$ from the variance of the differential image motion removes this bias from the data and increases the values of $r_0$.

The derivation of equation 3.26 assumed that the intensity of the light was constant in all the frames contributing to the variance calculation. However, in reality, the intensity is changing in each frame due to scintillation effects. By simulating a star image of either varying or constant intensity, the effect can be studied. The image is assumed to be Gaussian with a FWHM similar to real images. Readout noise is added and the variance of the centroid in the $x$ and $y$ direction is calculated for either constant or varying intensities in each frame. The intensity variations are simulated using samples of real data. In either case, the average intensity over all the frames is increased within a chosen range and the relevant variation in $x$ centroid recorded. The results are presented in figure 3.3 showing that the variance of the $x$ centroid $\sigma^2_R$ is inversely proportional to the square of the total intensity in the stellar image $I_{tot}^2$. When the intensity is held constant in each frame, the slope of the straight line is the same no matter what range
of average intensities is used, as expected from Tokovinin’s equation. When the intensities vary, the slope ranges from 1.3 to 1.78 times the constant slope. So, the error due to readout noise would be larger if the intensity levels were varying. As the variance of the differential image motion is of order $10^{-2}$ (CCD pixels)$^2$ and the variance due to readout noise is of order $10^{-4}$(CCD pixels)$^2$, increasing this second variance by a factor of 1.3 to 1.78 does not have a significant impact on the corrected image motion variances.

The centroid is also affected by photon noise and non-uniformity of the sensitivity of the CCD pixels. It has been found [71] that the variance due to the photon noise is small compared to the variance due to the readout noise for a direct imaging CCD. In modern CCDs, the variation in sensitivity of the pixels has a smaller effect than Poisson noise so the flat-field effect can be neglected along with Poisson noise.

In addition, there are two sources of systematic error in determining the centroids of the images: the effect of a gap between the pixels mentioned in chapter 2 and the sensitivity of the size of detector pixels used. Hardy[27] investigated the problem of gaps between the pixels of an Interline camera and discovered that there was little effect on the centring algorithm if the gaps were less than 10% of the pixel scale, which they are in this case. Hence, the effect of gaps is assumed to be negligible and not
considered any further. In the fast2d mode, the pixels are quite large because of the 4x4 binning. The typical size of a DIMM image is about 1.5 CCD binned pixels in fast2d mode. This has been determined by fitting a gaussian to both star images in several frames of a fast2d file and taking the average. The accuracy of the barycentre algorithm then depends on where the image falls, with the most favourable location at the junction between 4 CCD pixels [27]. A simple simulation determined the error introduced into the centroid when the actual centre of the image fell at various points within a fast2d mode CCD pixel (see figure 3.4). The FWHM of the Gaussian was set to a range of multiples of CCD pixels and the centroiding box was 5 pixels in length. The image was moved across the x-axis of the array by 0.1 of a CCD pixel at a time and the centroiding box was kept in the centre of the array. Plotted in figure 3.4 are the results for the deviation of calculated centroids from true centroids for FWHM values of 1.4 CCD pixels and 1.0 CCD pixels, which shows that for FWHM images of greater than 1.4 CCD pixels (the relationship starts to become increasingly non-linear for images smaller than this), the centroiding algorithm is accurate. Typical Fast2d modes images have FWHM of 1.5 CCD pixels. So, the conclusion was that the resolution, or the size of the pixels, in the fast2d mode did not adversely affect the performance of the centering algorithm.

![Figure 3.4: Plot showing deviation of the calculated image centres from the actual image centres for a simulated gaussian image moved across the x-axis of a simulated array of fast2d CCD pixels by small increments. The deviation is considerably worse for the smaller image, while the deviation is almost flat for the larger image.](image-url)
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3.2.5 Effect of Finite Exposure Time

The relationship between the variance of differential image motion and Fried’s parameter assumes that the image motion has been completely frozen during the short exposures. A lengthy exposure time can artificially reduce the motion variance and overestimate the seeing conditions \[61, 63\]. The effect of exposure time on differential motion depends on the ratio \(v\tau/D\) of distance travelled during an exposure to the aperture diameter. Theoretical models \[45\] predict that sub-apertures of diameter 10 cm require exposure times of 5-10 ms in low wind speeds and 2 ms in high winds of about 30 ms\(^{-1}\) in order to measure the seeing conditions with 10% accuracy. Soules et al \[66\] also recommended that exposures of 2 ms or less are used at all times regardless of the atmospheric conditions. The DIMMWIT uses smaller apertures of about 6 cm but it was hoped that because of the low wind conditions characteristic of the COAST site, exposure times of 2 to 3 ms would avoid any overestimation of the seeing. The degradation of the differential motion also depends on the direction of measurement with respect to the aperture separation. Hence, any differences between measurements made on the longitudinal and transverse axes may be because of some attenuation due to exposure time or anisotropic turbulence that is not yet fully developed \[45\].

To ensure that the exposure time used was short enough, any overestimation of \(r_0\) due to exposure times longer than 1 ms can be investigated by plotting variance against exposure time and using an exponential model to extrapolate back to zero exposure time. The longer exposures are simulated by adding lines or frames in the fast files together. Examples of such fits for two nights are shown in figures 3.5 to 3.8. The wind speeds at COAST are characteristically low for most of the year (4/5 ms\(^{-1}\)) so there is no great difference between the two observation periods. On 25th of February, the variance of 3 ms exposures was 6% lower than zero exposure in the transverse direction and 7% lower in the longitudinal direction. From the fast1d files we see that the variance of 2 ms exposures was 7% lower than zero exposure. On 19th of April, the variance of 3 ms exposures was 2% lower for transverse and 1% lower for longitudinal and the variance of 2 ms exposures was 2% lower than zero exposure. These exponential fits imply that the variance of 2-3 ms exposures underestimates the true variance of zero-length exposures by under 10%. All the data recorded by the DIMMWIT has been analysed in this way and there was no night where the variance underestimated the variance of zero exposures by more than 10%. A 10% error is enough to determine the reliability of the DIMMWIT measurements, which was the aim of the seeing campaign at COAST. If a more accurate measurement was required, the exposure time effect could be removed by performing an extrapolation back to zero exposure for every fast1d/fast2d measurement of \(r_0\).
3.2 Analysis

Figure 3.5: Plot of variance versus exposure time for fast1d files on 25th February 2004. The cycle time of fast1d mode is 2ms and the longer exposure times are simulated by adding lines together. The results of the exponential fit are shown in the key. The variance underestimation at 2ms is 7%, which is an acceptable level of error.

\[ V = 5.29603 \exp(-0.03709 T) \]

Figure 3.6: Plot of variance versus exposure time for fast2d files on 25th February 2004. The cycle time of fast2d mode is 3ms and the longer exposure times are simulated by adding frames together. The results of the exponential fit are shown in the key. The variance underestimation at 3ms is 7% in both directions, which is an acceptable level of error.

\[ V = 0.19026 \exp(-0.02971 T) \]
\[ V = 0.27615 \exp(-0.03127 T) \]
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Figure 3.7: Plot of variance versus exposure time for fast1d files on 19th April 2004. The cycle time of fast1d mode is 2ms and the longer exposure times are simulated by adding lines together. The results of the exponential fit are shown in the key. The variance underestimation at 2ms is 2%, which is an acceptable level of error.

Figure 3.8: Plot of variance versus exposure time for fast2d files on 19th April 2004. The cycle time of fast2d mode is 3ms and the longer exposure times are simulated by adding frames together. The results of the exponential fit are shown in the key. The variance underestimation at 3ms is less than 2% in both directions, which is an acceptable level of error.
3.3 Alternate Methods of Estimating Fried’s parameter

One way to prove the reliability of a DIMM system is to compare the seeing measurements made by the system and by a large nearby telescope. In this case, the DIMMWIT could be compared with the interferometer COAST. The Cambridge Optical Aperture Synthesis Telescope is a 5 element optical interferometer with two of the unit telescopes on the same arm of the array. Measuring the coherence time is part of the reduction suite of the interferometer, as outlined in chapter 4. Extracting Fried’s parameter from this interferometric data proves to be a great deal more complicated. However, Bharmal [8] demonstrated how the COAST autoguiding system could also be used to deduce \( r_0 \) by modifying what data the autoguider recorded. Finally, the performance of the DIMMWIT can be tested by comparison with the FWHM of long exposure images recorded by a Starlight Xpress camera on another nearby COAST unit telescope.

3.3.1 COAST autoguider and Fried’s parameter

The COAST autoguider is responsible for correcting the tip/tilt aberrations in the light beams entering the optical system. The autoguider wavefront sensor uses spot mirror beam-splitters to divide the light by allowing the outer annulus of the incoming beams onto the autoguider CCD, while the central portion is diverted to the beam combiner. The WFS (wavefront sensor) consists of a CCD upon which four light beams are imaged. Each image falls on a Quadrant Cell (QC) that measures the tip/tilt error on each beam by reading out the pixels every 6 ms to calculate the instantaneous image position. The estimates of the tip/tilt aberration are sent in arbitrary units called pseudo-volts to amplifiers controlling the piezo-electric actuators attached to flat mirrors. The actuators allow the mirror to rotate about the vertical and horizontal axes and these flat mirrors are referred to as ‘fast mirrors’.

Fried’s parameter, \( r_0 \), can be determined from the variance of the recorded image motion, i.e. fast mirror angles, along one axis. The autoguider software has been modified to record the QC signals rather than the actual ‘fast mirror’ angles. The mirror angles must be reconstructed after the measurements have been taken using calibration factors determined experimentally. If the image position measured by the QC, and scaled to pseudo-volts, at a given time is \( x_n \) then, using a gain of \( \beta \) (normally 1), the mirror angle is \( y_n \),

\[
y_n = y_{n-1} + \text{Int}[\beta x_n],
\]

(3.27)
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where \( y_0 = 0 \) and the 'Int' operator represents rounding down to the nearest integer. The QC image positions are rounded down in equation 3.27 because the autoguider control software uses integer arithmetic for speed. Each pair of calibration factors gives the rotation of the fast mirrors per pseudo-volt for the vertical and horizontal axis of the QC belonging to a COAST telescope and so the fast mirror angles can be converted to radians.

Single image motion can be affected by tracking errors, which would adversely affect any seeing estimates. Due to the poor performance of the target tracking algorithm, the fast mirrors try to compensate for these low frequency tip/tilt errors and as a result reach their range limits. So, to eliminate error the mirror angles outside the range limits of the mirrors must be discarded and the remaining low frequency errors are further reduced by high-pass filtering the fast mirror angles. The equivalent to using a high-pass filter is to filter the angles with a low-pass Infinite Impulse Response (IIR) filter. The fast mirror angle, \( y_i \), is related to its filtered version, \( y_i^* \), by

\[
y_i^* = \exp^{-\alpha} y_{i-1}^* + (1 - \exp^{-\alpha}) y_i.
\]  

(3.28)

where \( \alpha \) is usually 117 (a function of the time constant \( \tau \) and the sampling time \( \Delta \)), leading to a -3dB response frequency of 0.02Hz.

The corrected angles, \( y_{corr,i} \), are then given by

\[
y_{corr,i} = y_i - y_i^*.
\]  

(3.29)

The variances of the corrected fast mirror angles are calculated by fitting a normal distribution to the angles and then scaled to radians rms. The variance, in rad rms, of the G-tilt along one axis is then related to Fried’s parameter by[8]

\[
\sigma_G^2 = 0.598 \left( \frac{D}{r_0} \right)^\frac{5}{3},
\]  

(3.30)

where \( D \) is the diameter of the aperture. Here \( D \) is assumed to be the average of the inner and outer diameters of the annuli of the spot mirror beam-splitters. The \( D/r_0 \) ratios were calculated from the fast mirror angles assuming a wavelength of 855 nm, so the resulting \( r_0 \) values are scaled down from this to 500 nm for comparison with DIMMWIT measurements.

In order to calculate a correlation between the two sets of measurements, both the autoguider and DIMMWIT data are binned into boxes 1.2 seconds in length. A correlation will allow us to determine if the two instruments are measuring the same spatial
movement, even if the scale of the measurements differ slightly, i.e. the autoguider \( r_0 \)s are higher than the DIMMWIT \( r_0 \)s. Using a high pass filter and removing any image motion at frequencies around 0.02 Hz, results in a narrowed distribution and hence a decrease in \( D/r_0 \). Thus, the autoguider analysis could systematically overestimate \( r_0 \). The fraction of the atmospheric tilt power removed by the high-pass filter can be quantified using

\[
\frac{P_{\text{msed}}}{P_{\text{tot}}} = 1 - \frac{8}{9} \left( \frac{f_0}{f_1} \right)^{2/3},
\]

where \( f_0 \) is the cut-off frequency of the filter and \( f_1 \) is the break frequency in the single image motion power spectrum. For a break frequency of 2.1Hz and a cut-off frequency of 0.02Hz, the power removed by the high-pass filter is 4%, which might explain part of the discrepancy between the autoguider and DIMMWIT measurements.

<table>
<thead>
<tr>
<th>Date</th>
<th>Ratio of ( r_0 )s</th>
<th>Correlation between Autoguider and COAST</th>
<th>Statistical Significance*</th>
<th>Coherence Time of COAST fringes</th>
</tr>
</thead>
<tbody>
<tr>
<td>01.03.04</td>
<td>1.0 ± 0.09</td>
<td>0.3</td>
<td>10%</td>
<td>9.8 ± 0.4 ms</td>
</tr>
<tr>
<td>02.03.04</td>
<td>1.3 ± 0.03</td>
<td>0.6</td>
<td>10%</td>
<td>6.6 ± 0.3 ms</td>
</tr>
<tr>
<td>16.04.04</td>
<td>1.2 ± 0.08</td>
<td>0.5</td>
<td>5%</td>
<td>6.7 ± 0.4 ms</td>
</tr>
<tr>
<td>19.04.04</td>
<td>1.2 ± 0.04</td>
<td>0.6</td>
<td>1%</td>
<td>6.0 ± 0.3 ms</td>
</tr>
<tr>
<td>22.04.04</td>
<td>1.4 ± 0.04</td>
<td>0.4</td>
<td>10%</td>
<td>3.7 ± 0.2 ms</td>
</tr>
<tr>
<td>24.05.04</td>
<td>1.3 ± 0.08</td>
<td>0.3</td>
<td>30%</td>
<td>7.5 ± 0.5 ms</td>
</tr>
</tbody>
</table>

Table 3.1: Summary of comparison between autoguider and DIMMWIT measurements of \( r_0 \). 'Ratio' refers to the factor by which the autoguider are larger than the DIMMWIT \( r_0 \)s. 'Correlation' is the calculated correlation between the two sets of data - the nearer +1 or -1, the better the correlation. *A student t-test was performed to calculate the level of significance with which the null hypothesis (i.e. no correlation) could be rejected and the results are shown in 4th column. The average coherence time as measured from COAST fringes is also given in the last column. Note that the aperture of the DIMMWIT is 6.4cm and the aperture of the Autoguider is 32cm.

Table 3.1 is a summary of the comparison between autoguider and DIMMWIT data. Although there appears to be no high correlation between the DIMMWIT and autoguider, the two data sets are moderately correlated (i.e. a correlation between 0.4 and 0.6) on 4 out of the 5 nights when the correlation was statistically significant (i.e. t-test resulted in a better than 10% significance). On 19.04.04, scatter in the autoguider data results in autoguider values 20% higher than DIMMWIT values. However, most of the DIMMWIT data still falls within the range of error in the autoguider data. In addition, there is a correlation of 0.6 between the two sets of data. In contrast, on 01.03.04, there is a low but statistically significant degree of correlation between the two sets of data at 0.3 although the data sets more or less follow the same trend in \( r_0 \) over the course of the night.
Any overestimation of Fried’s parameter by the autoguider could be due to the high-pass filter used in the analysis, which removes some of the low-frequency variation of the image motion. In order to test this, the ratio of autoguider $r_0$ over DIMMWIT $r_0$ and the correlation are plotted against the coherence time measured by COAST. In figure 3.9 (a) the ratio of $r_0$s apparently decreases with increasing coherence time, which is the opposite of the expected relationship and in (b) the correlation increases with increasing coherence time, although you would expect a longer $t_0$ to result in a worse overestimation of $r_0$ by the autoguider, i.e. the correlation to decrease with increasing coherence time. In conclusion, it appears that the autoguider and DIMMWIT are measuring the same spatial motion but the autoguider may be overestimating the length scale of spatial turbulence. This is not caused, however, by the high-pass filter.

Two other possible reasons for the autoguider overestimating the seeing are

1. The mis-measurement of the calibration factors used to convert the pseudo-volts to fast mirror angles.
   
   There is a 4% error in the horizontal calibration factor used for the East telescope, and these results were used to determine the spatial seeing. The values for Fried’s parameter calculated on the vertical axis and on a different channel were substantially different from the DIMMWIT, which might imply that although the calibration factors on the East horizontal axis were accurate enough to give results similar to the DIMMWIT, they were not accurate enough to agree completely with the DIMMWIT.

2. Vignetting of the outer-parts of the annulus pupil used to feed the auto-guider leading to a smaller effective aperture diameter than 32cm, which results in overestimating $r_0$ when using 32cm in the calculations.
   
   There are two causes of vignetting at COAST. Firstly, it has been proven that misalignments can cause vignetting. Secondly, the 50cm siderostat mirror is not sufficiently over-sized for glancing incidence when viewing high-declination targets, causing vignetting of the outer parts of the annulus pupil.
3.3 Alternate Methods of Estimating Fried’s parameter

(a) Comparing the ratio of autoguider to DIMMWIT $r_0$ with the coherence time measured by COAST.

(b) Comparing the correlation between the two sets of data (autoguider and DIMMWIT $r_0$s) with the coherence time measured by COAST.

Figure 3.9: Comparing the ratio of autoguider to DIMMWIT $r_0$ and the correlation between the two data sets with the coherence time measured by COAST.
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Figure 3.10: Comparing measurements of Fried’s parameter $r_0$ made by the COAST autoguider and the DIMMWIT on 19th of April and 1st March 2004. The ratio of the average $r_0$ from autoguider data and DIMMWIT data is also given as well as the coherence time measured by the Interferometer. The rms change in $r_0$ over 3 minutes is also calculated and is consistent with zero on both nights.

(a) 190404 - ratio is 1.2, correlation is 0.6 (20 binned measurements) and coherence time is 6ms. Note that the rms change in $r_0$ over 3 mins is 0.9%±4.7%.

(b) 010304 - ratio is 1.0, correlation is 0.3 (42 binned measurements) and coherence time is 9.8ms. Note that the rms change in $r_0$ over 3 mins is 4.1%±4.4%.
3.3 Alternate Methods of Estimating Fried’s parameter

3.3.2 FWHM of Long Exposure Images

The reliability of the DIMMWIT can also be demonstrated by comparing DIMMWIT measurements of Fried’s parameter with the Full-Width-Half-Maximum of long exposure images. The Fried parameter is related to the FWHM of a long exposure image obtained with a large aperture by [75]

\[ \text{FWHM} = 0.98 \lambda/r_0. \]  \hspace{1cm} (3.32)

where Fried’s parameter has been corrected for zenith as previously described.

300ms and 1s exposure images were recorded using the acquisition Starlight Xpress camera on one COAST unit telescope and DIMMWIT files were recorded on the unit telescope converted into a DIMM. The long exposure images were added together to simulate long exposures of 9s or 50s as the tracking at COAST performs well enough not to move the image significantly in this time. Using equation 3.32, the DIMMWIT measurements are transformed into FWHM values and plotted with the FWHM of the long exposure images as shown in figures 3.11 and 3.12.

![Comparing Long Exposure Image FWHM with DIMMWIT FWHM 29.03.04](image)

Figure 3.11: FWHM of 50s long exposure images and DIMMWIT images recorded on two COAST unit telescopes in close proximity to each other on 29th March 2004.
Both sets of data take into account the different pixel scale on the two telescopes. Given that the starlight reaching the two telescopes has not travelled along precisely the same path through the turbulent atmosphere, there is good agreement between the two sets of data (for example, the DIMMWIT values had mean $2.08 \pm 0.44$ and the long exposure images had mean $1.48 \pm 0.21$ on 29th of March). It would have been preferable if the DIMMWIT camera had allowed exposures of more than 1 second, but summing the images and discarding those with skewed shapes seems to have worked well enough for this comparison. Note that images with skewed shapes were discarded because tracking errors may have been the cause of the shape rather than motion due to the atmospheric turbulence.
3.4 Observed Coherence Length at Lords Bridge

A seeing campaign was carried out from February to May 2004 at the COAST site. The campaign would need to continue for one to two years for a detailed study of the characteristic behaviour of the atmosphere. However, the campaign is sufficient to test the performance of the DIMMWIT system and the new readout modes in particular. A comparison with results from slow mode data would test the results of the fast modes and satisfy concerns about the resolution of the fast2d readout mode. The overall results of the campaign are then presented to give a sample of the typical seeing conditions at COAST.

3.4.1 Reliability of Fast Readout Modes

In figures 3.13 to 3.15, the Fried parameter has been calculated from FITS files recorded in all three modes. We can see that the results from the fast readout modes compare favourably with those from the slow readout mode. Note that a zenith correction has also been applied by multiplying the variance in either direction by $\cos \gamma$, where $\gamma$ is the zenith distance of the target star.
Chapter 3: Spatial Structure of Turbulence

Figure 3.14: Comparing measurements of Fried’s parameter $r_0$ calculated from FITS files recorded in all three readout modes - slow2d, fast1d and fast2d - on 1st March 2004.

Figure 3.15: Comparing measurements of Fried’s parameter $r_0$ calculated from FITS files recorded in all three readout modes - slow2d, fast1d and fast2d - on 2nd March 2004.
3.4 Observed Coherence Length at Lords Bridge

3.4.2 Behaviour of $r_0$ at COAST

As temporal information can only be gleaned from files recorded in the fast modes, the slow mode was no longer used once the fast modes could be trusted. The fast2d mode has less resolution than the fast1d mode but this does not seem to adversely affect the measurements of $r_0$, as shown in the following figures. Hence, both fast modes can be trusted to measure the temporal behaviour of the atmosphere, which is the subject of the following chapter. We can also study the spatial behaviour of the atmosphere at COAST from the figures. On some nights, the atmosphere is stable and Fried’s parameter changes little over the course of the night. On 25th February $r_0$ only varies over a range of 29% of the average $r_0$ on that night. From the 19th April, when more files were recorded in an hour, we can see the atmosphere begins to settle after a while. This contrasts with the behaviour on other nights. We can see that Fried’s parameter changes by a maximum of 55% of the average $r_0$ on 22nd April and $r_0$ changes even more rapidly on the 27th May - every 15 minutes or so. Overall, Fried’s parameter does not seem to change by more than 55% of the average $r_0$ on each night of observation.

![Figure 3.16](Image)

Figure 3.16: Measurements of Fried’s parameter $r_0$ calculated from FITS files recorded in fast1d and fast2d readout modes on 25th February 2004.
Figure 3.17: Measurements of Fried’s parameter $r_0$ calculated from FITS files recorded in fast1d and fast2d readout modes on 19th April 2004.

Figure 3.18: Measurements of Fried’s parameter $r_0$ calculated from FITS files recorded in fast1d and fast2d readout modes on 22nd April 2004.
3.4 Observed Coherence Length at Lords Bridge

Figure 3.19: Measurements of Fried’s parameter $r_0$ calculated from FITS files recorded in fast1d and fast2d readout modes on 27th May 2004.

Figure 3.20: Fried’s Parameter values recorded at 500nm during a seeing campaign between February and May 2004.
Chapter 3: Spatial Structure of Turbulence

3.4.3 Overall seeing campaign

Finally, the results of the 4 month seeing campaign at COAST are presented in figure 3.20. Fried’s parameter has been calculated from files recorded in all three readout modes and corrected for zenith as well. The median seeing at COAST was 4.9 cm at 500 nm over this period.

There are many publications on the characteristic seeing at other astronomical sites around the world. In table 5.4, the values for Fried’s parameter at various sites around the world are given at a wavelength of 500 nm.

<table>
<thead>
<tr>
<th>Location</th>
<th>Dates</th>
<th>Median $r_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albuquerque</td>
<td>1987 – 1990</td>
<td>6.0 [76]</td>
</tr>
<tr>
<td>La Silla</td>
<td>1987 – 1997</td>
<td>10.7 [61]</td>
</tr>
<tr>
<td>Paranal</td>
<td>1987 – 1997</td>
<td>14.7 [61]</td>
</tr>
<tr>
<td>Devasthal</td>
<td>1999</td>
<td>9.2 [57]</td>
</tr>
<tr>
<td>Mt Wilson</td>
<td>1986 – 1990</td>
<td>12.9 [76]</td>
</tr>
<tr>
<td>South Pole</td>
<td>1995</td>
<td>6.4 [41]</td>
</tr>
<tr>
<td>Maidanak, Uzbekistan</td>
<td>1996 – 1999</td>
<td>14.7 [23]</td>
</tr>
<tr>
<td>La Palma</td>
<td>1990</td>
<td>12.0 [46]</td>
</tr>
</tbody>
</table>

Table 3.2: Median values of Fried’s parameter measured at 500nm at sites around the world.

The lower $r_0$ values measured in New Mexico and the South Pole were found to be due to a highly turbulent boundary layer associated with a strong temperature inversion. As was expected, the COAST site, with a median of 4.9 cm, does not compare favourably with other astronomical sites around the world. The seeing may not be so poor that the site is unusable but it does explain why the visibilities measured by the interferometer are so low. The aperture size typically used at COAST is 16cm, which is too large a multiple of $r_0$ in order to maximise the signal-to-noise ratio at COAST[12]. Too large an aperture allows more photons in, which may reduce the photon noise but the phase variance across the aperture increases and this then causes a reduction in fringe visibility even though you have increased the light level.
3.5 Conclusions

The ability of a seeing monitor to measure the characteristic seeing at any site depends on the performance of its detector. The detector must be able to read out sub-frames with a cycle time less than the coherence time of the atmosphere in order to measure the coherence time. To test new readout modes, a unit telescope of the COAST interferometer was adjusted to form a DIMM system. This was an attractive design because the telescope already had a well tested and accurate tracking system. This setup allowed comparisons with the autoguider and FWHM of long exposure images in order to test the reliability of the DIMMWIT measurements.

3.5.1 Seeing Monitors for Interferometers

Setting up a seeing monitor that is part of an interferometer is straightforward and is a simple matter of using the acquisition camera of one of the unit telescopes as a DIMM. The only problem with this arrangement is that the unit telescope then stands alone and cannot be part of the normal array used for observations. This restricts the operation of the interferometer if the observer wishes to make seeing measurements at the same time as normal observations. Hence, other interferometers may only be interested in this design if losing a telescope in the array is acceptable. For example, they may wish to observe a target using the rest of the array while simultaneously measuring the seeing conditions in order to see the effect on the mean visibility. If the array cannot operate without all telescopes, it would be an inefficient use of observing time to change back the function of one unit telescope from acting as DIMM back to being part of the array of telescopes throughout the night. The most efficient method of operating the DIMMWIT in this fashion is to adjust the angle of the incoming beam so that it falls on the middle of both a fast1d and fast2d readout box (see chapter 2). So, there is no need to readjust the position of the image and, with the autoguider running in fast guiding mode, the images stay within the readout regions for long periods of time. The observer has only to select the mode and length of a file to be recorded. In addition, if the observer is interested in data from only one type of readout mode, they can choose to record as many files as they like in succession and leave the entire instrument alone for an hour or so.
3.5.2 Performance of the DIMMWIT

Although an attempt was made to compare the performance of the DIMM to another instrument, the COAST autoguider, the results were not satisfactory. Deriving Fried’s parameter from autoguider data is not as straightforward as deriving it from DIMM data and so it is not clear which instrument is responsible for any disagreement between the autoguider and DIMMWIT measurements. The autoguider $r_0$ was consistently higher than the DIMMWIT values by 10% to 50% on all nights except the 1st of March 2004. The new fast readout modes compared favourably with the conventional slow mode and the end result is that an observer can decide which mode gives the information they need and simply choose only one mode in which to measure the seeing conditions. Finally, the standard comparison with long exposure measurements was carried out and the DIMMWIT values for Fried’s parameter agreed with the long exposure values on both nights. While the DIMMWIT FWHM were 30% larger on average on the 29th of March, they were only 3% larger on average on the 31st of August. The DIMMWIT sampling of $r_0$ on shorter timescales (due to the use of only fast mode files) resulted in better agreement with the FWHM of long exposure images.

3.5.3 Optimum Aperture Size at COAST

The amount of light received in one exposure at COAST is very small because of the small aperture sizes and short integration times imposed by the atmospheric perturbations. Hence, the optimum performance of an interferometer like COAST depends on minimising the impact of photon noise at these low light levels. Early literature [12, 13] on the COAST interferometer recommended aperture sizes of $3r_0$ at 500nm in order to maximise the signal-to-noise-ratio. However, it was still suggested that a smaller aperture than the optimum be used in order to minimise the effect of changing seeing conditions on the calibrating of visibility amplitudes. Further investigation [31] that included the delay in the response of the autoguider and the effect of using an annulus would suggest a lower value of $2r_0$ at 500nm as the optimum aperture size. With some knowledge of the typical values of $r_0$ at the site, it is now possible to decide if the aperture size should be selected knowing $r_0$ and if it should be changed during observations. At the present time, the normal procedure is to use a 16 cm apertures at all times, unless the seeing is poor (i.e. $r_0$ is short and less than 3cm) when 11 cm apertures are used instead. If the median seeing at COAST is about 5 cm at 500nm, then the standard apertures are about $3.3r_0$, which means the apertures are slightly larger than they need to be on average. Using larger apertures may let more light into the
system, but there will be larger phase fluctuations across the aperture which decreases the mean visibility. Also, using the 16 cm aperture will badly affect the visibilities measured when $r_0$ falls below 5 cm, a common occurrence at COAST. We can also see that the value of $r_0$ can change from 4 to 8 cm over the course of one night. In this case, the optimum performance of the instrument would require measuring the value of $r_0$ and choosing an aperture size of 11 cm for seeing above 5 cm and switching to smaller apertures (8 cm) when the values fall below 5 cm. In practise, the light levels with these small apertures might be too low and the observer would have to compromise with apertures of $3r_0$ and the resulting visibility variations that would lead to mis-calibrations.
Chapter 3: Spatial Structure of Turbulence
In astronomy, high resolution imaging has two requirements: that there be sufficient light levels in the recorded images and that the exposures be short enough so that the signal is not blurred by changes in wavefront corrugations. Hence, the performance of high angular resolution instruments is greatly constrained by the temporal variations of the atmospheric wavefront perturbations. To overcome this, it is important to know the statistical parameter that describe the time scale over which the wavefronts evolve.

This chapter explains how this parameter is estimated from DIMMWIT data. Lopez [38] first introduced the concept of differential image velocities in 1992 from which the temporal behaviour of the wavefront fluctuations can be calculated. St-Jacques [58] then refined the method for a Shack-Hartmann sensor. Analysis of DIMMWIT files is based on a simplified 2-aperture version of his method. The velocity of the turbulent layers can also be derived from differential power spectra [17].

As was outlined in chapter 2, most Differential Image Motion Monitors are not able to record the image positions at a high enough rate to be able to estimate the temporal scale of turbulence. Since the coherence time is routinely measured by the COAST interferometer, the ability of the DIMMWIT to estimate the temporal scale can be thoroughly tested by operating it at COAST. Further, no measured differential power spectra have been published, to the author’s knowledge, and so the features of spectra of DIMMWIT data are discussed and compared with theoretical predictions because it has been suggested that one can estimate the coherence time from them.
Chapter 4: Temporal Structure of Turbulence

4.1 Timescales of turbulence

According to the Taylor hypothesis of frozen turbulence, the variations of the turbulence caused by a single layer can be modelled as a frozen pattern that is blown across the aperture of the telescope by the wind in that layer. As has been described in chapter 1, there are actually several turbulent layers in the atmosphere. However, it can be assumed that this bulk motion follows Taylor’s hypothesis with an additional component due to internal changes in the layers. Hence, the total time derivative of the wavefront phase at a given point \( r \) on the ground is given by

\[
\frac{d}{dt} \phi(r, t) = \frac{dr}{dt} \cdot \partial_r \phi(r, t) + \partial_t \phi(r, t),
\]  

(4.1)

where the first term is the translational component due to Taylor’s hypothesis and the second is the dispersive component describing the evolution of the phase due to boiling. If \( v \) is the composite of the individual wind velocities of the different layers in the bulk motion and \( \Delta v \) is defined to be the rate of dispersive phase evolution, then equation 4.1 becomes

\[
\frac{d}{dt} \phi(r, t) = v \cdot \nabla \phi(r, t) + \frac{1}{r_0} \Delta v(r, t),
\]  

(4.2)

where \( \Delta v \) has the same units ms\(^{-1}\) as the wind speed and is normalised by Fried’s parameter \( r_0 \). Here one can see that the more dominant the translational term, the more valid Taylor’s hypothesis (that \( \tau_0 = r_0/v \)). Analysis of the structure functions of the translational and boiling components has shown that for short time intervals the translational component does dominate [58]. Hence, we can assume Taylor’s hypothesis is acceptable for the conditions under which DIMMWIT measurements are made.

4.1.1 Roddier’s multiple-layer turbulence model

Roddier’s model [52] differs from the model in section 4.1 by being entirely based on Taylor’s hypothesis. In this model, the starlight wavefronts are corrugated by passing through several turbulent layers, each with its own wind speed and direction. It is assumed that Taylor’s hypothesis holds for each layer but the model also allows for wavefront dispersion by assuming that there is also a turbulent mixing of the layers.
4.1 Timescales of turbulence

The spatio-temporal correlation of the wavefront is

$$B_\tau(\xi, \tau) = \langle \psi(x, t) \psi^*(x + \xi, t - \tau) \rangle. \quad (4.3)$$

This can be related to the spatial correlation function

$$B(\xi, 0) = B(\xi) = \langle \psi(x, t) \psi^*(x + \xi, t) \rangle, \quad (4.4)$$

by assuming that a wind, uniform throughout the turbulence, drives the wavefront perturbations without dispersion. Hence, the spatial covariance undergoes a simple translation:

$$B_\tau(\xi', \tau) = \langle \psi(x, t) \psi^*(x + \xi' - \tau v, t) \rangle = B(\xi' - v \tau). \quad (4.5)$$

Using this and equation 1.19 leads to an expression for the spatio-temporal correlation function of the wavefront perturbations in one turbulent layer:

$$B_\tau(\xi, \tau) = \exp \left\{ -3.44 r_{0, j}^{\frac{5}{3}} |\xi - \tau v_j|^{\frac{5}{3}} \right\}. \quad (4.6)$$

An average is taken over all the layers with a different $r_0$ and wind velocity associated with each layer (Note that layers are multiplied because of the relationship between the coherence at the output of the layer and the coherence of the input to the layer, see reference[52]):

$$B_\tau(\xi, \tau) = \prod_j \exp \left\{ -3.44 r_{0, j}^{\frac{5}{3}} |\xi - \tau v_j|^{\frac{5}{3}} \right\}, \quad (4.7)$$

which can also be written as

$$B_\tau(\xi, \tau) = \exp \left\{ -3.44 r_{0, j}^{\frac{5}{3}} \sum_j r_{0, j}^{\frac{5}{3}} |\xi - \tau v_j|^{\frac{5}{3}} \right\}, \quad (4.8)$$

It can be assumed that a layer of thickness $\delta h$ at an altitude $h$ has a refractive index structure coefficient $C_N^2(h)$. Knowing that $r_{0, j}^{\frac{5}{3}} \propto C_N^2(h_j) dh_j$ and $r_{0}^{\frac{5}{3}} \propto \sum_j C_N^2(h_j) dh_j$ gives

$$B_\tau(\xi, \tau) = \exp \left\{ -3.44 r_{0}^{\frac{5}{3}} \int \frac{C_N^2(h) |\xi - \tau v(h)|^{\frac{5}{3}} dh}{\int C_N^2(h) dh} \right\}. \quad (4.9)$$
4.1.2 Wavefront boiling time

As soon as the various layer velocities change, the correlation will decay over a time \( \Delta \tau \) called the wavefront boiling time \([53]\). In order to derive an expression for the decay time of the correlation function, the correlation function is approximated by a Gaussian function, i.e. 5/3 is replaced with 6/3 = 2. Hence, equation 4.9 becomes

\[
B_{\tau}(\xi, \tau) \simeq \exp \left\{ -3.44 r_0^2 \frac{\int C_N^2(h) |\xi - \tau \nu(h)|^2 dh}{\int C_N^2(h) dh} \right\}. \tag{4.10}
\]

and by expanding \(|\xi - \tau \nu|^2\),

\[
B_{\tau}(\xi, \tau) \simeq \exp \left\{ -3.44 r_0^2 (|\xi|^2 - 2\xi \cdot \nu \tau + |\nu|^2 \tau^2) \right\}. \tag{4.11}
\]

where

\[
\nu = \frac{\int C_N^2(h) \nu(h) dh}{\int C_N^2(h) dh} \quad \text{and} \quad |\nu|^2 = \frac{\int C_N^2(h) |\nu(h)|^2 dh}{\int C_N^2(h) dh}. \tag{4.12}
\]

Here \( \nu \) is an averaged velocity for the turbulent layers with \( \overrightarrow{\theta} \) as its direction. To calculate the variance,

\[
(\Delta v)^2 = |\nu|^2 - |\nu|^2 \tag{4.13}
\]

\[
\Rightarrow \Delta v = \left\{ \frac{\int C_N^2(h) |\nu(h)|^2 dh}{\int C_N^2(h) dh} - \left| \frac{\int C_N^2(h) \nu(h) dh}{\int C_N^2(h) dh} \right|^2 \right\}^{\frac{1}{2}}. \tag{4.14}
\]

where \( \Delta v \) is the standard deviation of the distribution of the wind velocities or the average dispersion velocity. Hence, equation 4.11 becomes

\[
B_{\tau}(\xi, \tau) \simeq B(\xi - \tau \nu) \exp \left\{ -3.44 (\Delta v)^2 \tau^2 / r_0^2 \right\}. \tag{4.15}
\]

If the wavefront boiling time \( \Delta \tau \) is defined as the value of \( \tau \) at which the temporal correlation of the speckles (which is found by integrating \( B_{\tau}^2 \) over all the frequency plane) has decayed by a factor 1/e then

\[
\Delta \tau = 0.38 \frac{r_0}{\Delta v}. \tag{4.16}
\]

However, this relation is based on the approximation of the correlation function with a Gaussian function. If the more complicated 5/3 law is analysed, the wavefront boiling time becomes \([52, 53, 54]\)

\[
\Delta \tau = 0.36 \frac{r_0}{\Delta v}. \tag{4.17}
\]
4.1 Timescales of turbulence

4.1.3 Coherence Time of Michelson Interferometry

In the case of adaptive optics or interferometry through small apertures, the exposure times are also short enough to freeze the instantaneous fringe pattern. Assuming that the spatial frequency is much smaller than $r_0/\lambda$, the correlation of two wavefronts at the same point is

$$B_\tau(0, \tau) = \exp \left\{ -3.44 \frac{\tau^\frac{3}{5}}{r_0} \frac{\int C_N^2(h) |v(h)|^\frac{5}{3} dh}{\int C_N^2(h) dh} \right\},$$

$$= \exp \left\{ -3.44 (v^* \tau/r_0)^\frac{3}{5} \right\}, \quad (4.18)$$

where $v^*$ is an average wind speed defined as

$$v^* = \left[ \frac{\int C_N^2(h) |v(h)|^\frac{5}{3} dh}{\int C_N^2(h) dh} \right]^\frac{3}{5} \quad (4.19)$$

If $\tau_0$ is the coherence time defined in chapter 1, then $B_\tau^2(0, \tau)$ decays by a factor $1/e$ when

$$\tau_0 = 0.314 \frac{r_0}{v^*}. \quad (4.20)$$

By definition, you get the useful expression [53] that

$$|\nabla| \leq v^* \leq \sqrt{|\nabla|^2 + \Delta v^2}. \quad (4.21)$$

4.1.4 Translation and Dispersion Time Scales

The two mechanisms, translation and dispersion, that are responsible for wavefront evolution have different timescales [58]. In speckle interferometry\(^1\), the evolution of the speckle patterns depend on a dispersive velocity and is mainly unaffected by bulk translation. In high-resolution imaging techniques, like interferometry, the timescale of interest is the time over which the wavefront phase error changes significantly and this depends on a translational velocity. For Michelson interferometry, the coherence time $\tau$ depends on the average wind speed $v^*$ of the turbulent layers.

\(^1\)Speckle interferometry freezes the atmosphere using a sequence of short-exposure frames to get an image at the diffraction limit of a telescope. Roddier found that a single layer model did not fit measured speckle lifetimes and instead studied the impact of a dispersion velocity among several turbulent layers.
Chapter 4: Temporal Structure of Turbulence

4.2 Measuring Wind Velocities

With this simple translation-dispersion model in mind, how can the various velocities be measured by an apparatus like the DIMMWIT? The classical approach is to infer the translation velocity from the centroid motion power spectrum, assuming Taylor’s hypothesis is completely valid. However, errors can be introduced by tracking glitches, etc that affect single image motion. There are two new approaches based on the differential image motion positions recorded by a system like the DIMMWIT. The first infers the velocity from measurements of the mean square value of the differential centroid velocity and the second infers the average velocity of the turbulent layers from the differential centroid motion power spectrum.

4.2.1 Variance of Differential Motion Velocity

Assuming Taylor’s hypothesis is valid, the variance of the motion velocity of a star image at the focus of a telescope with diameter $D$ can be calculated by considering the phase variations of the wavefront at ground level [38]. Let $c_\alpha(x, y)$ be the image centroid measured along an axis $a$ at an angle $\alpha$ to the $x$ axis as shown in figure 4.1:

$$c_\alpha(a, b) = -\frac{\lambda}{2\pi} \frac{\partial}{\partial a} \phi_0(a, b), \quad (4.22)$$

where $\phi_0(a, b)$ represents the phase on the wavefront at the ground level and $\lambda$ is the wavelength.
4.2 Measuring Wind Velocities

Taylor’s frozen wavefront corrugation is horizontally translated along the layer as described in section 4.1. So, the angular velocity of the centroids is

$$\dot{c}_\alpha(a, b) = \frac{da}{dt} c_\alpha(x, y) = \frac{da}{dt} \frac{\partial}{\partial a} c_\alpha(a, b) + \frac{db}{dt} \frac{\partial}{\partial b} c_\alpha(a, b)$$

(4.23)

If the average wind has a velocity vector $\mathbf{v} = (v, \theta)$ as shown in figure 4.2, then according to Lopez[38] and Saint-Jacques[58]

$$\frac{da}{dt} = |\mathbf{v}| \cos(\theta - \alpha) \quad \text{and} \quad \frac{db}{dt} = |\mathbf{v}| \sin(\theta - \alpha),$$

(4.24)

The quantity of interest is the structure function for the motion velocity of an image given by

$$D_\dot{c}(\mu, \eta) = \langle |\dot{c}_\alpha(a, b) - \dot{c}_\alpha(a + \mu, b + \eta)|^2 \rangle,$$

(4.25)

which is by definition related to the covariance $B_\dot{c}$,

$$D_\dot{c}(\mu, \eta) = 2(B_\dot{c}(0, 0) - B_\dot{c}(\mu, \eta))$$

(4.26)

The process is considered stationary and ergodic, so the covariance $B_\dot{c}(\mu, \eta)$

$$B_\dot{c}(\mu, \eta) = \langle \dot{c}_\alpha(a, b) \dot{c}_\alpha(a + \mu, b + \eta) \rangle$$

(4.27)

is equal to its autocorrelation function.
Chapter 4: Temporal Structure of Turbulence

The autocorrelation function is calculated from the inverse Fourier transform of the power spectral density of the angular velocity filtered by the pupil function, \( W_{\dot{\epsilon}, \text{filt}}(f_x, f_y) \), where \( f = (f_x, f_y) \) is a spatial frequency vector in m\(^{-1}\) and

\[
W_{\dot{\epsilon}, \text{filt}}(f_x, f_y) = W_{\dot{\epsilon}}(f_x, f_y) \cdot \left[ \frac{2J_1(\pi D f)}{\pi D f} \right]^2
\]

where \( f = (f_x, f_y) \) is a spatial frequency vector in m\(^{-1}\) and

\[
W_{\dot{\epsilon}, \text{filt}}(f_x, f_y) = W_{\dot{\epsilon}}(f_x, f_y) \cdot \left[ \frac{2J_1(\pi D f)}{\pi D f} \right]^2
\]

Hence,

\[
B_{\dot{\epsilon}}(\mu, \eta) = \int \int_{\infty} W_{\dot{\epsilon}, \text{filt}}(f_x, f_y) \cos(2\pi \mu \cdot f) \, df
\]

Taking into account that the atmosphere consists of many thin turbulent layers, the covariance becomes

\[
B_{\dot{\epsilon}}(\mu, \eta) = \langle \sum_{h=0}^{H} \hat{\epsilon}_\alpha(a, b) \sum_{h=0}^{H} \hat{\epsilon}_\alpha(a + \mu, b + \eta) \rangle
\]

and the covariance \( B_{\dot{\epsilon}}(\mu, \eta) \) perpendicular and parallel to the separation of the sub-apertures is found to be negligible to \( B_{\dot{\epsilon}}(0, 0) \). Hence, the structure function is found to be

\[
D_{\dot{\epsilon}}(\mu, \eta) = 0.256\lambda^2 r_0^{-\frac{5}{3}} D^{-\frac{7}{3}} [\nabla(1 + 2 \cos^2(\theta - \alpha)) + 2\Delta v^2]
\]

The covariance term \( B_{\dot{\epsilon}} \) can be neglected by assuming that the structure function \( D_{\dot{\epsilon}} \) is independent of the aperture separation, i.e. the two apertures are considered infinitely distant from each other. Thus, the final expression for the structure function of differential motion velocity is [58] [59]

\[
D_{\dot{\epsilon}} = 2\sigma_{\text{Taylor}}^2 (\cos^2(\theta - \alpha) + \frac{1}{2}) + 2\sigma_{\text{boiling}}^2
\]

where \( \sigma_{\text{Taylor}}^2 = 0.256\lambda^2 D^{-\frac{2}{3}} r_0^{-\frac{5}{3}} \Delta v^2 \)

and \( \sigma_{\text{boiling}}^2 = 0.256\lambda^2 D^{-\frac{2}{3}} r_0^{-\frac{5}{3}} \Delta v^2 \).

Here we can see that the structure function has two components: a translational component, \( \sigma_{\text{Taylor}}^2 \) that is related to an anisotropic distribution of centroid velocities and a dispersive component, \( \sigma_{\text{boiling}}^2 \) that is related to an isotropic distribution.
4.2 Measuring Wind Velocities

4.2.2 Differential Power Spectra

The theoretical temporal power spectra for differential image motion can be determined from the spatial power spectrum that has been found from the Kolmogorov theory of atmospheric turbulence. Consider a pair of circular apertures of diameter $D$ and separation $s$ observing through a phase screen $\phi(r)$ of frozen turbulence moving with velocity $v$ as described in Taylor’s hypothesis. The separation vector $s$ is at an angle of $\psi$ to the axis along which the motion is measured and the wind vector $v$ is at an angle of $\theta$ to the axis along which the motion is measured as shown in figure 4.3. The centroid of the image, $a(r)$ is related to the average phase gradient across the aperture or G-tilt as discussed in chapter 3 and the power spectrum can be calculated by applying various filters to the phase screen $\phi(r)$.

$$a(r) = G(r) \ast \phi(r)$$

where

$$G(r) = [\delta(s/2) - \delta(-s/2)] \ast P(r/D) \ast \frac{\partial}{\partial x},$$

(4.33)

where $P(r/D)$ is the aperture function for a circular aperture of diameter $D$ defined by

$$P(r/D) = \begin{cases} 
4/(\pi D^2) & \text{if } r \leq D/2, \\
0 & \text{elsewhere.}
\end{cases}$$

(4.34)
Chapter 4: Temporal Structure of Turbulence

The spatial power spectrum $W_a(f)$ of this function $a(r)$ is

$$W_a(f) = W_\phi(f) \cdot W_G(f). \quad (4.35)$$

$W_G(f)$ is G-tilt spectral filter given by

$$W_G(f) = 4 |f_x \lambda|^2 \sin^2(\pi f \cdot \delta) \left| \frac{2f_1(\pi D f)}{\pi D f} \right|^2 \quad (4.36)$$

where $f$ is the modulus of the spatial frequency vector and $W_\phi(f)$ is the two-dimensional power spectrum of $\phi$.

In the case of fully developed Kolmogorov turbulence at the near field approximation, it has been found [56] that $W_\phi(f)$ is

$$W_\phi(f) = 0.33 (2\pi)^{-\frac{5}{3}} (2\pi/\lambda^2) C_2^2 d f^{-1.5} \quad (4.37)$$

where $C_2^2$ is the structure constant of refractive index fluctuations (introduced in chapter 1) in a turbulent layer of thickness $dh$.

To determine the temporal power spectrum, the Wiener Khinchin theorem is used which states that the spatial covariance is the inverse two-dimensional Fourier transform of $W_a(f)$,

$$B_a(\rho) = \int W_a(f) \exp(2i\pi \rho \cdot f) df \quad (4.38)$$

According to the Taylor hypothesis, a frozen screen of turbulence moves past the aperture at a wind velocity, $v$. Hence,

$$B(\tau) = \langle a(r, t) a(r, t + \tau) \rangle = \langle a(r, t) a(r - v \tau, t) \rangle = B_a(v \tau) \quad (4.39)$$

and so the temporal power spectrum $w_a(\nu)$ is given by

$$w_a(\nu) = \int B(\tau) \exp(2i\pi \nu \tau) d\tau \quad (4.40)$$

$$= \int \int W_a(f) \exp(-2i\pi f \cdot v \tau) \exp(2i\pi \nu \tau) df d\tau. \quad (4.41)$$
4.2 Measuring Wind Velocities

Letting $f_\parallel$ be the component of $f$ in a direction parallel to the wind and $f_\perp$ the component in the perpendicular direction [56]:

$$w_a(\nu) = \int W_a(f) \frac{1}{\nu} \delta \left( f_\parallel - \frac{\nu}{\nu} \right) df = \frac{1}{\nu} \int W_a \left( \frac{\nu}{\nu}, f_\perp \right) df_\perp. \quad (4.42)$$

Thus, the one-dimensional temporal power spectrum is calculated from the integration of the two-dimensional spatial power spectrum along an axis perpendicular to the wind direction.

In the case of G-tilt, a simple axis rotation changes equation 4.36 to [45]

$$W_G(f) = 4\lambda^2 (f_\parallel \cos \theta + f_\perp \sin \theta)^2 \times \sin^2[\pi s(f_\parallel \cos(\theta - \psi) + f_\perp \sin(\theta - \psi))] \times \left[ \frac{2J_1(\pi D f)}{\pi D f} \right]^2 \quad (4.43)$$

and substituting $W_a(f_\parallel, f_\perp)$ gives the temporal power spectrum

$$w_a(\nu) \propto \int \left[ \frac{\nu}{\nu} \cos \theta + f_\perp \sin \theta \right]^2 \sin^2 \left[ \pi s \left( \frac{\nu}{\nu} \cos(\theta - \psi) + f_\perp \sin(\theta - \psi) \right) \right] \times \left[ \frac{2J_1(\pi D \sqrt{\nu^2/\nu^2 + f_\perp^2})}{\pi D} \right]^2 \left[ \frac{\nu^2}{\nu^2 + f_\perp^2} \right]^{-\frac{17}{6}} df_\perp \quad (4.44)$$

where the temporal frequency $\nu = \nu f_\parallel$. The integrand is then numerically analysed to determine the shape of the temporal power spectrum. For the longitudinal spectrum (i.e. parallel to the axis along which the motion is measured) $\psi = 0$ and for the transverse spectrum (perpendicular to the axis) $\psi = \pi/2$. The integrand is then evaluated for different values of $\theta$ to see how the shape of the temporal power spectrum changes for different directions of the wind speed.
In chapter 3, the idea that performing a windowing technique meant the centroids became Z-tilts, the best least-squares fit to the wavefront across the aperture, was introduced. For differential Z-tilt, the gradient and aperture averaging is replaced by the Zernike polynomial number 2 (the tilt along the x-axis) \([71]\). Hence,

\[
a(r) = Z(r) \ast \phi(r)
\]

where \(Z(r) = [\delta(s/2) - \delta(-s/2)] \ast z_2(r),\) \hspace{1cm} (4.45)

and the spatial power spectrum \(W_a(f)\) is now

\[
W_a(f) = W_\phi(f) \cdot W_Z(f),
\] \hspace{1cm} (4.46)

where \(W_Z(f)\) is given by

\[
W_Z(f) = 64f^2 \left[ \frac{\lambda}{\pi D} \right]^2 \left[ \frac{I_2(\pi D f)}{\pi f^2 D} \right]^2 \sin^2(\pi f \cdot s)
\]

\[
= 64 \left[ \frac{\lambda}{\pi D} \right]^2 [f_\parallel \cos \theta + f_\perp \sin \theta]^2
\]

\[
\times \sin^2 \left[ \pi s (f_\parallel \cos(\theta - \psi) + f_\perp \sin(\theta - \psi)) \right] \left[ \frac{I_2(\pi D f)}{\pi f^2 D} \right]^2
\] \hspace{1cm} (4.47)
4.2 Measuring Wind Velocities

Figure 4.5: Log-log plot of predicted differential power spectra for wind speeds at various angles $\theta$, and image motion measured along the longitudinal ($\psi = 0$) or transverse ($\psi = \pi/2$) axis [11]. The first break appears at $v_1 \sim 0.2v/s$ and the second break appears at $v_2 \sim 0.3v/D$. Before the first break the spectrum follows a $\nu^{1.5}$ power law, between the two breaks it follows a $\nu^{-1.5}$ power law and after the second break it follows a $\nu^{-1.7}$ power law. If the separation vector is parallel to the image motion axis, the spectrum follows a $\nu^0$ law, except for when the wind direction is orthogonal to the axis along which the motion is measured.

In this case, the temporal power spectrum is determined by numerically evaluating equation 4.44 at different values of $\theta$ as shown in figure 4.5.

$$w_a(\nu) \propto \int \frac{1}{D^2} \left[ \frac{\nu}{v} \cos \theta + f_\perp \sin \theta \right]^2 \sin^2 \left[ \pi s \left( \frac{\nu}{v} \cos(\theta - \psi) + f_\perp \sin(\theta - \psi) \right) \right]$$

$$\times \left[ J_2 \left( \frac{\pi D}{\sqrt{\nu^2/v^2 + f_\perp^2}} \right) \right]^2 \left[ \frac{\nu^2}{v^2 + f_\perp^2} \right]^{-23/6} df_\perp \tag{4.48}$$

The temporal power spectrum of differential image motion has been numerically analysed by other authors [17] [45] [16]. There are two characteristic breaks in the differential power spectrum, regardless of whether G-tilt or Z-tilt is being measured. They are caused by spatial averaging over the aperture and over the separation of the apertures. The behaviour of the Bessel functions in equations 4.44 and 4.48 at low frequencies and high frequencies determines the shape of the G-tilt and Z-tilt spectra.
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For low frequencies, the first term of the power series expansion of the Bessel function provides a reasonable approximation[74]

\[ J_j(z) \approx \frac{(1/2z)^j}{j!}. \] (4.49)

At high frequencies the Bessel function can be approximated by

\[ J_j(z) = \sqrt{\frac{2}{\pi z}} \cos \left( z - \frac{1}{2} j \pi - \frac{1}{4} \pi \right) \] (4.50)

Hence, below the second break at \( \nu_2 \sim 0.3v/D \), the spectrum follows a power law \( \nu^{-\frac{2}{3}} \) and above the second break it follows a \( \nu^{-\frac{11}{3}} \) power law for G-tilt and a \( \nu^{-\frac{17}{3}} \) for Z-tilt.

Below the first break, \( \nu_1 \sim 0.2v/s \), the spectrum follows a power law \( \nu^0 \). The shape of the spectrum at frequencies less than \( 0.2v/s \) relies heavily on the assumption of frozen turbulence and this behaviour would not be expected in practice. If the separation of the two apertures is parallel to the axis along which the image motion is measured, then the low-frequency spectrum follows a \( \nu^0 \) law, except for when the wind direction is orthogonal to the axis along which the motion is measured. The \( \nu^0 \) law also occurs when the separation vector is perpendicular to the axis of measurement and the wind direction is at 45° to the axis along which the motion is measured.

In reality there are multiple turbulent layers and not one single layer. The spectrum from each individual layer will have breaks at slightly different frequencies according to the conditions in that layer. Hence, if the observed light has passed through many layers, the breaks are smeared out as all the individual spectra overlap. The two characteristic break frequencies could thus appear as one break in the mean layer spectrum at a frequency somewhere between the two theoretical frequencies.
4.3 Calculating Differential Velocities

The method of deriving wind speeds from differential image motion requires two dimensional information in order to separate translational and dispersive effects, which means only the fast2d fits files described in chapter 2 can be used. This analysis is included in the same software package that calculates Fried’s parameter values from the fast2d files.

4.3.1 Application to DIMMWIT measurements

Section 4.2.1 described the theoretical model for differential velocities. Now we must analyse the real data recorded by the DIMMWIT. The variance of the difference in image velocity

\[ D\dot{c} = \left\langle (\dot{c}_{\alpha(0)}(t) - \dot{c}_{\alpha(1)}(t))^2 \right\rangle_t \]  

is evaluated for the two apertures 0 and 1 separated by a distance \( d \). Let \( \Delta c \) denote the two-dimensional difference in centroid displacement over a time \( \Delta t \) (which is the inverse of the frame rate):

\[
\Delta c(t) = \Delta c_x(t)i + \Delta c_y(t)j = \\
\left[ (c_{x(0)}(t+\Delta t) - c_{x(0)}(t)) - (c_{x(1)}(t+\Delta t) - c_{x(1)}(t)) \right] i \\
+ \left[ (c_{y(0)}(t+\Delta t) - c_{y(0)}(t)) - (c_{y(1)}(t+\Delta t) - c_{y(1)}(t)) \right] j, \tag{4.52}
\]

where \( i \) and \( j \) are two unit vectors defining the x and y axis respectively. For a particular value of the velocity measurement angle \( \alpha \), \( D\dot{c}(\alpha) \) is given by

\[
D\dot{c}(\alpha) = \frac{1}{N\Delta t^2} \sum_t |\Delta c(t)|^2 \cos(\alpha - \arctan(\Delta c_y(t)/\Delta c_x(t))), \tag{4.53}
\]

where \( N \) is the number of frames used in the calculation (8000 in this case). The result is a peanut shaped function of \( \alpha \) where \( \alpha \) lies in the range 0 to \( 2\pi \). A least square fit of this data to equation 4.32, allowing the parameters \( \sigma_{\text{Taylor}}^2 \), \( \sigma_{\text{boiling}}^2 \) and \( \theta \) to vary, will give the average wind speed \( \dot{v} \) of the turbulent layers, the direction in which the wind is blowing \( \dot{\theta} \) and the wind dispersion speed \( \Delta v \) of the turbulent layers. Note that the average \( r_0 \) of both transverse and longitudinal directions is used to calculate the velocities. A typical peanut shaped \( D\dot{c} \) function is shown with the fit to the model in figure 4.6.
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Figure 4.6: Example of fit to Equation 4.51 of the differential centroid velocity variance $D_c(\alpha)$ computed from DIMMWIT data. The results of the fit were 5 m/s wind speed at a direction of 283° and a wind dispersion speed of 4 m/s. The anisotropic Taylor component is peanut shaped and the isotropic boiling component is circular. If the dispersion velocity is higher than the average velocity then the resulting $D_c$ function is closer to circular rather than peanut shaped. Here, the central width of the peanut suggests that the dispersion velocity and average velocity were of similar scale. Theta then determines the 'slant' from the vertical axis of the peanut shape.

4.3.2 Error Analysis

The variance of the differential velocities is affected by the readout noise of the CCD in the same manner as the differential positions. From equation 4.51, we have

$$\langle \Delta^2 D_c \rangle = 2\langle \Delta^2 \dot{c} \rangle = 2\left(\frac{\Delta^2 \dot{c}}{\Delta t^2}\right) = 2v^2\langle \Delta^2 \dot{c} \rangle = 2v^2(2\sigma_R^2) \Rightarrow \langle \Delta^2 D_c \rangle = 4v^2\sigma_R^2, \quad (4.54)$$

where $v$ is the frame rate. The variance of the differential velocities due to readout noise is calculated in a similar manner and subtracted from each value of $D_c(\alpha)$ before fitting to the model. As this error is isotropic, it will only affect $\sigma_{\text{boiling}}^2$ and hence $\Delta v$.

In addition, the assumption that the correlation of the velocity of the two images is negligible has been found[39] to lead to an error of 4% in the estimate of the $\sigma_{\text{Taylor}}^2$ component and an error of plus or minus 15 degrees in the estimate of the direction $\theta$. Hence, there is an error of 11% (including the error in the measured $r_0$, in the estimate of the average wind velocity of the turbulent layers, $\mathbf{v}$.)
4.4 Alternate Methods of Estimating Wind Speeds

The method of calculating wind speeds from differential velocities has rarely been compared with the results from another seeing instrument. The temporal behaviour of turbulence can also be estimated from data recorded at COAST, an optical interferometer. At COAST, the ‘characteristic coherence time’ $\tau_c$ is defined as the length of time taken for the autocorrelation of a datastream containing fringes to fall by a factor of 2, or the length of time over which the fringes remain coherent [9]. This coherence time is estimated from the inverse of the width of the peak in the power spectrum (see section 6.2.1 for explanation of power spectrum) that is calculated by taking the Fourier Transform of the signal data recorded by COAST, i.e. $\tau_c = \frac{\sqrt{2 \log 2}}{2\pi \sigma_v}$ (where $\sigma_v$ is the width of the fringe peak where FWHM = $2\sigma_v \sqrt{2 \log 2}$). The error in the coherence time is calculated from the scatter among estimates from different segments of the fringe data file. The relationship between the COAST coherence time and the coherence time of a Michelson interferometer, defined in section 4.1.3, is [9]

$$\tau_0 = \frac{0.530}{0.441} \tau_c. \quad (4.55)$$

Hence, the mean characteristic velocity of the turbulent layers can be derived from the coherence time of the COAST interference fringes using equation 4.20.

4.4.1 Comparing DIMMWIT and COAST wind speeds

Equation 4.21 [39] relates Michelson interferometry and differential image motion. The characteristic velocity derived from COAST interference fringes can be compared with the average wind speed and dispersion speed calculated from DIMMWIT data. The value of $r_0$ used in equation 4.20 is that recorded by DIMMWIT data at the same time. COAST fringes were recorded on 10 nights of DIMMWIT observations and there were 6 nights where there was a good match between COAST and DIMMWIT data, i.e. the COAST wind speeds (calculated from COAST coherence times) lay in between the translational wind speed and the combination of translational and dispersive wind speeds measured by the DIMMWIT. Two examples are shown in figures 4.7 and 4.8. The lines are the upper and lower limits for $v^*$ given in equation 4.21 and the dots are the values of $v^*$ calculated from COAST fringe data with the associated error bars.
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Figure 4.7: Wind speeds derived from the coherence time of interferometric fringes recorded by COAST are compared with those recorded by the DIMMWIT on 29th February 2004.

Figure 4.8: Wind speeds derived from the coherence time of interferometric fringes recorded by COAST are compared with those recorded by the DIMMWIT on 19th April 2004.
4.4 Alternate Methods of Estimating Wind Speeds

On 4 of these 10 nights, the wind speeds measured by the DIMMWIT did not fall on either side of the velocity from the coherence time of COAST fringes. On 24th May and 2nd March, the coherence time was on average 7.5ms and 6.6ms respectively, according to COAST measurements, and the translational wind speeds measured by the DIMMWIT were higher than the COAST wind speeds instead of lower (the average DIMMWIT translational wind speed was 1.1 and 1.5 times the COAST wind speeds). On the 23rd and 25th February, the coherence time was on average 4.9ms and 4.3ms respectively and on both nights, the translational wind speed was higher than the COAST wind speeds instead of lower (the average translational wind speed was 2.2 and 1.1 times the COAST wind speed). In order to compare the two sets of data, the \( r_0 \) values were ‘binned’ around the time of each coherence measurement in 10 second bins. This extrapolation of what \( r_0 \) might have been at that time seemed satisfactory on all nights - the spatial seeing did not appear to change significantly over a 10 second period. One possible explanation for the DIMMWIT overestimating the wind speed is that the seeing was different for the COAST baselines and the DIMMWIT. The DIMMWIT was attached to a COAST unit telescope on a long arm of the array, while the coherence time measurements were made on a short arm of the array.

The image may have moved around significantly within the 3ms sampling time of the DIMMWIT so that the monitor did not record that motion and the coherence time thus appears shorter. Hence, the DIMMWIT would underestimate the wind speeds. In order to assess if the DIMMWIT underestimated the wind speed because of its sampling time of 3 ms, the analysis of the image positions was repeated using every other frame. The ratio of the velocities with 3ms sampling time to the velocities with 6ms sampling time could then be used to judge the validity of each data point. Thus, only valid data points below a critical ratio would be used in the comparison with the COAST coherence times. In order to determine the critical ratio, the ratios of each data point were plotted on the graph comparing COAST and wind speeds. The highest ratio for data points that obeyed the inequality could then be identified by studying plots of all observations. This ‘critical ratio’ could then, in principle, be used to identify cases where the sampling time turned out to be too short. However, all of the fast2d measurements during this campaign were made when the coherence time was sufficiently long that the velocities weren’t underestimated. Hence, the value of this critical ratio could not be determined.
4.5 Observed Time Scales at Lords Bridge

As already stated in chapter 3, the seeing campaign covered four months from February to May 2004 at the COAST site. Fast readout mode files were recorded on all 13 useful nights from this period allowing measurements of the wind speed at the site as well as Fried’s parameter. The image motion spectra are dealt with first by comparison with the COAST autoguider single image motion spectra, before presenting the differential image motion spectra and discussing the possibility of deriving wind speeds from these spectra. Finally, the wind speeds derived from fast2d files using the differential velocity method are presented in the form of a histogram to allow an estimation of the coherence time at the COAST site.

4.5.1 Comparing DIMMWIT and Autoguider Single Image Motion Spectra

Before the differential image motion spectra are presented in the following section, the single image motion spectra can be compared with autoguider image motion spectra to test the performance of the DIMM. The COAST autoguider has already been described in chapter 3. The suite of software that analyses the motion of the image in a quadcell contains a program that fits power laws to the spectrum of the image motion. In theory, low frequency behaviour of single image motion follows $\nu^{-\frac{2}{3}}$ and high frequency follows $\nu^{-\frac{11}{3}}$ [14, 70, 60]. Spatial averaging over the aperture then causes a turnover in the spectrum at a break frequency $\nu_{\text{aperture}} \sim 0.2v/D$, where $v$ is the wind speed and $D$ is the diameter of the aperture. Because the autoguider and DIMMWIT systems have different sized apertures, it is the wind speeds that each spectrum measures that must be compared and not the break frequencies. As an example, the four single image spectra from fast2d files and two single image spectra from fast1d files recorded on 16 April 2004 are shown in figure 4.9 and figure 4.10. Here, image 0 refers to the first image in the readout region and image 1 refers to the second. ‘Long’ and ‘trans’ then refer to longitudinal and transverse motion respectively. As expected, the single image spectra for the DIMMWIT image positions appear to be the same no matter what direction or what image you choose to analyse. Two power laws are then fitted to the spectrum in order to calculate the break frequency, and hence the wind speed, as shown in figure 4.11.
4.5 Observed Time Scales at Lords Bridge

Figure 4.9: All four single image spectra from fast2d files recorded on 16 April 2004. There is good agreement between the different spectra so an average of the fit to each line is used as a comparison to the autoguider results.

Figure 4.10: Both single image spectra from fast1d files recorded on 16 April 2004.
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Figure 4.11: An example of a fit to the single image motion spectra from fast2d files recorded on 16 April 2004. The break frequency is 10.8Hz and the slopes are -0.9 and -1.7. For an autoguider spectrum recorded on the same night, the break frequency is 2.1Hz and the slopes are -0.83 and -3.5. The wind speed according to the DIMMWIT is 4.22m\(\text{s}^{-1}\) and according to the autoguider it is 4.24m\(\text{s}^{-1}\).

The model of two straight lines fitted to the single image motion spectra was defined by

\[
f(x) = \begin{cases} 
  a_1 x + b_1 & \text{if } x \leq x_t \\
  a_2 x + (a_1 - a_2) x_t + b_1 & \text{if } x \geq x_t
\end{cases}
\]  

(4.56)

where \(a_1\) and \(a_2\) are the slopes of the first and second straight lines in the model, \(b_1\) is the offset of the first straight line and \(x_t\) is the break frequency of the single image motion spectra. Note that a model with three straight lines was used, in order to fit a power law to the centroiding noise (see explanation in the last paragraph on the next page) at the high frequency end of the spectra. This was done to ensure that the high frequency slopes were not underestimated. However, the results of the fit were similar to the two line model. Hence, the two line model was deemed satisfactory for fitting to one image spectra.
From chapter 3, the best three nights on which the autoguider and DIMMWIT agreed on the value of Fried’s parameter have been chosen for this spectra comparison. The following tables contain the results of model fitting to the spectra from both instruments. The wind speeds are calculated using $\nu = 0.2v/D$ where the diameter of the DIMMWIT apertures are 6.4cm and the diameter of the autoguider aperture is 32cm (as outlined in section 3.3.1).

<table>
<thead>
<tr>
<th>Date</th>
<th>Type of File</th>
<th>Slope 1</th>
<th>Slope 2</th>
<th>Break $\nu$ / Hz</th>
<th>Wind Speed / m s$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.04.04</td>
<td>Fast2d</td>
<td>-0.8 ± 0.1</td>
<td>-1.7 ± 0.1</td>
<td>11.6 ± 0.6</td>
<td>3.7 ± 0.2</td>
</tr>
<tr>
<td></td>
<td>Fast1d</td>
<td>-0.95 ± 0.1</td>
<td>-1.7 ± 0.1</td>
<td>9.4 ± 0.8</td>
<td>3.0 ± 0.2</td>
</tr>
<tr>
<td></td>
<td>001 Auto</td>
<td>-0.8 ± 0.1</td>
<td>-3.5 ± 0.2</td>
<td>2.1 ± 0.1</td>
<td>3.4 ± 0.2</td>
</tr>
<tr>
<td>19.04.04</td>
<td>Fast2d</td>
<td>-0.7 ± 0.1</td>
<td>-1.8 ± 0.1</td>
<td>11.5 ± 0.7</td>
<td>3.7 ± 0.2</td>
</tr>
<tr>
<td></td>
<td>Fast1d</td>
<td>-0.8 ± 0.1</td>
<td>-1.8 ± 0.1</td>
<td>8.3 ± 0.4</td>
<td>2.7 ± 0.1</td>
</tr>
<tr>
<td></td>
<td>004 Auto</td>
<td>-0.7 ± 0.1</td>
<td>-3.5 ± 0.2</td>
<td>2.1 ± 0.1</td>
<td>3.5 ± 0.2</td>
</tr>
<tr>
<td>22.04.04</td>
<td>Fast2d</td>
<td>-0.4 ± 0.1</td>
<td>-1.5 ± 0.1</td>
<td>6.1 ± 0.3</td>
<td>2.0 ± 0.1</td>
</tr>
<tr>
<td></td>
<td>Fast1d</td>
<td>-0.4 ± 0.1</td>
<td>-1.5 ± 0.1</td>
<td>8.3 ± 0.4</td>
<td>2.7 ± 0.1</td>
</tr>
<tr>
<td></td>
<td>001 Auto</td>
<td>-0.4 ± 0.1</td>
<td>-3.5 ± 0.2</td>
<td>1.5 ± 0.1</td>
<td>2.5 ± 0.1</td>
</tr>
</tbody>
</table>

Table 4.1: Summary of fitting power laws to the single image motion spectra derived from fast1d and fast2d files and autoguider image positions. File number '00* Auto' refers to the file used from that night of autoguider data to calculate the break frequency. Slope 1 refers to the low frequency behaviour and slope 2 the high frequency behaviour. The wind speed is then calculated from the break frequency knowing that $D$ is 6.4cm for the DIMMWIT and $D$ is 32cm for the Autoguider. The error in the DIMMWIT power spectrum is calculated from the standard deviation of the averaged spectra and the model fitting process takes this error into account.

There is fairly good agreement between the wind speeds derived from both types of spectra, especially between the fast2d spectra and the autoguider. The autoguider always follows the theoretical behaviour at high and low frequencies, while the DIMMWIT high frequency behaviour is always shallower than expected. The single image motion of the DIMMWIT data may have been affected by some small scale motion that did not affect the autoguider.

Note that the flattening of the spectra at high frequencies is due to the white noise in the centroid estimates due to photon noise and detector readout noise. The result of this ‘centroiding noise’ is the superposition of a flat horizontal line and the power spectrum of the image motion due to the atmosphere.
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4.5.2 Wind Speeds from Differential Spectra

Differential power spectra were constructed by averaging the spectra from an entire night of files to reduce the noise in the final power spectrum making any trends or breaks easier to see. The power spectra of the differential image positions in the longitudinal and transverse direction are shown in figures 4.12 to 4.14 with the results of power law fitting shown in table 4.2. A model consisting of three straight lines was used as the fit to the spectra in order to calculate the break frequencies:

\[
g(x) = \begin{cases} 
   a_1 x + b_1 & \text{if } x \leq x_1 \\
   a_2 x + (a_1 - a_2) x_1 + b_1 & \text{if } x_1 \leq x \leq x_2 \\
   a_3 x + x_1 (a_1 - a_2) + b_1 + x_2 (a_2 - a_3) & \text{if } x \geq x_2 
\end{cases}
\]  

(4.57)

where the \(a_i\) are the slopes of the first and second straight lines in the model, the \(b_i\) are the offsets of the first and second straight lines and the \(x_i\) are the break frequencies of the differential image motion spectra. Note that the third straight line in the model is used to fit a power law to the centroiding noise (mentioned in section 4.5.1) at the high frequency end of the spectra. Hence, the break frequency mentioned in table 4.2 is \(x_1\), the only apparent break frequency in the differential spectra, contrary to the two break frequencies, \(\nu_1 \sim 0.2v/s\) and \(\nu_2 \sim 0.3v/D\), predicted from the theoretical model. As discussed in section 4.2.2, the presence of multiple turbulent layers has resulted in one smeared break frequency that lies somewhere in between the two breaks of all the individual layers. In table 4.2, it has been assumed that the break frequency is \(\nu_2\) for the sake of investigating if wind speeds can be derived from power spectra.

<table>
<thead>
<tr>
<th>Date</th>
<th>Type of File</th>
<th>Slope 1</th>
<th>Slope 2</th>
<th>Break (\nu) / Hz</th>
<th>Wind Speed / (\text{ms}^{-1})</th>
<th>Ave. Diff vel, (v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.02.04</td>
<td>Fast1d l</td>
<td>-0.4 ± 0.1</td>
<td>-1.5 ± 0.1</td>
<td>19.2 ± 1.0</td>
<td>4.1 ± 0.2</td>
<td>5.7 ± 1.9</td>
</tr>
<tr>
<td></td>
<td>Fast2d l</td>
<td>-0.4 ± 0.1</td>
<td>-2.1 ± 0.1</td>
<td>40.5 ± 3.2</td>
<td>8.6 ± 0.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fast2d t</td>
<td>-0.6 ± 0.1</td>
<td>-1.9 ± 0.1</td>
<td>40.0 ± 3.0</td>
<td>8.5 ± 0.8</td>
<td></td>
</tr>
<tr>
<td>01.03.04</td>
<td>Fast1d l</td>
<td>-0.5 ± 0.1</td>
<td>-1.8 ± 0.1</td>
<td>3.5 ± 0.2</td>
<td>0.7 ± 0.1</td>
<td>2.8 ± 1.7</td>
</tr>
<tr>
<td></td>
<td>Fast2d l</td>
<td>-0.6 ± 0.1</td>
<td>-1.9 ± 0.1</td>
<td>6.1 ± 0.5</td>
<td>1.3 ± 0.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fast2d t</td>
<td>-0.6 ± 0.1</td>
<td>-1.9 ± 0.1</td>
<td>6.1 ± 0.6</td>
<td>1.3 ± 0.1</td>
<td></td>
</tr>
<tr>
<td>14.04.04</td>
<td>Fast1d l</td>
<td>-0.6 ± 0.1</td>
<td>-1.8 ± 0.1</td>
<td>9.8 ± 0.5</td>
<td>2.1 ± 0.1</td>
<td>3.2 ± 0.7</td>
</tr>
<tr>
<td></td>
<td>Fast2d l</td>
<td>-0.6 ± 0.1</td>
<td>-1.7 ± 0.1</td>
<td>11.9 ± 0.7</td>
<td>2.5 ± 0.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fast2d t</td>
<td>-0.3 ± 0.1</td>
<td>-2.1 ± 0.1</td>
<td>31.8 ± 1.9</td>
<td>6.8 ± 0.3</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2: Wind Speeds calculated by fitting power laws to the differential image motion spectra and the average wind speed and standard deviation, calculated by the differential velocity method.
4.5 Observed Time Scales at Lords Bridge

Figure 4.12: Differential Image Motion Spectra from COAST fast1d files recorded on 25th February, 1st March, 14th April 2004.

Figure 4.13: Longitudinal Differential Image Motion Spectra from COAST fast2d files recorded on 25th February, 1st March, 14th April 2004.
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Figure 4.14: Transverse Differential Image Motion Spectra from COAST fast2d files recorded on 25th February, 1st March, 14th April 2004. Any difference between the longitudinal and transverse spectra is most apparent on the 14th April because the transverse spectrum is flatter at low frequencies than the longitudinal spectra.

Figure 4.15: Example of how two power laws are fit to the power spectra. In this example with data from the fast2d files recorded on the 1st March 2004 in the longitudinal direction, the slopes of the two power laws are found to be -0.6 and -1.9 with a break frequency at 6.1Hz giving a wind speed of $1.3\text{ms}^{-1}$.
4.5 Observed Time Scales at Lords Bridge

The shape of the longitudinal spectra is similar for fast2d and fast1d files, with only one clear break between the shallow and steep parts of the graph. The predicted zero or positive slope before the first break at low frequencies does not appear. The low frequency slopes were steeper than expected (-0.6), close to the expected slope of $-\frac{2}{3}$ after the first break. The experiment was repeated with longer segments of data, in case the original segments weren’t long enough to measure frequencies lower than 5 or 6 Hz in the spectra as the first break for an individual layer moving at 5ms$^{-1}$ would be at 5Hz. However, even with longer segments, a break was still not evident. The final part of the spectrum is much shallower than the -11/3 predicted slope, implying that something is affecting differential image motion at high frequencies. Wind speeds were calculated from the break frequencies of the spectrum assuming $\nu_2 \sim 0.3v/D$. The fast1d values were lower than the range of values derived from the differential velocity method on two of the three nights in table 4.2 while the fast2d longitudinal wind speeds lay within this range on two out of three nights and the fast2d transverse wind speeds were too high on two out of three nights. Overall, the comparison between break frequency wind speeds and differential velocity method wind speeds was poor.

4.5.3 Overall results of COAST seeing campaign

The results of a 4 month campaign at COAST are presented in the form of two histograms shown in figures 4.16 and 4.17. The wind speeds have been calculated from fast2d files recorded on 13 nights over this 4 month period. The median wind speed, $|v|^2$ is 4.5ms$^{-1}$ and the median of the average is 4.96ms$^{-1}$ (the average wind speed for each night is calculated and then the median of that is calculated). The median for the dispersion wind speed is 4.8ms$^{-1}$ with the median of the average being 5.5ms$^{-1}$. Returning to chapter 4, the wind speed $v^\ast$ and Fried’s parameter are related to the coherence time of Michelson fringes by

$$\tau_0 = 0.314 \frac{r_0}{v^\ast}, \quad (4.58)$$

and the average wind velocity of the turbulent layers $\overline{v}$ and the dispersion velocity $\Delta v$ that are measured by the DIMMWIT can be interpreted as limits on the estimate of $v^\ast$ [38] using equation 4.21

$$|\overline{v}| \leq v^\ast \leq \sqrt{|\overline{v}|^2 + \Delta v^2}. \quad (4.59)$$

Hence, the expected coherence time of interferometric fringes would be around 2.3 to 3.4ms.
Figure 4.16: Average velocity of turbulent layers $|\overline{v}|^2$ measured by the DIMMWIT over 13 nights at COAST. The median wind speed is 4.5 ms$^{-1}$ (averaged wind speed median is 4.9 ms$^{-1}$).

Figure 4.17: Dispersion velocity, $\Delta v$, measured by the DIMMWIT over 13 nights at COAST. The median wind dispersion speed is 4.8 ms$^{-1}$ (averaged wind dispersion speed median is 5.5 ms$^{-1}$).
4.6 Conclusions

As described in chapter 3, very few existing DIMMs have detectors that are capable of measuring the temporal behaviour of the atmosphere, i.e. they don’t have cycle times short enough. The setup at COAST allowed the DIMMWIT to be compared with the measurements of the COAST interferometer to prove the reliability of the temporal measurements. Only then could the DIMMWIT be transported to another site of unknown temporal behaviour in order to characterise the site with confidence.

13 nights over a 3 month campaign is not enough to characterise the site at COAST. However, calculating the coherence time of the fringes is part of the usual data analysis at the interferometer. Hence, the temporal behaviour of the site is well understood and the DIMMWIT can be compared with reliable measurements at COAST. According to Baldwin et al [3], the median coherence time of the fringes at COAST is 7ms at 830nm, which is 3.8ms at 500nm. So, the median of the 3 month campaign at COAST of 2.3 to 3.4ms is is only slightly lower than the median of a year of $\tau_0$ measurements at
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COAST. In addition, section 4.4 demonstrated the agreement between the wind speeds measured by the DIMMWIT and the coherence times measured by COAST (converted into wind speeds knowing Fried’s parameter). Overall, the DIMMWIT seems capable of measuring the temporal characteristics of the atmosphere using the differential velocity method of analysis.

The results from the differential power spectra were not as satisfactory as those from the differential velocity method. There is only one apparent break in the spectra. The first break should be around 5Hz if the average wind speed at the COAST site is indeed 5ms$^{-1}$, as measured by the differential velocity method. This break does not appear in any of the spectra, even when longer segments (8000 data points as opposed to 2000) are used to sample lower frequencies. The breaks in the single image motion spectra gave wind speeds similar to that calculated from breaks in the autoguider spectra, suggesting that the DIMMWIT spectra should give good results for wind speeds. On 2 out of 3 nights, the fast2d gave a wind speed 1.1 times the wind speed from an autoguider spectrum on the same night, and the fast1d gave a wind speed 0.7 to 0.8 times the wind speed from an autoguider spectrum. The wind speeds from the fast files did not lie within the range of error in determining a wind speed from an autoguider spectrum. However, they were judged to be close enough due to the effect of averaging the spectra over an entire night rather than analysing a few spectra recorded at approximately the same time as the autoguider file.

The breaks (assumed to be $\sim 0.3v/D$) in the differential spectra gave wind speeds that were only sometimes in agreement with the differential velocity method results (taking into account the errors in both quantities). In order to take into account the effect of averaging the spectra over an entire night, the spread of wind speeds calculated from the differential velocity method was noted. The wind speed from a fast1d file lay within the range of wind speeds on only one night, while the wind speeds from a fast2d file in the longitudinal direction lay within the range of wind speeds on all three nights presented (taking into account the error in determining the break frequency). The low frequency slopes appear to be similar to that predicted between the first and second break, while the high frequency behaviour was much shallower than expected. This implies that there is something in the instrumentation that affects both single and differential image motion at high frequencies. Since the differential velocity method has shown that there is a significant dispersive component in the turbulence at the COAST site, it implies that a more complicated model with many turbulent layers and with mixing between the layers may be needed to predict the break frequencies in differential spectra. It is hoped that this would lead to more accurate wind speed measurements from the spectra. It would also be advisable to use very long segments
of data (over 10,000 data points with maximum length DIMM files) in order to sample the low frequencies at which a first break might be expected.
Chapter 4: Temporal Structure of Turbulence
5 DIMMWIT measurements at MROI

The Magdalena Ridge Observatory is a US Department of Defence-funded project being built by the New Mexico Institute of Mining and Technology (NMT) in collaboration with the University of Cambridge (UK) and several other collaborators, and is located in the Magdalena Mountains near Socorro, New Mexico [34] [21]. Two instruments are to be built on this site - a single telescope (MROST) and a stellar interferometer (MROI). The southern edge of the ridge is suitable for an interferometer because it is flat with only small changes (3m) in elevation along any of the arms of the proposed interferometer and there is also the advantage of a large proportion of clear nights. The COAST group is a collaborator on the interferometer project and because of this partnership, the DIMMWIT became the seeing monitor at MRO in 2003. The DIMMWIT measurements were part of the Astronomical Site Monitoring System for MRO in which meteorological, environmental and sky quality data were collected to characterise the observing conditions at the site.

DIMMWIT was designed to be easily transportable to another site. At MRO, the DIMMWIT would be attached to an amateur telescope with an accompanying autoguiding system and would have to operate in conditions unlike that at COAST. On a mountain site, the DIMMWIT would have to cope with higher wind speeds than at COAST. The developments in its design for this new site are outlined here, as well as a discussion of possible designs for the continuing campaigns at MRO. The seeing campaign at MRO has been running for two years using other DIMMs. Although the DIMMWIT was established at MRO in 2003, it only became fully operational recently, measuring both Fried’s parameter and the wind speed of the turbulent layers at the site. The results of a short campaign in May 2004 are presented here along with a comparison to other weather data available at the site, a facility not available at the COAST site. Overall, the main question addressed here is the viability of the DIMMWIT in seeing campaigns at different sites around the world and the cost of equipment capable of recording good quality seeing data.
5.1 Transportable aspects of the DIMMWIT

At the COAST site, the DIMMWIT was able to take advantage of a non-amateur telescope with an autoguiding system. To operate at another site, it would have to use an amateur telescope and autoguiding system. Fortunately, both of these instruments were available at the MRO, thus avoiding the cost of ordering and transporting these vital parts of a DIMM. The old observatory was still standing at the time of my observations in the location of the MROST at the northern end of the ridge. It consisted of two domes, one of which housed a 16” Schmidt telescope and the other a 14” Celestron telescope on a Software Bisque mount (the mount provides the tracking system). An STV autoguiding system was also available. The STV camera was slotted into the guidescope on the telescope while the Starlight Xpress camera was put into the telescope eyepiece. In addition, the Celestron was fitted with a remote control focuser, which would allow the observer to change the separation of the images remotely and would minimise any backlash in the focus mechanism of the telescope. Controlling the system required four separate computers as described in chapter 2. The software for the telescope mount was run on a PC inside the observatory building and the autoguiding system was standalone with its own control box. The camera was connected to a two computer network (single-board and laptop) in order to operate remotely and to overcome timing delays in the new CCD modes. The single board computer was housed in the dome with the telescope while the user’s computer sat in the observer room. Due to a malfunction of the Celestron, a 12” Meade telescope was used to acquire the seeing data for the first few nights of the observing campaign. The Meade’s mount was controlled by a hand remote only instead of control software on a PC and the guidescope had a much shorter focal length and aperture size than that of the Celestron.
5.1 Transportable aspects of the DIMMWIT

Working with amateur telescopes does require some experience. In order to obtain optimum tracking, the telescope must first be balanced properly on its mount to avoid drift during the night. This can require a few hours work. However, once the correct balance is found, the telescope pointing will not drift significantly throughout observations for several weeks if left undisturbed. In addition, the telescope must be pointed accurately at the start of each observing run. This is easily achieved with the PC software that accompanies the mount. Selecting a target is then as simple as using a COAST unit telescope. The presence of a dome contributed significantly to the performance of the telescope. At the top of a mountain, the MRO site is hardly sheltered and a telescope would not be stable in the outdoors - the wind shake would ruin any seeing measurements made. Inside the dome, the Celestron is sheltered from strong winds and is placed on solid foundations separate from the floor of the dome. Overall, the telescope is in the optimum conditions for an amateur telescope. The Meade, unfortunately, had to rest on a tripod stand on the dome floor and so was not in as stable a position as the Celestron.

The main problem with the new set up was the autoguiding system. The difficulty lay not with the STV autoguider but with the guidescope attached to the telescope. With the Meade, the pixel scale using the guidescope was too coarse, and hence the observer could not set the guiding centre (corner of four STV pixels) accurately enough for the autoguider corrections to keep the position of the star images from the telescope in the centre of a fast2d, and in high winds, a fast1d box (see chapter 2).
Chapter 5: DIMMWIT measurements at MROI

The Celestron had a larger guidescope giving a finer pixel scale, so the autoguider was able to keep the two star images within the fast2d or fast1d boxes. However, the centre of the quad cell in the autoguider still did not correspond precisely to the centre of a fast2d or fast1d box, reducing the number of good frames in each file (i.e. images were more likely to fall outside the subframe and have to be discarded). The guidescope mount to the Celestron was not sturdy enough to adjust the position of the centre of the quad cell relative to the optical axis of the Celestron. The autoguiding system did perform well enough to acquire fast readout files, but it would have been preferable for the autoguiding system to be more sturdy and reliable. The optimum situation would have been to insert a beamsplitter before the eyepiece of the telescope to feed both the autoguider and the camera. Because the autoguider would then have pixels of a similar size to the Starlight Xpress, the pixel scales would be about the same for both, overcoming any difficulties with locking star images into place.

Figure 5.3: Layout of DIMMWIT system at MROI, New Mexico, composed of a 14” Celestron telescope on a Software Bisque mount controlled by a STV autoguiding system. The guidescope is the smaller telescope attached to the 14” body. Also shown is the computer network that controls the system.
5.2 Fried’s Parameter and Wind Speeds at MROI

A small seeing campaign with the fully operational DIMMWIT was carried out from 12 May 2004 to 29 May 2004 at the MROI site in New Mexico. Continuing this seeing campaign for 2 years would result in enough data to characterise the statistical behaviour of $r_0$ and $\tau_0$ at the site before the interferometer begins its scientific observations. In this section, the wind speeds and $r_0$ measurements are compared in order to investigate any correlation between the temporal and spatial seeing at the site and the wind speeds are also compared with those measured by the weather station at MROST, just outside the observatory. Several nights have been chosen for this purpose. The 27th and 29th of May have been chosen as examples of fairly stable seeing conditions. The 13th and 20th are examples of nights where the seeing conditions have changed drastically during the night. Finally, the overall results of the campaign are presented in the form of histograms to demonstrate the typical seeing behaviour at the site.

Figure 5.4 is a comparison of the different readout modes at the MROI site. As in chapter 3, the results from the three readout modes are in good agreement and follow the same trends. For example, when the seeing changed between 6 and 7 UT, the results from all three modes increase by the same amount.

Figure 5.4: Comparing zenith corrected measurements of Fried’s parameter $r_0$ at 500nm calculated from FITS files recorded in all three readout modes - slow2d, fast1d and fast2d - on 27th May 2004 at the site of the MROI in New Mexico.
Note that the data in figure 5.4 imply that the seeing can change substantially on very short-timescales. In the fast CCD modes, the minimum interval between seeing measurements is 24 seconds – 8000 fast2d frames or 120000 fast1d frames are used in each measurement of the seeing and this corresponds to a measurement every 24 seconds within the same fast2d or fast1d file. On the 27th May 2004, the fast2d mode shows an apparent increase from one measurement to the next of 40%±2% and the fast1d mode a decrease of 25%±1% between 5 and 6 UT. The fast1d mode also records an increase of 29%±1% from one measurement to the next between 8 and 9 UT. These figures are the maximum increases or decreases recorded. However, the average change from one measurement to the next for the whole night for fast2d measurements is 7% (error not significant) and for fast1d measurements it is 7% (error not significant). So, although there were some large changes a few times during the night, overall the changes were more slight. Similar behaviour to that seen on this night was also recorded on other nights during May as can be seen from figures 5.9 to 5.11.

5.2.1 Spatial and temporal correlation at MROI

The translational and dispersive wind speeds are inversely proportional to a coherence time $\tau$ (hence, an increase in wind speed means a shorter coherence time and vice versa). The linear Pearson correlation coefficient, $r$, is used to judge the correlation between the wind speeds measured by the DIMMWIT and the weather station as shown in figures 5.5 and 5.6.

$$r = \frac{\text{covariance of } X \text{ and } Y}{(\text{standard deviation of } X)(\text{standard deviation of } Y)} \quad (5.1)$$

The nearer the coefficient is to +1 the better the correlation. Any result close to 0 implies a weak or non-existent correlation and a negative value implies an anti-correlation (i.e. as one increases the other decreases) between the data sets. The correlation between the translational wind speed and Fried’s parameter on 28th May is -0.45, implying that a moderate anti-correlation between the temporal and spatial seeing exist. For the dispersive wind speed, the correlation coefficient is -0.38, implying a moderate anti-correlation between the time scale of boiling and the spatial scale of seeing. These observed anti-correlations imply that good temporal seeing (lower velocities) is correlated with good spatial seeing (higher $r_0$).

Figures 5.7 and 5.8 are examples of the correlation between the wind speeds measured by the weather station at MROST and the translational wind speeds measured
by the DIMMWIT. These two instruments are measuring the wind speeds of different parts of the atmosphere. The weather station measures the ground wind speed and the DIMMWIT the average wind speed of the turbulent layers above the telescope. Table 5.1 summarises the various correlation results for the seven nights of acceptable data recorded in fast2d readout mode at MROI.

<table>
<thead>
<tr>
<th>Date</th>
<th>Correlation between Weather Wind Speed and Translational Wind Speed / ms$^{-1}$</th>
<th>Correlation between Weather Wind Speed and Dispersive Wind Speed / ms$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.05.04</td>
<td>0.62 (0.1%)</td>
<td>-0.10 (not sig.)</td>
</tr>
<tr>
<td>19.05.04</td>
<td>-0.10 (not sig.)</td>
<td>0.19 (not sig.)</td>
</tr>
<tr>
<td>20.05.04</td>
<td>-0.80 (0.1%)</td>
<td>-0.20 (not sig.)</td>
</tr>
<tr>
<td>23.05.04</td>
<td>0.01 (not sig.)</td>
<td>-0.35 (30%)</td>
</tr>
<tr>
<td>27.05.04</td>
<td>0.67 (0.1%)</td>
<td>0.31 (20%)</td>
</tr>
<tr>
<td>28.05.04</td>
<td>0.20 (5%)</td>
<td>0.12 (30%)</td>
</tr>
<tr>
<td>29.05.04</td>
<td>-0.23 (20%)</td>
<td>0.40 (2%)</td>
</tr>
</tbody>
</table>

Table 5.1: Summary of correlations between weather station wind speeds and translational and dispersive wind speeds measured by the DIMMWIT. A student t-test was performed to calculate the level of significance with which the null hypothesis (i.e. no correlation) could be rejected and the results are shown in brackets (not sig. means the confidence level was under 50%).
Figure 5.5: Translational wind speeds plotted against Zenith corrected measurements of Fried’s parameter \( r_0 \) at 500nm. The correlation here is -0.45. The data points are calculated from FITS files fast2d readout mode on 28th May 2004 at 500nm using the Celestron 14” telescope and apertures of diameter 6.9cm separated by 27.3cm.

Figure 5.6: Dispersive wind speeds plotted against Zenith corrected measurements of Fried’s parameter \( r_0 \) at 500nm - The correlation here is 0.38. The data points are calculated from FITS files recorded in fast2d readout mode on 28th May 2004 at 500nm using the Celestron 14” telescope and apertures of diameter 6.9cm separated by 27.3cm.
5.2 Fried’s Parameter and Wind Speeds at MROI

Figure 5.7: Wind speeds measured by the weather station at MROST plotted against translational wind speeds measured by the DIMMWIT on 28th May 2004. The correlation is 0.20.

Figure 5.8: Wind speeds measured by the weather station at MROST plotted against translational wind speeds measured by the DIMMWIT on 27th May 2004. The correlation is 0.67.
Chapter 5: DIMMWIT measurements at MROI

There was a moderate to strong correlation or anti-correlation between the weather station and the DIMMWIT translational wind speeds on only 3 out of 7 nights. This could imply that the turbulence in layers at high altitude dominates the seeing behaviour more often than ground layer seeing but more than 7 nights of data would be needed to confirm this.

<table>
<thead>
<tr>
<th>Date</th>
<th>Translational Wind Speed / ms(^{-1})</th>
<th>Dispersive Wind Speed / ms(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.05.04</td>
<td>0.22 (5%)</td>
<td>0.05 (not sig.)</td>
</tr>
<tr>
<td>19.05.04</td>
<td>-0.05 (not sig.)</td>
<td>-0.13 (not sig.)</td>
</tr>
<tr>
<td>20.05.04</td>
<td>-0.89 (0.1%)</td>
<td>-0.45 (10%)</td>
</tr>
<tr>
<td>23.05.04</td>
<td>0.39 (10%)</td>
<td>0.34 (20%)</td>
</tr>
<tr>
<td>27.05.04</td>
<td>-0.52 (0.1%)</td>
<td>0.40 (0.1%)</td>
</tr>
<tr>
<td>28.05.04</td>
<td>-0.45 (0.1%)</td>
<td>0.38 (0.1%)</td>
</tr>
<tr>
<td>29.05.04</td>
<td>-0.07 (not sig.)</td>
<td>-0.14 (20%)</td>
</tr>
</tbody>
</table>

Table 5.2: Summary of correlations between translational and dispersive wind speeds measured by the DIMMWIT and \(r_0\) measured by the DIMMWIT.

As seen from table 5.2, the observed anti-correlations between \(r_0\) and the differential velocity wind speeds were generally statistically significant (particularly for the translational wind speed), whereas the one night of positive correlation was not really statistically significant. However, there were three nights of positive correlation for the dispersive wind speed, two of which were significant at the 0.1% level. In order to clarify whether the relationship between wind speeds and \(r_0\) is actually a correlation or anti-correlation would require significantly more data.

<table>
<thead>
<tr>
<th>Date</th>
<th>Weather Station Wind Direction</th>
<th>DIMMWIT Wind Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.05.04</td>
<td>161.4 SE</td>
<td>60.6 SW</td>
</tr>
<tr>
<td>19.05.04</td>
<td>69.7 SW</td>
<td>60.9 SW</td>
</tr>
<tr>
<td>20.05.04</td>
<td>66.3 SW</td>
<td>58.5 SW</td>
</tr>
<tr>
<td>23.05.04</td>
<td>49.9 SW</td>
<td>60.9 SW</td>
</tr>
<tr>
<td>27.05.04</td>
<td>117.2 SE</td>
<td>98.3 SE</td>
</tr>
<tr>
<td>28.05.04</td>
<td>127.8 SE</td>
<td>66.0 SW</td>
</tr>
<tr>
<td>29.05.04</td>
<td>128.7 SE</td>
<td>79.1 SW</td>
</tr>
</tbody>
</table>

Table 5.3: Summary of wind directions measured by the DIMMWIT and the weather station at MROST.
Table 5.3 shows the comparison between wind directions measured by the weather station at MROST and the DIMMWIT. On the nights where there was a strong correlation between the wind speeds, the two instruments seem to agree on a wind direction most of the time. Any differences in the median wind direction may be due to the DIMMWIT sampling of the wind speeds. If the DIMMWIT wasn’t sampling the wind speed as often as the weather station, it might lead to a different median wind direction from that measured by the weather station.

### 5.2.2 Behaviour of $r_0$ at MROI

As seen in figures 5.9 to 5.11, the nights of the 13th, 20th, and 29th of May 2004 are good examples of the varying behaviour of $r_0$ at the site. The slight reduction in $r_0$ on the 29th May may have been due to the fact that cloud had just passed by. The sudden drop in $r_0$ on the 13th may have been due to a sudden increase in wind speed, if the conditions in the turbulent layers were similar to those on the ground. The same behaviour appears on the night of the 20th May when the wind speeds start to increase around 8 UT (according to measurements made by both the DIMMWIT and the weather station) and $r_0$ begin to fall until 11 UT when the winds started to die down again and $r_0$ increases. The drop in $r_0$ when switching to another target is not a feature seen on all 14 nights in May. It is only apparent on the 13th, 20th and 23rd of May, when there was a delay between observing targets and the seeing could have changed. Other nights show conditions similar to 27th May 2004 in figure 5.4, where the target has changed but the seeing doesn’t start to decrease for a while after the new target has been acquired.

### 5.2.3 Overall seeing campaign

The results of the month long campaign at MROI are presented in figures 5.12, 5.13 and 5.14. Fried’s parameter has been calculated from files recorded in all three readout modes (where available) and corrected for zenith as well. The median value of Fried’s parameter over 14 nights is 7.1cm but if the results are analysed by taking a mean for each night, then the median value is 7.5cm. The median wind speed, $\sqrt{v^2}$, over 7 nights where good fast2d files were recorded is $6.9\text{ms}^{-1}$ and the median of the nightly averages is $8.2\text{ms}^{-1}$. The median for the dispersion wind speed is $7.9\text{ms}^{-1}$ with the median of the nightly averages being $8.2\text{ms}^{-1}$. Returning to chapter 4, the wind speed $v^*$ and Fried’s parameter are related to the coherence time of interferometric fringes.
Figure 5.9: Zenith corrected measurements of Fried’s parameter $r_0$ at 500nm calculated from FITS files recorded in fast1d, fast2d and slow2d readout modes on 29th May 2004 using the Celestron 14” telescope and apertures of diameter 6.9cm separated by 27.3cm.

Figure 5.10: Zenith corrected measurements of Fried’s parameter $r_0$ at 500nm calculated from FITS files recorded in fast1d and slow2d readout modes on 13th May 2004 using the Meade 12” telescope and apertures of diameter 6.1cm separated by 24.4cm.
5.2 Fried’s Parameter and Wind Speeds at MROI

Figure 5.11: Zenith corrected measurements of Fried’s parameter \( r_0 \) at 500nm calculated from FITS files recorded in fast1d, fast2d and slow2d readout modes on 20th May 2004 using the Celestron 14” telescope and apertures of diameter 9.7cm separated by 24.5cm.

by \( \tau_0 = 0.314(r_0/v^*) \), and the average wind velocity of the turbulent layers \( \sqrt{|v|^2} \) and the dispersion velocity \( \Delta v \) that are measured by the DIMMWIT can be interpreted as limits on the estimate of \( v^* \) [38]. So, expected coherence times might typically be 2.1 to 2.9ms sometimes at the MROST site. Weather stations have since been set up at both sites (MROST and MROI) [33] and the measurements have shown that it is one and half times windier at ground level at MROST, implying that the behaviour of the turbulent layers at MROI could be different. The seeing campaign needs to be continued of course to confirm this and to characterise the seeing all year round.
Chapter 5: DIMMWIT measurements at MROI

Figure 5.12: Zenith corrected Fried’s Parameter values recorded at 500nm during a seeing campaign of 14 nights in May 2004 at MROI, New Mexico. The median is 7.1 cm at 500nm.

<table>
<thead>
<tr>
<th>Location</th>
<th>Dates</th>
<th>Median $r_0$ /cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>MROST</td>
<td>May 2004</td>
<td>7.1</td>
</tr>
<tr>
<td>APO</td>
<td>2000-2002</td>
<td>12.6 [77]</td>
</tr>
<tr>
<td>Albuquerque</td>
<td>1987 – 1990</td>
<td>6.0 [76]</td>
</tr>
<tr>
<td>La Silla</td>
<td>1987 – 1997</td>
<td>10.7 [61]</td>
</tr>
<tr>
<td>Paranal</td>
<td>1987 – 1997</td>
<td>14.7 [61]</td>
</tr>
<tr>
<td>Devasthal</td>
<td>1999</td>
<td>9.2 [57]</td>
</tr>
<tr>
<td>Mt Wilson</td>
<td>1986 – 1990</td>
<td>12.9 [76]</td>
</tr>
<tr>
<td>South Pole</td>
<td>1995</td>
<td>6.4 [41]</td>
</tr>
<tr>
<td>Maidanak, Uzbekistan</td>
<td>1996 – 1999</td>
<td>14.7 [23]</td>
</tr>
<tr>
<td>La Palma</td>
<td>1990</td>
<td>12.0 [46]</td>
</tr>
</tbody>
</table>

Table 5.4: Median values of Fried’s Parameter measured at 500 nm at sites around the world. Magdalena Ridge Observatory compares favourably with some of these sites and it is hoped that a longer campaign would result in a higher median value of $r_0$. 
5.2 Fried’s Parameter and Wind Speeds at MROI

Figure 5.13: Average velocity of turbulent layers $|\overrightarrow{v}|^2$ measured by the DIMMWIT over 7 nights at MROI, New Mexico. The median wind speed is $6.9\text{ms}^{-1}$.

Figure 5.14: Dispersion velocity, $\Delta v$, measured by the DIMMWIT over 7 nights at MROI, New Mexico. The median wind dispersion speed is $7.9\text{ms}^{-1}$. 


Figure 5.15: Wind directions measured by the DIMMWIT over 7 nights at MROI, New Mexico. The median wind direction is 128.3°. Note that there is 180° ambiguity in the wind directions measured by the DIMMWIT.
5.3 Differential Spectra at MROI

Returning to the topic of differential image motion spectra introduced in chapter 4, would the spectra recorded at MROI with the DIMMWIT compare favourably with the predicted plots for G and Z tilts? The spectra were again calculated by adding together spectra from all the fast2d or fast1d files from one night of observing together. The spectra from fast1d files are longitudinal differential image motion spectra and are shown in figures 5.16 and 5.17. Fast1d files measure Z-tilts as outlined in chapter 4. The fast2d files have been split into transverse and longitudinal spectra as shown in figures 5.18 and 5.19. As outlined in chapter 4, the fast2d readout mode records G-tilts because of the centroiding algorithm employed. Overall, the fast1d and fast2d modes appear consistent, with similar slopes in fast1d and fast2d longitudinal spectra. Only one break frequency appears between 18 to 36 Hz which gives wind speeds of around 6 to 12 ms\(^{-1}\), the same range of wind speeds as the differential velocity method (assuming this break is the \(v_2 \sim 0.3v/D\) predicted from models). The low frequency slopes are between -0.2 and -0.4 and the high frequency slopes are between -1.0 and -2.5, similar values to those in the spectra recorded at COAST.

There is a strange dip in the transverse spectra from the 18th May. According to the DIMMWIT measurements, the wind direction was about 135° to the x-axis on 18th and the spectra have the same shape as the \(\theta_{45}, \psi_{90}\) in figures 4.4 and 4.5. The wind direction was also around 135° on the 28th, 29th, and 30th and a lot more fast2d files were available to produce the spectra but this dip feature does not reappear. Lengthening the segment of data used in overlapping the spectra (see chapter 4 for discussion on segment length) reveals a positive slope at low frequencies before this ‘dip’, contrary to theoretical predictions. This could be a result of the one turbulent layer having a dominant affect, giving a negative slope in the spectra, but the remaining layers resulting in a positive slope at lower frequencies. As was observed at COAST, section 5.2.3 shows that a significant dispersive component is characteristic of the MRO site. A more complex model simulating many turbulent layers with a strong dispersive component is needed in order to understand the shape of this particular spectrum from 18th May.
From table 5.5 it is apparent that the wind velocity (as measured by the differential velocity method) vary over quite a wide range throughout the night. Although the average wind velocity on the 27th May over the whole night of measurements was 6.1 ms$^{-1}$, there was quite a wide range (9.7 ms$^{-1}$) of velocities measured throughout the night. In addition, $r_0$ was on average 9.6 cm that night but varied over a range of 6.4 cm. Hence, averaging the power spectra would not allow a measurement of the average wind speed on a night when the temporal and spatial seeing were so unstable. For example, on 19th the temporal behaviour was more stable (range of 3.8 ms$^{-1}$) and there is closer agreement between the fast mode longitudinal break wind speeds and the differential velocity wind speed, implying that averaging did not have an adverse impact on the results.

There is a range of slopes at high frequencies in table 5.5 for the differential spectra and they do not agree with the theoretical predictions discussed earlier. A possible reason could be that the DIMMWIT did not sample the high-frequency wavefront evolution properly either at low altitude or at high altitude in the free atmosphere. The cycle time of the DIMMWIT determines how well the high frequency changes in the atmosphere are sampled. 3ms may not have been short enough to sample the atmosphere behaviour accurately if there were fast wind speeds (over 20 ms$^{-1}$ calculated assuming that the median $r_0$ is 7.1 cm, the cycle time is 3 ms and that $v = r_0$/cycle time) at various altitudes. At low altitudes, fast ground wind speeds might mean the ground layer was under-sampled and thus the high frequency part of the image motion spectrum would be affected. However, according to the weather station, the wind speeds were on average 10 ms$^{-1}$ on all 4 nights that a comparison could be made between the weather station and the DIMMWIT. In addition, a moderate correlation between the weather station wind speeds and the high frequency slopes was not statistically significant and neither was the moderate anti-correlation between the wind direction and the high-frequency slopes (we would expect a bigger effect when the wind had travelled
over the hill nearby before reaching the DIMM). So, the ground layer wind speeds are unlikely to be the cause of the range of high-frequency slopes. At high altitudes, fast winds could also impact the high frequency part of the image motion spectrum. A scan of jet stream maps (on the California Regional Weather Server) over the month of May shows that the wind speeds at 300mbar pressure altitude (which is approximately 10km) were under $60\text{ms}^{-1}$ from 19th to 24th May and in a range of $70\text{ms}^{-1}$ to $90\text{ms}^{-1}$ from around the 24th May to the 29th May. The high frequency slopes decreased from -1.52 on the 19th of May to -1.49 or -1.29 on 28th and 29th May when the high altitude wind speeds increased. This might imply that there is a relationship between the high altitude wind speeds and the high-frequency behaviour of the image motion spectra. However, the jet stream maps do not give enough detail below $60\text{ms}^{-1}$ to be able to confirm a relationship. Sampling wind speeds at high altitudes would require extending a seeing campaign at MRO to include balloon flights as well as DIMMWIT measurements. Having considered the impact of the DIMMWIT sampling rate due to fast wind speeds at various altitudes, there could also be another explanation for the range of high-frequency slopes. Local seeing in the ground layer may have been caused by site topography as a small hill was located near the observatory and this would result in possible non-Kolmogorov behaviour of the atmosphere being sampled by the DIMMWIT. It has been found\cite{37} that non-Kolmogorov behaviour of the atmosphere results in a decrease in the high frequency slopes of the phase and hence differential image motion spectra. The low frequency behaviour was found\cite{37} to be similar to that of the classic case and hence, non-Kolomogorov and Kolmogorov turbulence are indistinguishable at lower frequencies. If the seeing at the MRO site was non-Kolmogorov due to local seeing in the ground layer, there should be a correlation between the high frequency slopes of the DIMMWIT spectra and the ground wind direction. As has already been stated, there does not seem to be a definite correlation between the two at the site.
Chapter 5: DIMMWIT measurements at MROI

Figure 5.16: The longitudinal differential image motion spectra of files recorded in fast1d readout mode on nights where no fast2d files were recorded to give wind directions.

Figure 5.17: The longitudinal differential image motion spectra of files recorded in fast1d readout mode on nights where wind directions were known from DIMMWIT measurements.
5.3 Differential Spectra at MROI

Figure 5.18: The longitudinal differential image motion spectra of files recorded in fast2d readout mode.

Figure 5.19: The transverse differential image motion spectra of files recorded in fast2d readout mode.
Chapter 5: DIMMWIT measurements at MROI

5.4 Conclusions

Many DIMMs have been in operation over the years at MROST but none of the designs before the DIMMWIT were capable of measuring the temporal seeing. Neither was an interferometer already in place, as was the situation at COAST. So the wind speed of the turbulent layers above South Baldy was an unknown parameter. As expected the spatial seeing at MROI was better than that measured at COAST. The median $r_0$ is 7cm at MROI as opposed to the 5cm measured at COAST. With only one month’s data, no real comparison can be made between MROI and the sites of other interferometers around the world or with COAST. The short campaign has shown that the seeing conditions should be useful for interferometry, i.e. Fried’s parameter is 7cm or above and the coherence time is about 2 or 3ms. A 2 year campaign with the DIMMWIT would be required to confirm that these values are characteristic of the site all year round and not just in May. The optimum situation at MROI would be to install substantial domes (or at least a telescope that doesn’t shake) at the three arms of the proposed interferometer to properly characterise the seeing at the site.

5.4.1 Performance of the DIMM

A short seeing campaign may not be able to characterise the site, but it demonstrated that a transportable DIMM made of shop-bought instruments aimed at amateur astronomers is capable of measuring the temporal seeing. However, this success was reliant on the presence of a dome at the site. With the high wind speeds characteristic of this mountain site, an amateur telescope would be too badly affected by wind shake to record files in fast2d mode. The images would constantly be moved outside of the readout area on the CCD. Another important element of the success of the DIMMWIT is a well mounted telescope. It is possible that a telescope mounted on solid foundations without a dome might be able to measure the temporal seeing in calmer conditions. If wind speed measurements are not required, a flimsy tripod stand like the Meade’s mount is acceptable but it will affect the acquisition of fast1d files because the number of good quality files is reduced considerably as mentioned in section 5.3. So there would less chance of getting any temporal information out of the break frequencies in these spectra. The third important part of the system for recording fast2d files was the autoguider. An STV autoguider was a simple instrument for autoguiding the telescope. If a guidescope is the only choice of the autoguiding system, a long focal length is needed for a fine pixel scale and the guidescope must be securely fixed to the telescope tube. The wind speeds at MROI are of a similar range to those at COAST as
5.4 Conclusions

the median average velocity measured by the DIMMWIT was $6.9 \text{ms}^{-1}$, so we would expect the error due to the exposure time of the cameras (discussed in chapter 3) to be less than 10%. The aim of the seeing campaign at MROI was to assess whether DIMMWIT could be operated efficiently on a remote high-altitude site. Thus, potentially overestimating the spatial seeing by approximately 10% was deemed acceptable for our purposes. In addition, the mean measured seeing from such a small number of nights is likely to differ by more than 10% from the year-round average. Hence, in this light, the systematic error due to finite exposure time is insignificant.

5.4.2 Specifications of a DIMMWIT system

In conclusion, what are the specifications of a DIMMWIT system for future sites or the continuing campaign at MROI? The observer’s first decision must be the seeing parameters of interest. If the only parameter of interest is $r_0$, then the cost of a DIMM system is made very low by choosing not to record files in fast1d and fast2d mode. There is no need for a dome or stable foundations for the mount of the telescope as long as the wind conditions at the site are not too extreme. A simple 12 or 14” telescope can be setup at the site with no autoguiding system required unless the observer wishes to leave the instrument alone for long periods. The camera’s software can be set to acquire slow2d subframes of a large size since it does not seem to matter how quickly the frames are acquired in order to measure $r_0$, as seen from the agreement of all three readout modes in figure 5.4 (but precision is reduced). In fact, the STV comes with a setting where the image positions can be saved so that it can act as both DIMM camera and autoguiding system (the STV is capable of exposure times as short as 1ms).

If the observer wishes to measure the wind speeds, and hence the temporal seeing, on scales of 1 to 5ms, then the budget for a DIMMWIT system increases. The optimum set up for measuring the temporal seeing is to have a proper dome built on the site with the foundations for the telescope separate from the rest of the building. Such a dome may already be available, which would reduce the cost of the system. There are portable domes available that can be attached to a truck and driven from site to site. If the telescope is detached from the floor of the portable dome then this setup might suffice in reducing the wind shake and fast2d files could be recorded. (This has recently been tested and proven at MROI). The mount of the telescope not only improves the performance of the system but the ease with which it can be used. The Software Bisque mount was incredibly easy to set up and little pointing was needed each night. The telescope could then be controlled remotely, with the additional help of
a focuser control, and the observer can sit in a nice warm building a distance away (the importance of this should not be underestimated). The use of a dome and a reliable telescope mount are essential parts of the DIMMWIT in order to be able to keep the images in a box of approximately 60 arcseconds on the sky (i.e. the size of a fast2d or fast1d box) and achieve cycle times of less than 3ms, which should be adequate at most sites to measure the temporal seeing. Note that some sites may have very high winds where cycle times of 1ms or less are required in order to measure the temporal changes in the atmosphere.

The importance of the quality of autoguiding cannot be stressed enough when recording fast2d files. Using a guidescope for the tracking system is possible, but for future seeing campaigns it would be preferable for the chosen telescope to have a beam splitter so that the autoguiding system can be run using the main telescope. The autoguider cannot move during the night as it can if the guidescope is knocked and it will have a similar pixel scale to the DIMM camera (especially if the STV is the chosen autoguider). However, if a beam-splitter is used, it must be ensured that the image is in focus on the autoguider CCD or else the autoguider algorithm needs to be changed to be able to cope with two images. Having an autoguider and DIMMWIT CCD camera with the same pixel scale makes it easier to keep the images locked at the centre of the 60 arcsecond box per image. The better the ability of the system to keep the images at the centre of the 60” box, the more frames/lines of the fast2d/fast1d files can be used to calculate the seeing (if the images fall too close to the edge they are discarded), improving the results of the DIMMWIT calculations.

As for the choice of DIMM camera - the two fast readout modes were developed to get around the present limitations on CCD readout rates. However, the field of amateur cameras is ever-developing and at present there are CCD cameras that use UBS2 for faster data transfer rates (hence, faster cycle times). The cycle time would still be constrained by the size of the subframes, so the presence of a dome and autoguider would still be required to measure temporal seeing. However, the sensitivity would be better than that of the fast2d readout modes and better measurements of the wind speeds could be made in the future.

Recommended specifications of a DIMM system:

1. Maximum size of a binned pixel:
   A DIMMWIT can measure G-tilts and Z-tilts. The resolution of the binning mode must be fine enough to select a centroiding window of length the diameter of the Airy disk, \(2 \times 1.22\lambda/D\), without including any of the light from beyond the
5.4 Conclusions

first dark ring or cutting out any of the light from the centre of the Airy pattern. A simulation was used to determine that in order to have this resolution, each CCD pixel should be approximately 0.2r to 0.3r, where r is the radius of the Airy disk, i.e. the radius of the centroiding box should be at least 3 pixels. In the case of the DIMMWT, the fast2d mode pixels were 0.8r and the fast1d mode pixels were 0.2r and so the fast2d mode measured G-tilt and the fast1d mode measured Z-tilt. For the fast1d mode, the centroiding window was 11 pixels in length (an extra pixel is added to centre the window around the brightest pixel).

2. Minimum ratio of sub-aperture separation to diameter:

The differential velocity analysis assumes that the correlation of the two centroid velocities, which is a function of sub-aperture separation, is negligible. Hence, if the observer wishes to use the DIMM to measure the temporal seeing using this analysis, the sub-apertures must have sufficient separation compared to their diameter for this assumption to hold true. It has been shown [38] that for circular sub-apertures separated by five times their diameter, the correlation function contributes only 3% of the value of the structure function and hence, can be neglected. In addition, it has been found that if the sub-apertures are separated by 1.7 times their diameter, neglecting the correlation contributes an error of 4% to the measured layer velocities and an error of 15 degrees to θ [39]. In order to determine the optimum ratio of sub-aperture separation to diameter, the observer must numerically evaluate the correlation function for various ratios of sub-aperture separation to diameter. For our purposes a separation of 2 to 3 times their size was deemed sufficient to limit the error introduced by the assumption that the correlation between the two sub-apertures was negligible.

3. Maximum exposure time:

The exposure time of the camera limits how accurately the spatial and temporal seeing can be determined. If the exposure time is too long, $r_0$ will be overestimated because the variance of the differential image motion will be underestimated. Below is a table showing the systematic error in $r_0$ resulting from the exponential fits to two nights of data (see Chapter 3) where

$$\sigma^2 = A \exp(-bT)$$ (5.2)

and $A$ and $b$ vary by type of readout mode (fast1d or fast2d) and the different atmospheric conditions (such as wind speed and wind direction w.r.t. to the baseline between the two DIMM apertures). The exponential fits give the zero-exposure variance and hence the systematic error in using an exposure of 2 or 3ms instead. The values of the systematic error for the 1ms and 10ms exposure time are calculated using the fast2d exponential fit. The fractional error in Fried’s
parameter is then 0.6 times the fractional error in the variance.

<table>
<thead>
<tr>
<th>Date</th>
<th>Exposure Time /ms</th>
<th>Average Translational Wind Speed / ms⁻¹</th>
<th>Systematic Error in Variance /%</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.02.04</td>
<td>1ms</td>
<td>6.1</td>
<td>3%</td>
</tr>
<tr>
<td></td>
<td>2ms</td>
<td></td>
<td>7%</td>
</tr>
<tr>
<td></td>
<td>3ms</td>
<td></td>
<td>9%</td>
</tr>
<tr>
<td></td>
<td>10ms</td>
<td></td>
<td>26%</td>
</tr>
<tr>
<td>19.04.04</td>
<td>1ms</td>
<td></td>
<td>1%</td>
</tr>
<tr>
<td></td>
<td>2ms</td>
<td>5.1</td>
<td>2%</td>
</tr>
<tr>
<td></td>
<td>3ms</td>
<td>5.1</td>
<td>2%</td>
</tr>
<tr>
<td></td>
<td>10ms</td>
<td></td>
<td>6%</td>
</tr>
</tbody>
</table>

Table 5.6: Systematic errors due to underestimating the variance of the differential image motion because of exposure time greater than 2ms.
4. Diameter of sub-apertures:

The diameter of the sub-apertures is calculated given the magnitude of a typical target assuming that the centroid will be accurate if the light levels are 4 times the RMS of the background level in the CCD. For an optical throughput efficiency product of the detector quantum efficiency ($QE$) and the total transmission of all of the optics ($T$), the number of photoelectrons detected in the stellar image is

$$n_e = N_\gamma 10^{-M^V/2.5} 0.25\pi D^2 t_{\text{exp}} B_{nm}[QE \times T]$$

where $B_{nm}$ is the effective passband in nanometres, $t_{\text{exp}}$ is the exposure time in seconds, $N_\gamma$ is the flux from zero magnitude star ($10^4$ photon s$^{-1}$ nm$^{-1}$ cm$^{-2}$) and $M^V$ is the apparent magnitude of the target star. If the star image covers 11 by 11 CCD pixels (see point 1 above) and the peak falls on the centre of 4 CCD pixels, then each of those 4 CCD pixels will contain 8% of the total light from the star. The optimal diameter for a sub-aperture allows enough light to fall on the CCD so that 8% of the total photons equals 4 times the RMS of the background level of the CCD in order for the centroiding algorithm to be reliable. If we assume that for an unfiltered CCD $B_{nm} = 300$nm and that the throughput $[QE \times T]$ of a typical DIMMWIT setup including a beamsplitter is 0.4, then we get the following values shown in table 5.7 for the minimum diameter of the sub-apertures for an exposure time of 1ms.

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>RMS of Background Level /electrons</th>
<th>Minimum Diameter/cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>50</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>2.4</td>
</tr>
<tr>
<td>1.0</td>
<td>50</td>
<td>2.6</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>2.8</td>
</tr>
<tr>
<td></td>
<td>70</td>
<td>3.1</td>
</tr>
</tbody>
</table>

Table 5.7: Minimum diameters for the sub-apertures assuming an exposure time of 1ms, $B_{nm} = 300$nm, throughput $[QE \times T]$ of 0.4.
For the Starlight Xpress camera with a RMS of the background level of 59 electrons (in fast1d mode with an exposure time of 2ms) and a magnitude 0.6 star, such as Betelgeuse, we found that 2.4cm would have allowed in adequate light levels. However, with scintillation effects, we choose 6 to 7cm sub-apertures instead.

5. Focal length of the telescope:
From item 1, we assume that the star image must cover 11 CCD pixels (as was the case with the fast1d mode of the DIMMWIT camera) and we can use the following equation to estimate the focal length of an appropriate telescope

\[
\text{focal length (cm)} = \frac{\text{length of CCD pixel (cm)} \times 11 \times D}{2 \times 1.22 \times \lambda (\text{cm})}
\]  

(5.4)

For the Starlight Xpress HX516, \( \lambda \) is 650nm, pixels are 0.00074 cm and the sub-aperture diameter was 6.4cm. Hence, the focal length of an appropriate telescope was approximately 330cm.

6. Length of data recording:
Although the DIMMWIT system was restricted to files of length 3 to 4 minutes due to the memory capacity of the singleboard computer, another DIMM system might be capable of recording much longer files. Increasing the number of frames used to calculate \( r_0 \) can reduce the statistical error in determining Fried’s parameter. To measure \( r_0 \) with an accuracy of 10% requires 76 frames and with an accuracy of 5% requires 289 frames (see equation 3.25). In Chapter 3, the changes in \( r_0 \) over 3 minutes were found to be consistent with zero implying that the seeing was stationary over 3 minutes. For the DIMMWIT, a frame is recorded every 3 milli-seconds (in the fast2d mode) so in 3 minutes 60,000 frames are recorded and if they are indeed independent, the resulting error in \( r_0 \) would be 0.4%. In addition to reducing the error in measurements of \( r_0 \), long recordings are potentially helpful for the differential image motion spectra, as they reduce the lowest measured frequency.
6 Calibration and Seeing Measurements

An interesting application of seeing measurements is their use in calibration. For an optical interferometer like COAST the phase perturbations in the atmosphere reduce the spatial coherence of the incoming wavefronts and when the beams are combined the resulting fringe visibility amplitude is reduced. This loss can be calibrated out by normalising the target visibility with the known visibility of a nearby star. Unfortunately, if the seeing conditions are different from when the target star was observed, then the calibrator measurements are less effective.

The theoretical relationship between visibility amplitude and the Fried parameter, \( r_0 \), can be used to correct the calibrator visibilities for changes in seeing conditions and hence improve the calibration technique. The introduction of a spatial filter at COAST allowed \( r_0 \) to be calculated from visibility amplitudes. Spatial filtering is a simple way of improving the spatial coherence of the light beams in an interferometer in order to increase the visibility amplitudes. As part of its introduction, a simulation was designed to show the behaviour of spatially filtered and unfiltered visibilities as a function of seeing. The unfiltered visibilities are strongly affected by changes in seeing and this could be corrected as follows. The DIMMWIT measurements give the \( r_0 \) values at the times the target and calibrator files were recorded and can be used along with the predicted unfiltered visibilities from the simulation to correct the unfiltered calibrator visibilities for changes in seeing.

A brief introduction to optical interferometry follows with an overview of the COAST array and how visibility amplitudes are derived from the interferometric data. The simulation for predicting spatially filtered and unfiltered visibilities is also described and the concept of using DIMMWIT measurements to improve calibration is investigated.
Chapter 6: Calibration and Seeing Measurements

6.1 Optical Interferometer

An optical stellar interferometer is an instrument that measures the brightness distribution of a distant source using the degree of spatial coherence of wavefronts[64]. A basic interferometer consists of two apertures separated by a distance $\bar{B}$, also called the baseline of the interferometer. The light from both apertures is combined to form interference fringes from which we can calculate the spatial coherence function or visibility, $V$, given by

$$V = |V| \exp i\phi$$

(6.1)

where $|V|$ is the normalised modulation depth (or amplitude) of the fringe pattern given by

$$|V| = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$

(6.2)

and $\phi$ is the phase of the fringes, i.e. the position of the fringes relative to some point. This complex visibility is related to the source brightness distribution function by the van Cittert-Zernicke theorem which states that visibility measured as a function of baseline is the normalised Fourier transform of the source brightness distribution. Hence, measuring the signal from interfering light beams (i.e. the visibility amplitude and phase) allows us to image the source.

Figure 6.1: Michelson Unit Visibility or Normalised Visibility Amplitude. Michelson defined the visibility of his fringes as the apparent contrast between light and dark areas of fringes visible in his telescope eyepiece. Hence, the fringe visibility is defined by equation 6.2.
6.1 Optical Interferometer

6.1.1 Visibility amplitude and phase

To derive a mathematical expression for the interference signal, the source is assumed to be at a sufficient distance that the incoming wavefronts are planar[31]. The light path length of each of the combined beams must be of equal length for interference fringes to become visible. To compensate for the geometric delay in the light path of the one of the beams, an additional optical path, \( \delta \), is added so that the total path of both beams is equal.

![Figure 6.2](image)

In figure 6.2 the two beams are entering at an angle given by direction vector \( \vec{\theta} \) from the zenith so the path difference is \( \delta - \vec{B} \cdot \vec{\theta} \). If the total electric field of each incoming beam is given by \( A(\vec{\theta})\exp(i\omega t) \), then the total field of the combined light is

\[
E = A(\vec{\theta})e^{i\omega t} \left[ 1 + e^{ik(\vec{B} \cdot \vec{\theta} - \delta)} \right]
\]

(6.3)

and thus the intensity measured by the interferometer is

\[
I = \left| A(\vec{\theta})e^{i\omega t} \left[ 1 + e^{ik(\vec{B} \cdot \vec{\theta} - \delta)} \right] \right|^2
= A(\vec{\theta})A^*(\vec{\theta}) \left[ 2 + e^{ik(\vec{B} \cdot \vec{\theta} - \delta)} + e^{-ik(\vec{B} \cdot \vec{\theta} - \delta)} \right]
= I(\vec{\theta}) \left[ 2 + e^{ik(\vec{B} \cdot \vec{\theta} - \delta)} + e^{-ik(\vec{B} \cdot \vec{\theta} - \delta)} \right]
= 2I(\vec{\theta}) + 2I(\vec{\theta}) \text{Re} \left( e^{ik\vec{B} \cdot \vec{\theta} e^{-ik\delta}} \right)
\]

(6.4)

where \( I(\vec{\theta}) \) is the intensity of each beam \( A(\vec{\theta})A^*(\vec{\theta}) \).
Chapter 6: Calibration and Seeing Measurements

As the target is in reality an extended source, the separate intensity components from each part of the target can be added together, assuming the source is incoherent across its surface, so the total output intensity is

\[ I(\vec{B}) = 2I_{tot} + 2|FT[I(k\vec{\theta})]| \cos(k\delta - \arg(FT[I(k\vec{\theta})])) \] (6.5)

As previously mentioned, measuring the complex visibility, \( V \), for a baseline \( B \) determines a Fourier component of the normalised intensity distribution of the source, so

\[ V(\vec{B}) = \frac{FT[I(k\vec{\theta})]}{I_{tot}} \] (6.6)

Hence, equation 6.5 can be written in terms of this complex visibility function, \( V \)

\[ I(\vec{B}) = 2I_{tot} + 2|V| \cos(k\delta - \phi) \] (6.7)

Now the light is not actually monochromatic because it enters the system across a finite bandwidth and so equation 6.7 must be integrated over the spectral bandpass of the instrument with respect to the wavenumber \( k \). For a top hat bandpass of \( \Delta k \) centred on wavenumber \( k \), small enough that \( FT[I(k\vec{\theta})] \) is constant across it, the total intensity becomes

\[ I(\vec{B}) = 2I_{tot} + 2|V| \text{sinc} \left( \frac{\Delta k \delta}{2} \right) \cos(k\delta - \phi) \] (6.8)

The resulting fringe pattern (see figure 6.3) is similar to that in the monochromatic case, but with a modulation depth reduced by a factor known as the coherence envelope. The coherence envelope is the Fourier transform of the bandpass function so, for example, if the bandpass is a top-hat, the coherence envelope will be a sinc function. The amplitude of the fringe pattern at the centre of the envelope is referred to as the visibility amplitude. Some interferometers are used to measure the visibility amplitude and phase of the fringe pattern on as many baselines as possible in order to image the source by performing an inverse Fourier transform.

Figure 6.3: An interference fringe pattern of unit visibility. Here the bandpass is a top-hat so the fringe pattern is modulated by a sinc coherence envelope.
6.2 High Resolution Imaging at COAST

In practice not enough baselines can be observed, leaving gaps in the Fourier transform. In addition, measurements made by optical interferometers of the phase of the fringe pattern contains no information about the brightness distribution of the target because the fringe phase is corrupted by the atmosphere. Instead interferometers measure a closure phase, which is a combination of the phase measured on three baselines. The disadvantage of this technique is that a fraction of the phase information corresponding to the measured amplitudes is missing and this can limit the quality of the image reconstruction. Imaging is therefore a more complicated process than described above.

6.2 High Resolution Imaging at COAST

COAST (Cambridge Optical Aperture Synthesis Telescope) is an Earth-rotation optical/IR synthesis array of five telescopes, each of which can be moved between a number of fixed stations to provide interferometer baselines between 2 and 100 m. Having more than 3 telescopes means that COAST can be used to make interferometric images because phase information can be derived from the observations[10]. The Earth-rotation approach allows the visibility on a small number of baselines to be measured repeatedly as the Earth rotates, providing different baselines projected on the sky. Each unit telescope comprises of a 50 cm siderostat feeding the light from a distant source onto a 40 cm primary and a smaller secondary mirror that make up a Cassegrain telescope. The telescopes are arranged in a Y shaped configuration so that the North and South-East arms have one telescope and the South-West arm has two telescopes, only one of which can be used at a time.

The light from each telescope is fed back via fast-guiding mirrors into the central laboratory where all the beams are combined using an arrangement of beam splitters. The light paths of the beams are equalised by reflecting the beams off roof mirrors mounted on trolleys that can move along rails on a long optical table. The combined beams at each output of the beam combiner then pass through an aperture stop to match the observations to the local seeing conditions and optical filters perhaps chosen to isolate emission or absorption features in the source-spectrum before being focused by a lens onto an optical fibre which leads to an Avalanche Photo-Diode (APD) detector that increments a counter every time a photon event is detected. The sum of the photons recorded by the APDs in each integration time is the intensity of the combined beam that varies over time. The amount of path delay giving maximum intensity at the centre of a white light fringe is constantly changing in an unpredictable way due
to atmospheric turbulence. To overcome this problem, the path delay is scanned with a linear sweep so that the fringe envelope is always within the length of a sweep. Each observation typically takes between 20 to 120 seconds (10 sweeps/second typically).

### 6.2.1 Data Reduction at COAST

In order to calculate visibilities from the observations, segments of data corresponding to single sweeps of the fringe envelope past one detector are fourier transformed into power spectra that are added together to reduce noise. Summing the raw data would wash out the fringe because the fringe position is not stationary along the sweep. So, instead the power spectra from the 10t sweeps obtained in any t second observation are summed to produce a power spectrum with high signal to noise. Each power spectrum in the sum is calculated from the time-series of photon counts corresponding to a single sweep of the fringe envelope past a given APD.
6.2 High Resolution Imaging at COAST

Figure 6.5: Mean power spectra, averaged over a 99s observation of high-visibility. Note: the peak at 715Hz is due to the mean scan rate of the fringes past the detector. Y axis units are photons$^2$/Hz (arbitrary scaling).

There are two features in the typical power spectrum shown in figure 6.5 - a background level mostly due to photon noise, which is partly low frequency noise due to source scintillation, and a fringe power peak. The width of the peak is due to the optical filter used and the speed of the path length modulation, and is further broadened by atmospheric fluctuations. Note that if the period of the path modulation is kept the same and the speed of the sweep is increased (i.e. the fringes are scanned in a shorter time) then the peak will be widened in Fourier space. Estimates of the fringe power are obtained by integrating the total power under the peak and above the background. The summed power is divided by the square of the mean photon rate (mean intensity) and the square root is taken to estimate the root mean square visibility amplitude, $\sqrt{\langle V^2 \rangle}$.

6.2.2 The Need for Calibration

Atmospheric turbulence introduces both phase and intensity perturbations into incoming starlight, so by the time the light reaches the ground, some of the spatial coherence has been lost. When the light beams are combined, this loss in coherence results in a loss in visibility because different parts of the individual beams have different intensities and phases. The visibility is further reduced by phase aberrations introduced by misalignments in instrumentation such as the beam combiner, by differing overall intensities in the interfered beams and by the finite bandpass of the optical filter[10] (when not at the white light fringe). Hence the need for calibration to cancel out the effects of instrumentation and atmosphere alike. A nearby calibrator star of unit (i.e. unresolved source) or known visibility is observed before and after the target star in order to factor out the losses due to instrumentation and the atmosphere. In addition, because instrumental changes evolve slowly and atmospheric effects occur on
all timescales, each observation is averaged over a long enough time to include most of the atmospheric changes so that calibration is more effective. The visibility amplitude measurements are calibrated by normalising the source visibility (i.e. the root mean square visibility amplitude) with a similarly calculated quantity for the calibrator source.

6.2.3 Imaging a Star

Information about the optical brightness distribution of a target can be extracted from interferometric data by model fitting. The visibility curve, which is the calibrated visibility against the projected baseline length as seen by the source, is compared with various models of the brightness distribution function until an acceptable fit is found. A model of an uniformly bright circular disk, i.e. a single (non-binary) star, would have a visibility curve similar to the one shown in figure 6.6. If the source has a non-uniform distribution due to limb darkening (note that the distribution might be non-uniform due to other types of asymmetries on the surface of the star), then the visibilities at projected baselines beyond the first null of the visibility curve are lower than the model values\[22\], as shown by the limb-darkened star’s visibility curve (figure 6.6). Limb-darkening is a term used to describe the difference in intensity between the centre and limb of a stellar source caused by the finite optical depth of a stellar atmosphere \[9\]. Existing interferometers, such as COAST, have baselines long enough to sample the visibility curve beyond the first null point\[9\] which allows the presence of limb darkening to be detected.

The limb darkening model used in figure 6.6 is the standard linear limb darkening model where the distribution of brightness across the star’s disk is given by\[81\]

\[
\frac{I(r)}{I_0} = 1 - \alpha (1 - \mu(r)) \\
\mu(r) = \sqrt{1 - \frac{r^2}{r_0^2}}
\]

(6.9)

where \(r\) is the angular radius of a particular point on the surface, \(r_0\) is the radius of the target, \(I(r)\) is the brightness of the source, \(I_0\) is the brightness at the centre and \(\alpha\) is the limb darkening coefficient. Although the use of this model has appeared in many publications\[26][51][10\], the best-fit parameters for this model in the case of a strongly limb-darkened disk correspond to a brightness distribution with unphysical behaviour.
6.3 Correcting Uncalibrated Visibility Amplitudes

![Visibility Curves](image)

Figure 6.6: The visibility curves of a uniform disk star and a limb darkened star. The presence of limb-darkening can only be detected on long baselines if the diameter is unknown. It is at these projected baseline values that the departures from the uniform disk model can be detected.

Towards the edge of the stellar disk[10]. Hence, the preferred model in this situation is the Hestroffer model[28]

\[
\frac{I(r)}{I_0} = \mu(r) \alpha
\]

(6.10)

The model can be further improved for some classes of target by including a hotspot in the form of a Gaussian of fixed FWHM[84]. In this case there are 5 different model variables fitted by Bayesian analysis to the interferometric data: the diameter of the disk, the flux and position (distance and angle with respect to the centre of the underlying disk) of the hotspot and finally the limb-darkening coefficient.

6.3 Correcting Uncalibrated Visibility Amplitudes

One of the interesting targets for an optical interferometer such as COAST is Betelgeuse, a cool M-supergiant of large angular diameter. The M supergiants are characterised by low (3000K) surface temperatures, extended atmospheres and large radii. The red supergiant Betelgeuse is a particularly attractive target for high-resolution imaging experiments, because it is large enough to be resolved at the diffraction limit of conventional optical telescopes. Previous studies of Betelgeuse have detected a non-uniform optical brightness distribution, suggesting the presence of limb-darkening and hot spots on the surface[15][78][35][73]. The hot spots contribute 10-20% of the total flux from the system and both their brightnesses and locations on the stellar disk.
appears to change on time-scales of months\cite{79}. However, there has still been no consensus as to the origin of these hot spots. Hence, the need for further study. Although Betelgeuse would be resolved at the diffraction limit of large optical telescopes, atmospheric turbulence prevents direct observation of these features. However, long baseline interferometers are capable of resolving surface structure on spatial scales that are diffraction and not seeing limited. Hence, an interferometer like COAST should be capable of detecting non-uniformities in the visibility curve reduced from Betelgeuse data. The only remaining difficulty is the quality of the calibrated visibilities.

Standard calibration procedures assume that the seeing statistics have not changed between the calibrator and target star measurements. The DIMMWIT results for seeing conditions at COAST show that this is sometimes untrue. If the calibration process mentioned in section 6.2.2 has been adversely affected by changes in seeing between observing the target and calibrator star, the goodness of fit to a particular model might also be affected leading to a misinterpretation of the results. By correcting the calibration technique for changes in seeing, the scatter in visibility values might be reduced allowing a better interpretation of the results.

6.3.1 Spatial Filtering

Spatial filtering by means of an pinhole was introduced at COAST in order to lessen the effect of the atmosphere on visibility amplitudes so that the amplitudes varied less often during an observation and hence improve the calibration process. The beam’s spatial coherence is improved by filtering out the random noise component, due to the atmosphere, of a corrupted wavefront, with the additional benefit of removing static aberrations due to internal misalignments.

The filtered and unfiltered visibility amplitudes, as a function of $d/r_0$, where $d$ is the diameter of the aperture, have been simulated for a turbulent atmosphere obeying Kolmogorov statistics\cite{32}. Two apertures observe a distant, unresolved source and it is assumed that the intensity of the light incident on the apertures is equal and the optical throughput to the beam combiner from both apertures is identical. Random phase perturbations are then generated and used to produce two corrupted wavefronts of constant intensity. The tilt component of the wavefront perturbations is compensated for by a simulated fast autoguider with a fixed time delay between measurement and correction, assuming a single layer of ‘frozen’ turbulence passing at speed $v$ over the telescope. The simulated wavefronts are then spatially filtered or left unfiltered and the
6.3 Correcting Uncalibrated Visibility Amplitudes

average visibility amplitudes over a large number of random wavefront realisations calculated.

The resulting visibility amplitudes are plotted in figure 6.7 showing the difference between spatially filtered and unfiltered visibilities (root mean squared visibility). From figure 6.7, it is apparent that small changes in Fried’s parameter have a negligible affect on spatially filtered visibilities compared to unfiltered visibilities. Hence, this theoretical model could be used to correct the measured unfiltered visibilities for changes in seeing between the acquisition of a target file and a calibrator file.

The change in \( r_0 \) from target star to calibrator is known from some DIMMWIT measurements that were recorded at the same time as the fringe pattern. Hence, the calibrated visibilities \( V_{UF,T} \) recorded on the unfiltered channel can be corrected by applying a correction factor, C.F., to the unfiltered visibilities, \( V_{UF,C} \), of the calibrator files.

\[
V_{\text{calibrated,corr}} = \frac{V_{UF,T}}{V_{UF,C} \cdot \text{C.F.}}
\]

where \( \text{C.F.} = \frac{V_{UF}(d/r_0^T)}{V_{UF}(d/r_0^C)} \)  

(6.11)
Chapter 6: Calibration and Seeing Measurements

6.3.2 Putting Theory into Practise

Part of the DIMMWIT seeing campaign was designed to overlap with a collaboration between COAST and IOTA to characterise the surface structure of Betelgeuse, the late-type supergiant star mentioned previously. It was hoped that using DIMMWIT data to correct the calibrated visibilities for changes in the seeing conditions would improve the fit of the interferometric data to the model of a limb-darkened plus hotspot star, i.e. reduce the scatter in the data caused by mis-calibration.

The night of the 1st of March 2004 was chosen as a suitable night for this experiment because a large number of DIMMWIT and COAST files were recorded that night and there was good agreement between the DIMMWIT and autoguider results (see chapter 3). The COAST data were reduced as normal as described in section 6.2. The only adjustment was to alter the time-stamp on the file so that it related to the start of the observation and not the middle as is the norm for COAST data. This is just for the purpose of matching up contemporaneous measurements and doesn’t affect the estimated visibility or the calculation of the projected baseline. In order to correct the unfiltered calibrator visibilities for changes in \( r_0 \), the DIMMWIT data were binned to smooth out the \( r_0 \) values on either side of the observation. Each bin was 1.8 minutes in length from the start of the visibility observation. The resulting DIMMWIT \( r_0 \) values were then scaled to 782nm (the central wavelength of the APD filter in the unfiltered channel) as shown in figure 6.8.
6.3 Correcting Uncalibrated Visibility Amplitudes

Figure 6.8: Binned DIMMWIT $r_0$ data scaled to 782nm, the wavelength of the APD filter in the unfiltered channel. The zero values correspond to times when there are no $r_0$ values available from DIMMWIT data to correct the calibrator visibilities. When no DIMMWIT data was available the calibrator visibilities were left uncorrected. The unfiltered visibilities are shown in the second plot. A subset of these are the calibrator visibilities which could only be corrected if there was a known value of $r_0$ at the time both the target and calibrator star were observed.
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<th>Reduced $\chi^2$</th>
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<th>L-d par. $\alpha$</th>
<th>Hotspot r/mas</th>
<th>$\theta$/deg</th>
<th>Frac. flux</th>
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<td>335±13</td>
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</tr>
</tbody>
</table>

Table 6.1: Fits of models to COAST visibility data for Betelgeuse from 1st March 2004. The table compares results obtained with and without correcting the calibrated visibilities for changes in seeing as measured with DIMMWIT. The model consists of a limb-darkened disk with a single unresolved bright feature (modelled as a 5mas FWHM circular Gaussian) superimposed. Hestroffer’s limb-darkening parameterisation was used for the disk component. The diameter of the disk component was allowed to vary, then the flux and/or the position of the hotspot with respect to the centre of the star. The $\chi^2$/degree of freedom of the corrected data is about twice that of the uncorrected data due to the increase in scatter. It is hoped that on a night of badly calibrated data, the opposite would occur with the corrected data giving better fits than the uncorrected data.

The results of the correction (equation 1.11) are shown in figure 6.9. In this instance, the data were already well calibrated before applying the correction so the main effect has been to increase the scatter in the calibrated visibilities. If the data is well calibrated then it is to be expected that the noise in the DIMMWIT data adds to the noise in the calibrated visibility measurements increasing the scatter in the results. Although only one night of data is not really enough to test a best fit (because of complicated source morphology), the uncorrected and corrected calibrated visibilities were run through the model-fitting process. The parameters used in the model were taken from the results of the overall Betelgeuse campaign[84] with the Hestroffer limb-darkening parameter $\alpha$ fixed at 9.0. The other four parameters mentioned in section 6.2 (diameter of the disk, flux of the hotspot, etc) were either kept fixed or allowed to vary in order to see how good the fit was before and after the correction. As expected, the $\chi^2$ of the best model fit to the corrected data was about twice that of the uncorrected data due to the increase in scatter. The size of the bin widths were also adjusted to see if a better correction could be made to the calibrator data. However, it was found that the 1.8 minute bins gave the smallest degree of scatter as this was the minimum length to have at least one $r_0$ measurement for nearly every visibility measurement (there were a few exceptions as seen in figure 6.8).
6.3 Correcting Uncalibrated Visibility Amplitudes

Figure 6.9: Comparing the calibrated visibilities from the unfiltered channel before and after the correction for changes in seeing. The scatter in the calibrated visibilities has increased leading to a worse fit to the limb-darkening model. The $\chi^2$ of the corrected data is twice that of the uncorrected data for all values of the four variables. Note that if the seeing conditions did not change much between observing the target and calibrator, then any reduction due to seeing was already well corrected for by normalising with the uncorrected calibrator visibilities.
6.4 Conclusions

Each target visibility is divided by that expected from an unresolved source to give a calibrated measurement. In previous COAST publications[82][83], the error estimates on the calibrated visibilities had to be adjusted to take the unknown changes in seeing conditions between observing the target and the calibrator stars into account. This was not always a satisfactory method of improving the fit of the data to the various models and could have led to misinterpretations of the best-fit results.

Recent work at COAST[31] used the simulated filtered and unfiltered visibilities to correct for changes in seeing. In this case, the value of $r_0$ at the time of the observation was inferred from the theoretical ratio of the filtered and unfiltered visibilities so that the calibrator visibilities could be corrected, leading to more accurate calibrated visibilities. There were two difficulties with this method. The first was that a single anomalous calibrator observation (i.e. the observation has been effected by the instrumentation in some way very different to the other observations) could affect two adjacent science target observations. The second difficulty lay in the assumption that the filtered and unfiltered channels were affected in the same way by misalignments in the instrumentation. Hence, taking a ratio of the two would cancel out the effects of these misalignments so that the ratio was a function of Fried’s parameter only. If, in fact, this assumption was inaccurate and additional effects did not simply cancel out, then the technique would be flawed. However, this method was deemed successful during initial trials because the calibrated visibilities improved after the correction was made.

For this experiment, actual measurements of Fried’s parameter were used to correct the calibrated visibilities. It was again assumed that taking a ratio of visibilities would cancel out the effects of misalignments and differing intensities in the two interfering beams. Unfortunately, the observing campaign for Betelgeuse was too successful and the data were already well calibrated. Hence, the effect of the correction was to increase the scatter in the calibrated visibilities so that it no longer optimally fit the model. The seeing campaign did not overlap with any badly-calibrated data sets and so there was no opportunity to correct badly calibrated data. It is hoped that DIMMWIT data will be simultaneously recorded with the visibility observations in future campaigns at COAST, especially on nights where the seeing is not stable throughout the night. Hence, there is a possibility that any poorly calibrated visibility measurements could be improved using the technique outlined in this chapter.
7 Conclusions

Differential Image Motion Monitors are useful compact instruments that can reliably measure the turbulent behaviour of the atmosphere. The two parameters most relevant to the field of interferometry are the Fried parameter, $r_0$, and the coherence time, $\tau_0$, which characterise the spatial and temporal behaviour of a turbulent atmosphere. Knowledge of these parameters can then be used to optimise the performance of an interferometer and the analysis of interferometric data. The aim of my PhD project was to design a simple cheap transportable DIMM system that was capable of measuring both of these atmospheric parameters.

7.1 Performance of the DIMM

In order to test the reliability of the DIMMWIT in measuring $r_0$, the results were compared with two other methods of estimating the spatial seeing. The first method used the image positions recorded by the COAST autoguider. The autoguider $r_0$ was consistently higher than the DIMMWIT values by 10% to 50% on all nights except the 1st of March 2004. However, most of the DIMMWIT data falls within the range of error in the autoguider data and the two data sets were moderately correlated on 4 out of 5 nights when the correlation was statistically significant (i.e. t-test resulted in a better than 10% significance). Deriving the Fried parameter from autoguider data is not as straightforward as deriving it from DIMM data and so it is not clear which instrument is responsible for any disagreement between the autoguider and DIMMWIT measurements. The second method used full-width-half-maximum measurements of long exposure images to estimate $r_0$ and there was fairly good agreement between both sets of data. While the DIMMWIT FWHM were 30% larger on average on the 29th of March, they were only 3% larger on average on the 31st of August. The DIMMWIT sampling of $r_0$ on shorter timescales (due to the use of only fast mode files) resulted in better agreement with the FWHM of long exposure images on the latter night. Given the acceptable performance of the DIMMWIT when compared with these two other
methods, it was concluded that the DIMMWIT could reliably measure Fried’s parameter.

However, the performance of the DIMMWIT was found to be limited by three types of random errors: measuring the pixel scale, statistical error in determining the variance from a number of frames and the bias in the variance of the image centroids caused by centroiding errors due to readout noise. The error due to measuring the pixel scale could be reduced by increasing the number of frames used to measure the separation of the binary stars. The statistical error can be reduced to less than 1% by using over 600 frames to calculate the Fried parameter as up to 60,000 frames can be independent according to section 3.2.4. The bias in the variance of the centroids due to readout noise can be subtracted out as described in Chapter 3. In addition, wind speeds impact the degree to which the DIMMWIT is systematically limited by the exposure time of the camera. With its present design capable of exposure times of 2 to 3ms, the DIMMWIT can only measure $r_0$ with less than an estimated 12% error if the wind speeds are less than 10ms$^{-1}$ (based on the same exponential fits used to derive the results in Table 5.6 (Eq. 5.2), with the assumption that the parameter b scales linearly with wind speed). To correct for this error, it is possible to extrapolate back to the value of Fried’s parameter at zero exposure. If the wind speeds were greater than 10ms$^{-1}$ and this extrapolation were not performed, the DIMMIT would overestimate Fried’s parameter by more than 10% and the measurements would no longer be reliable. In order to overcome this limitation, a new camera or readout mode should be designed that would allow exposure times of 1ms or less. Then, the reliability of the DIMMWIT in measuring $r_0$ would no longer be limited to low wind speeds.

Before the DIMM system could be used to carry out a seeing campaign at the site of a future interferometer, its ability to measure the temporal seeing had to be confirmed. There were two types of analysis used to deduce the wind speeds, and hence coherence times, from DIMM data. The first was a differential velocity method that produced translational and dispersive wind speeds and wind direction. These results compared favourably with the coherence times measured by COAST (as part of the analysis of interferometric fringes) – on 6 out of 10 nights of data, the COAST wind speeds (calculated from COAST coherence times) lay in between the translational wind speed and the combination of translational and dispersive wind speeds measured by the DIMMWIT. The discrepancy seen on 4 of the 10 nights might be explained by the seeing being slightly different for the COAST baseline between two array telescopes and the array telescope to which the DIMMWIT was attached.

The second was a differential image motion power spectrum method, similar to one
image motion spectrum analysis of temporal seeing. The breaks (assumed to be $\sim 0.3v/D$) in the differential spectra gave wind speeds that were only in agreement with the differential velocity method results on 2 out of 3 nights (taking into account the errors in both quantities). In order to take into account the effect of averaging the spectra over an entire night, the spread of wind speeds calculated from the differential velocity method was noted. The wind speed from a fast1d file lay within the range of wind speeds on only one night, while the wind speeds from a fast2d file in the longitudinal direction lay within the range of wind speeds on all three nights presented (taking into account the error in determining the break frequency). In addition it was noted that the two breaks predicted from theory ($0.3v/D$ and $0.2v/s$) did not appear in the differential spectrum. So, the results from the differential image motion spectra were not as satisfactory as expected.

Single image motion spectra from the DIMM data were also compared with the single spectra from COAST autoguider data. The breaks in the single image motion spectra gave wind speeds similar to that calculated from breaks in the autoguider spectra, suggesting that the DIMMWIT spectra should give good results for wind speeds. On 2 out of 3 nights, the fast2d gave a wind speed 1.1 times the wind speed from an autoguider spectrum on the same night, and the fast1d gave a wind speed 0.7 to 0.8 times the wind speed from an autoguider spectrum. The wind speeds from the fast files did not lie within the range of error in determining a wind speed from an autoguider spectrum. However, they were judged to be close enough due to the effect of averaging the spectra over an entire night rather than analysing a few spectra recorded at approximately the same time as the autoguider file. So, the comparison with wind speeds from the autoguider single image motion spectra was somewhat satisfactory.

In both the single image motion and differential image motion spectra, the high frequency behaviour was much shallower than expected (-1.8 instead of $-11/3$ or $-17/3$). This implied that something in the instrumentation was affecting both the single and differential image motion at high frequencies. Since the differential velocity method has shown that there is a significant dispersive component in the turbulence at the COAST site, it implies that a more complicated model with many turbulent layers and with mixing between the layers may be needed to predict the break frequencies in differential spectra. In conclusion, it was decided that the DIMMWIT could measure the temporal seeing using the differential velocity method, while the differential spectrum method required a more complicated theoretical model in order to estimate wind speeds.

A random source of error that can affect the accuracy of the DIMMWIT measurements
is the readout noise of the CCD. However, the bias introduced by the readout noise of the CCD can be easily subtracted out as shown in Chapter 4. The accuracy with which the DIMMWIT can measure the temporal seeing is systematically limited by the assumption in the differential velocity analysis that the correlation of the two centroid velocities, which is a function of sub-aperture separation, is negligible. Neglecting the correlation contributes an error of 4% to the measured layer velocities and an error of 15 degrees to $\vartheta$ [39] if the sub-apertures are separated by 1.7 times their diameter. Hence, the error can be reduced by changing the ratio of sub-aperture separation to diameter. The atmospheric conditions, in particular the wind speed, can also impact the accuracy of the results in a similar fashion to overestimating $r_0$ in high wind speeds. The cycle time must be kept short (under 3ms or less in high wind speeds) in order to continuously sample changes in the atmospherically-corrupted wavefront. If the cycle time is too long then the DIMMWIT will not be accurately sensing the changes over time.

### 7.2 Results of seeing campaigns

The median seeing at COAST over the 4 month seeing campaign was 4.9cm at 500nm which is an acceptable value but is lower than the seeing at most other astronomical sites in the world. The median wind speed at COAST during this time was 4.5ms$^{-1}$ and the median dispersion speed was 4.8ms$^{-1}$, implying that there is a substantial amount of mixing between the layers above COAST. This wind speed implies a coherence time of 2.3 to 3.4ms at COAST, which is shorter than expected from compiled COAST fringe data (the median coherence time measured by COAST in 1997 was 4ms at 500nm[3]). The underestimation of the coherence time could be due to the behaviour of the translational speed – it does not always behave as a proper lower limit to the wind speed derived from the coherence time of interferometric fringes. It was also concluded that the optimum performance of the COAST interferometer would require measuring the value of $r_0$ during observations and choosing an aperture size of 11cm for seeing above 5cm and switching to smaller apertures (8cm) when the seeing falls below 5cm at the observation wavelength.

MROI is a next generation optical interferometer that will be built at the Magdalena Ridge Observatory in New Mexico. Although several DIMMs have been measuring the Fried parameter at the site, no previous designs have been capable of measuring temporal seeing so the coherence time was an unknown quantity. Thus, the DIMMWIT
was to be set up in order to carry out a more detailed seeing campaign. Weather stations were also set up as part of a more detailed study of weather conditions at the site. This allowed some comparison between the DIMMWIT wind speeds and the weather station wind speeds. There was a statistically significant (i.e. t-test resulted in a better than 10% significance) moderate correlation to strong correlation between the weather station and the DIMMWIT translational wind speeds on 2 out of 7 nights and a strong anti-correlation on 1 out of 7 nights. This could imply that the turbulence in layers at high altitude dominates the seeing behaviour more often than ground layer seeing but more than 7 nights of data would be needed to confirm this. The median Fried parameter at MRO from 14 nights recorded in one month is 7.1cm, the median wind speed 6.9ms$^{-1}$ and the median dispersive wind speed was 7.9ms$^{-1}$, implying a coherence time of 2.1 to 2.9ms. However, sampling the seeing during one month is hardly sufficient to characterise the site and to determine if such values are truly typical of the atmosphere above MRO and hence compare the values for $r_0$ at MRO with other astronomical sites.

To make more use of the DIMMWIT data, an attempt was made to see if measurements of Fried’s parameter could be used to adjust interferometric data for changes in seeing conditions. Each target visibility is divided by the visibility measured on an unresolved source to give a calibrated measurement. If the seeing conditions have changed between observing the target and the calibrator star, then the calibration will not have worked as well as it should. Unfortunately, in this case the Betelgeuse data were too well calibrated and any corrections made for changes in seeing simply added noise to the measurements and resulted in a worse fit (i.e. doubled the chi-squared) to the model. It is hoped that if the experiment were repeated with data that were not calibrated as well, then the corrections made knowing the changes in $r_0$ would improve the fit to the model, rather than worsen it.

### 7.3 Specifications of a future DIMMWIT system

The suggested design of a DIMMWIT consists of a 14” Schmidt-Cassegrain telescope with an equatorial mount and a beamsplitter at the eyepiece in order to send some of the light into an autoguider camera that is part of a tracking system with the remainder entering a CCD camera. An STV is the recommended choice for an autoguider as it has its own control box and does not require a computer to control it. The choice of camera is a Starlight Xpress camera (HX516) that has freeware software that can be adjusted to readout regions of the CCD array at cycle times of less than 3ms. A mask made of thin
metal with two small apertures is then attached to the front of the telescope, which is
defocused so that two images of the star fall on the CCD camera. The system must be
sheltered in a dome for the DIMMWIT to be able to measure temporal seeing. Without
shelter telescope vibrations make tracking a star accurately enough for fast readout
modes impossible.

The simplest and cheapest way to design a DIMM system that will measure $r_0$ only,
is to have a Schmidt Cassegrain telescope of large aperture size (minimum 12 inches
in order to facilitate the optimum ratio of sub-aperture separation to diameter) with a
mount capable of receiving a signal from an autoguider. The STV autoguider is capable
of acting as a DIMM camera, with exposure times of 1ms, in addition to tracking so
there would be no need for an additional camera. An autoguider is only required if
the observer wishes to collect data for long periods of time without user input. A
dome would also not be required as the observer can select a large area of the CCD
for readout and telescope vibrations would not affect the quality of the data collected.
However, if the wind speeds at the site were particularly high (over 20-30ms$^{-1}$) at
times, then the observer might find operation of the system easier with the presence of
a dome. The important specification of the DIMM system is the exposure time of the
camera, which must be 1ms or less if the site has characteristically high wind speeds
(10ms$^{-1}$ or higher), in order to avoid overestimating Fried’s parameter.

If the aim of the seeing campaign is to measure the temporal scale of turbulence as well
as the spatial scale, then the constraints on the design of the system increase. There
must be a dome set up at the site in order to protect the telescope from wind shake. A
portable version has been found to be acceptable if the floor of the dome is decoupled
from the telescope pier. The DIMM system must be capable of tracking a star with
precision (i.e. within a 60 arcsecond region on CCD) for long periods of time to reduce
the lowest measured frequency in the differential image motion spectra. A high frame
rate, i.e. a short cycle time, is required to detect short-timescale seeing changes. Hence,
the CCD camera may have to be capable of cycle times of less than 2 or 3 ms.

The performance of the DIMM can be further improved by designing the DIMM to
measure Zernike tilts rather than G-tilts. It has been found[71] that the DIMM can
become impervious to atmospheric coma aberrations if it measures Z-tilts rather than
G-tilts, as discussed in Chapter 3. A 1 rad coma moves the G-centroid by $0.8192\lambda/D$
so the difference between G- and Z-tilts approaches 0.1” at 10 cm apertures (assuming
that $D/r_0=1$ when the rms coma coefficient is 0.08 rad). If a 4x4 centroiding box of CCD
pixels is used to centroid an image, each pixel must be approximately 0.25$r$ where $r$ is
the radius of the first dark ring in an Airy disk, $1.22\lambda/D$. If the pixels (binned or not)
7.3 Specifications of a future DIMMWIT system

are larger than this, then the DIMM is measuring G-tilts. Thus, the choice of diameter
for the sub-apertures may impact the binning mode used to achieve optimum exposure
and cycle times. For example, in the case of the DIMMWIT, the diameter of the sub-
apertures meant that only the pixels for the fast1d mode, which were unbinned in the
horizontal direction, were small enough in size to measure Z-tilts.
Chapter 7: Conclusions
Bibliography


Bibliography


[77] Apache Point Observatory Website, 2005.


