Fitting and Comparison of Models of Radio Spectra

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R60

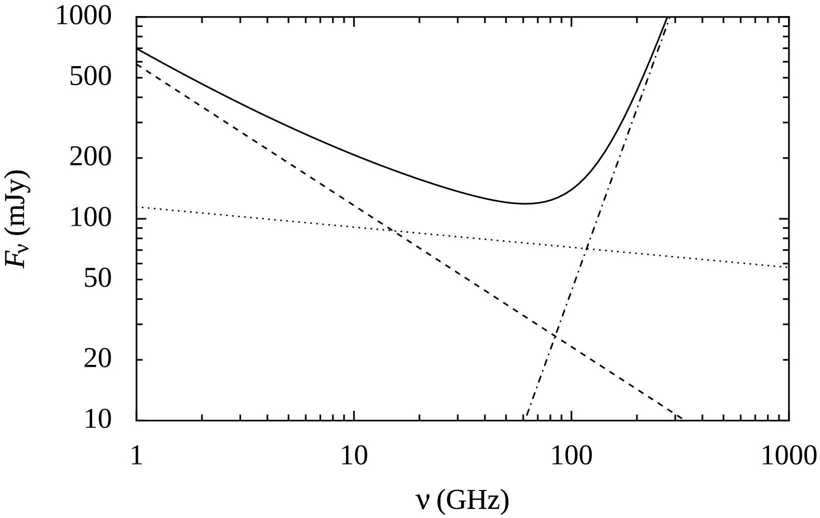
Outline

Introduction

- This work was done in preparation for analysing forthcoming data from GBT+MUSTANG
- It is a little bit of a “spare-parts” project in which I reused various software components I developed for other purposes (e.g., phase calibration for ALMA)
- The approach is described in full detail in Nikolic (2009)
- All of the source code available for download (GPL license) from website:

  `http://www.mrao.cam.ac.uk/~bn204/galevol/speca/index.html`

Schematic radio spectrum of a star-forming galaxy



Schematic & **hypothetical** (continuum-only) spectrum of NGC 3627: the dashed line is the synchrotron the component; the dotted line is the free-free component; the dash-dot-dash line is the dust component; the solid line is the total emission.

Why analyse radio spectra

- Energetics
  - Reconstruct the total energy balance from few/sparse measurements of the spectrum
- Inference of properties of the source:
  - Geometry (Filling factor from low-frequency turnover)
  - Dynamics (e.g., through electron ageing)
- Redshift determination $\equiv$ radio “photometric” redshifts
  - Currently mostly used for sub-millimetre selected (“SCUBA”) sources
- Physics:
  - Free-free emission
  - Slope of the dust continuum – physics of interstellar dust

Analysis strategy

Model fitting

In radio, sub-mm and far-IR, the physics is fairly well understood and candidate models are computational easy. So analysis often consists of "fitting" a set of models to the observations.

- Synchrotron radiation (analytic or 1-D integral)
- Thermal free-free (analytic)
- Modified black-body emission from dust (analytic or 1-D integral)
- Spinning dust models (analytic)

Requirements for model fitting

- Objective measure of how well the model fits observed data
- For all model parameters:
  - Unbiased estimates
  - Error on these estimates
  - Correlations between the errors
  - Full probability distributions if significantly non-Gaussian
- Objective way of **comparing** how well *different* models fit the data
- A mechanism to incorporate already known constraints on model parameters
- Visualisation of the fit in comparison to observations

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1. Introduction
2. Method
    - Bayesian analysis
    - Implementation
    - Visualisation
3. Simple examples
    - NGC 628
    - NGC 3627
    - NGC 7331
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Bayesian analysis in a nutshell

1. Can handle “nuisance” parameters
2. Fully describes non-Gaussian distributions
3. Unbiased
4. Objective model (or hypothesis) selection

Bayes equation & the evidence

$$\boxed{p(\theta|D,H)} = \frac{p(D|\theta,H)p(\theta|H)}{p(D|H)}$$

- D: Observed data → flux density at several frequencies
- H: Hypothesis → model for emission & priors for parameters
- $p(D|\theta,H)$: Likelihood → given a model *and* its parameters, how likely are the observed data?
- $p(\theta|D,H)$: Posterior → given a model, what we know about it's parameters
- $p(D|H)$: "Evidence", objective measure of how good the model is

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Calculating the evidence

Evidence is an integral over the likelihood surface

$$p(D|H) = \int \mathrm{d}\theta p(D|\theta, H) p(\theta|H)$$

- Evidence is not available from standard Markov Chain Monte Carlo calculations
- I use a new implementation of the nested sampling algorithm by Skilling (2006). Compared to MCMC, this algorithm is:
    - Efficient (fewer likelihood function evaluations)
    - Reliable (less chance of getting stuck in local maxima)
    - The output is both the evidence and the posterior distribution

Inputs/outputs

Inputs

1. The model: a C++ class. Possible to compose different models in the Python layer
2. Priors: only flat, independent priors supported. Supplied as a dictionary in the Python layer
3. Observed data: supplied as simple list in Python layer

Outputs

1. The evidence value
2. Histograms of marginalised distributions of each model parameter
3. Fan-diagram of flux vs frequency
4. Maximum likelihood plot of flux vs frequency

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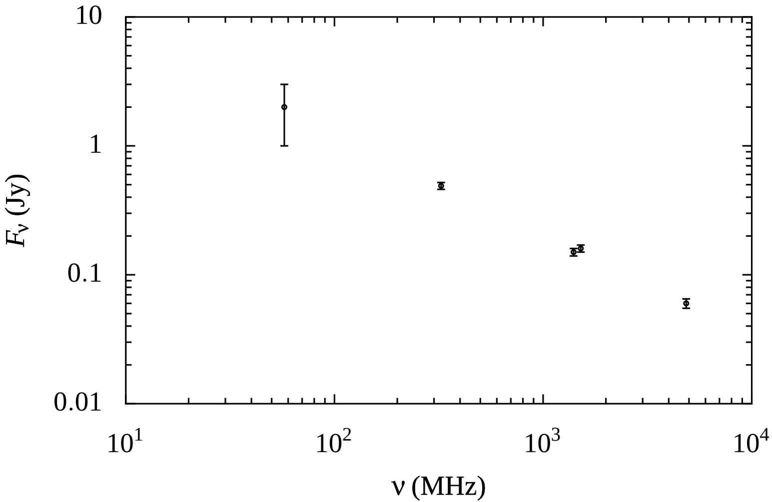
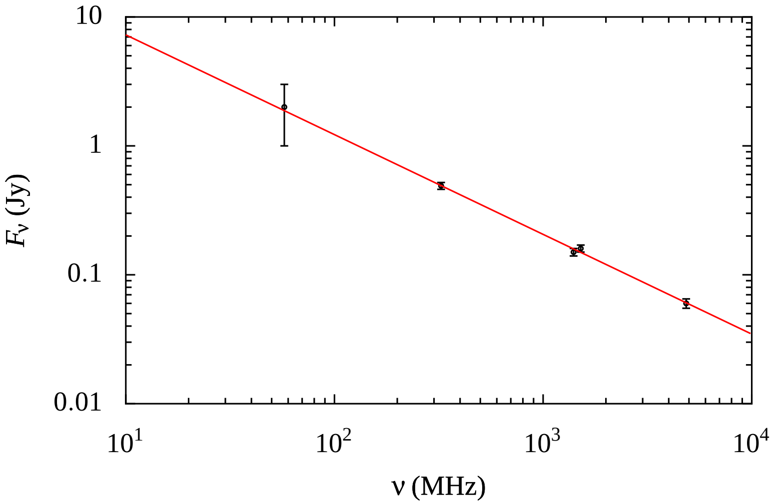
NGC 628 observations



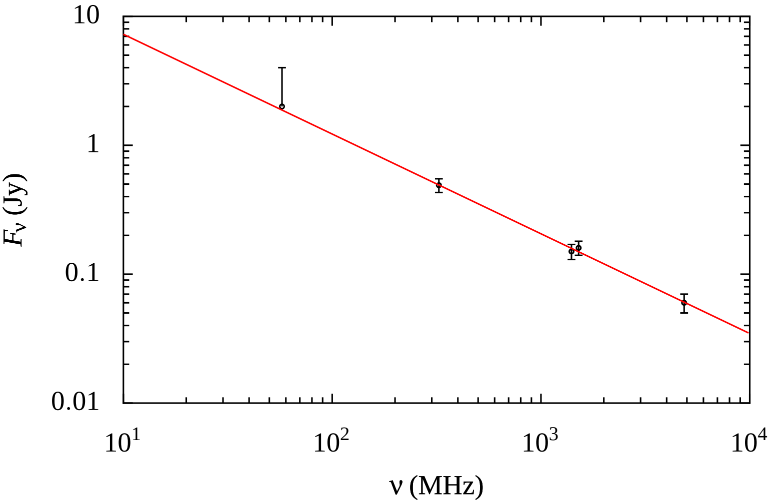
Observations at five frequencies of the near-by galaxy NGC 628 collected by Paladino et al. (2009)

NGC 628 – max. likelihood line fit



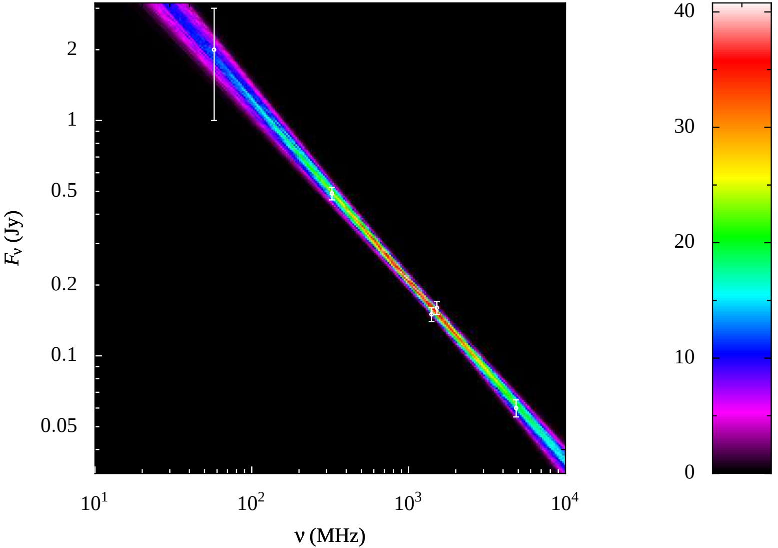
Observations at five frequencies of the near-by galaxy NGC 628 collected by Paladino et al. (2009)

NGC 628 – doubled errors & max. likelihood line fit

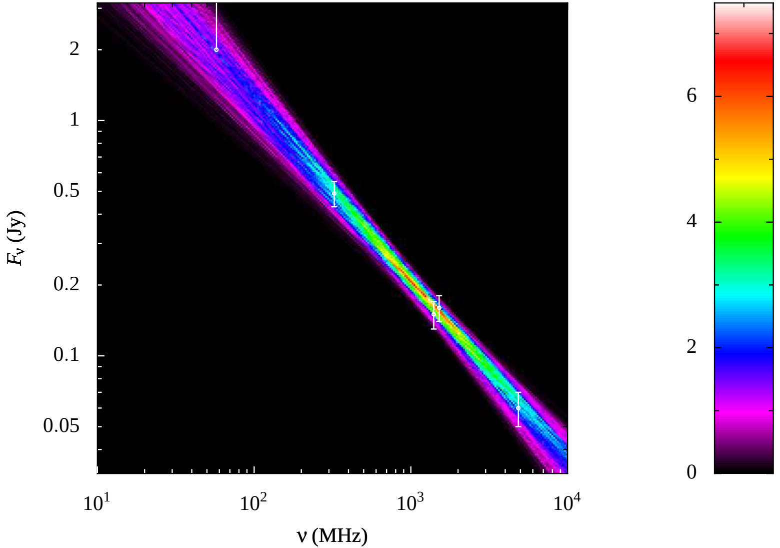


I have scaled up the error estimates by a factor of two

NGC 628 – original errors & fan-diagram



NGC 628 – doubled errors & fan-diagram



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Four simple models for synchrotron emission

Underlying synchrotron spectrum

- Power law
  $F_\nu(\nu) = F_\nu^0 \cdot \left(\frac{\nu}{1\,\mathrm{GHz}}\right)^\alpha$
- Continuous injection of electrons → very approximately broken power law

$$\begin{array}{ll} F_\nu^0 \cdot \left(\frac{\nu}{1\,\mathrm{GHz}}\right)^\alpha & \nu \le \nu_\mathrm{br} \\ F_\nu^0 \cdot \left(\frac{\nu}{1\,\mathrm{GHz}}\right)^\alpha \left(\frac{\nu}{\nu_\mathrm{br}}\right)^{-1/2} & \nu > \nu_\mathrm{br}. \end{array} \quad (1)$$

$\otimes$

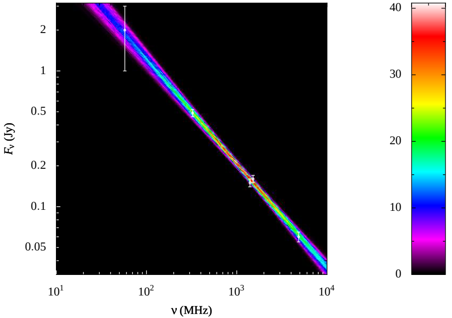
Low-frequency optical depth effects

- None
- Synchrotron self-absorption

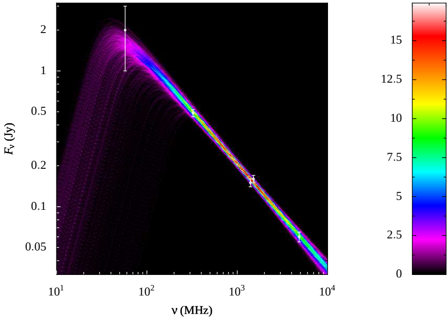
$$x = \nu/\nu_\mathrm{pk}$$
$$A_\mathrm{s} = x^{-\alpha+5/2} \times \left[1 - \exp\left(1 - x^{\alpha-5/2}\right)\right]$$

For the purposes of these examples, I've taken the models from Paladino et al. (2009) to go with their data – both more complex and more physical models could be used

Model fits for NGC 628 I

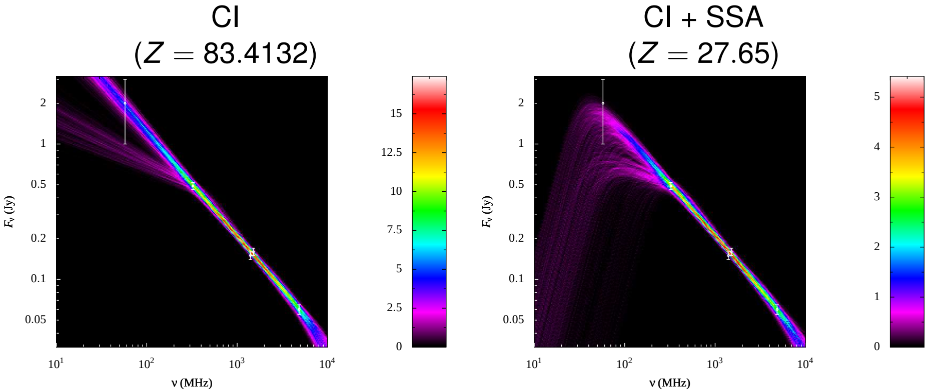


Power-law
($Z = 185.874$)



Power-law + SSA
($Z = 76.3034$)

Model fits for NGC 628 II



CI

($Z = 83.4132$)

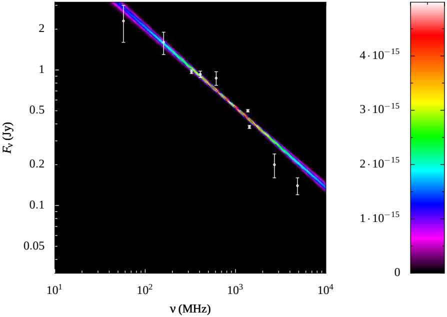
CI + SSA

($Z = 27.65$)

Model fits for NGC 3627 I

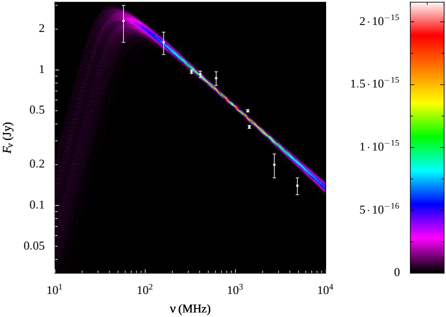
Power-law

$(Z = 9.7 \times 10^{-15})$



Power-law + SSA

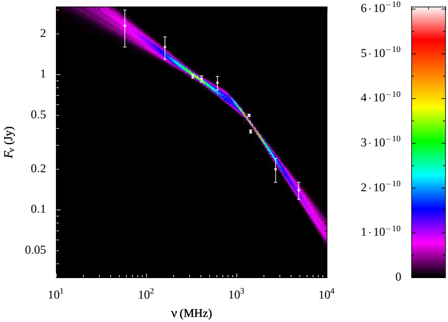
$(Z = 3.9 \times 10^{-15})$



Model fits for NGC 3627 II

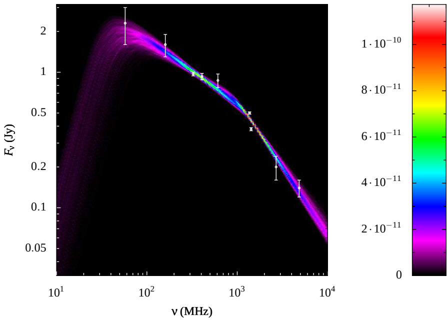
CI

($Z = 1.3 \times 10^{-09}$)

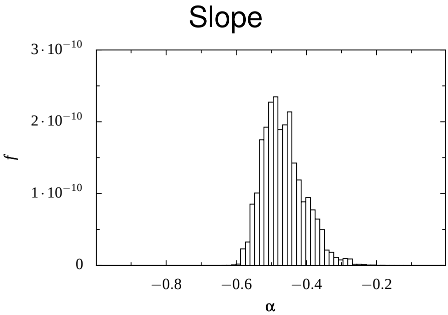


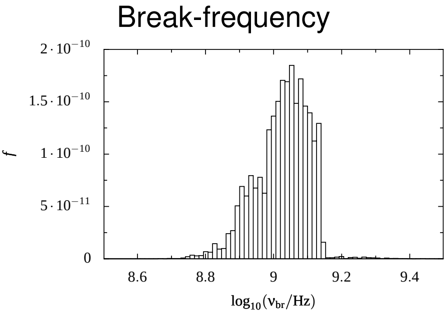
CI + SSA

($Z = 2.6 \times 10^{-10}$)



Probability distributions of parameters

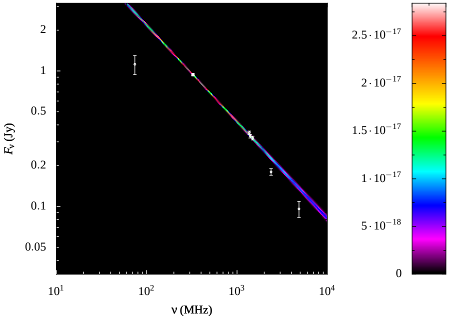




Model fits for NGC 7331 I

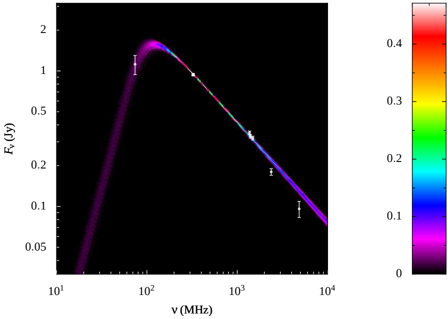
Power-law

$(Z = 2.9 \times 10^{-17})$

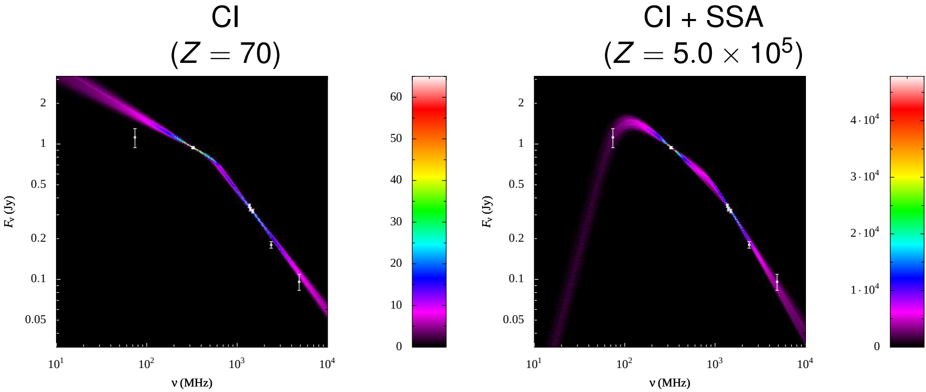


Power-law + SSA

$(Z = 0.5)$



Model fits for NGC 7331 II



Outline

Introduction

Data courtesy of D. A. Green in Cambridge

- Analysis of spectrum of supernova remnant HB3
- Is there evidence for flattening of the spectrum?
    - Could be interpreted as a thermal free-free component due to interaction of shock with the molecular cloud
- See Urošević et al. (2007), Green (2007), Onic & Urosevic (2008)

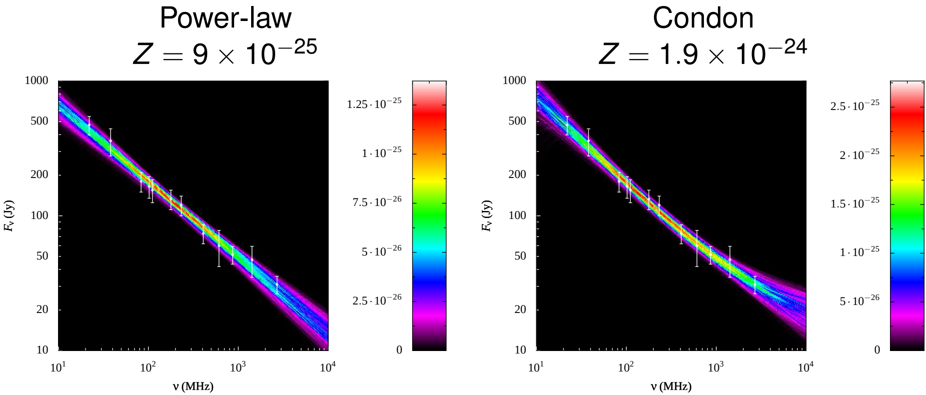
Condon model

- Single power-law synchrotron with slope (α) as a free parameter
- Free-free emission component (H is the thermal fraction at 1 GHz)
- Thermal free-free absorption at low frequencies (τ is the optical depth at 1 GHz)

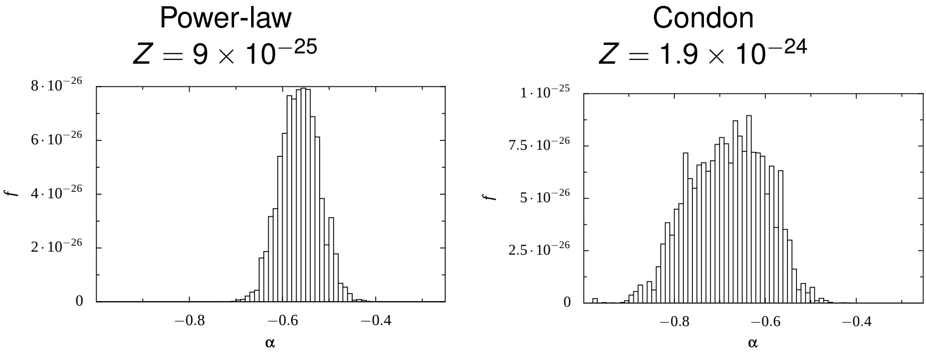
$$A(\nu;\tau^*) = 1 - \exp\left[-10^{\tau^*}\left(\frac{\nu}{1\,\mathrm{GHz}}\right)^{-2.1}\right]$$

$$F_\nu(\nu;H,\alpha) = \frac{A(\nu)}{A(1\,\mathrm{GHz})}\left(\frac{\nu}{1\,\mathrm{GHz}}\right)^2\left[H + (1-H)\left(\frac{\nu}{1\,\mathrm{GHz}}\right)^{0.1+\alpha}\right]$$

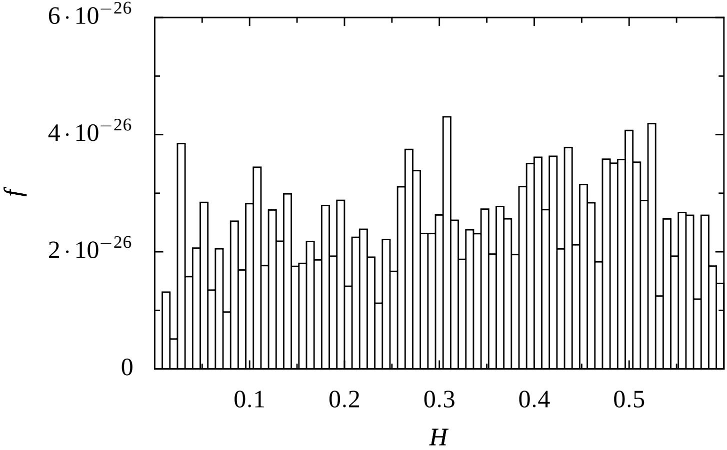
Model fits for HB3



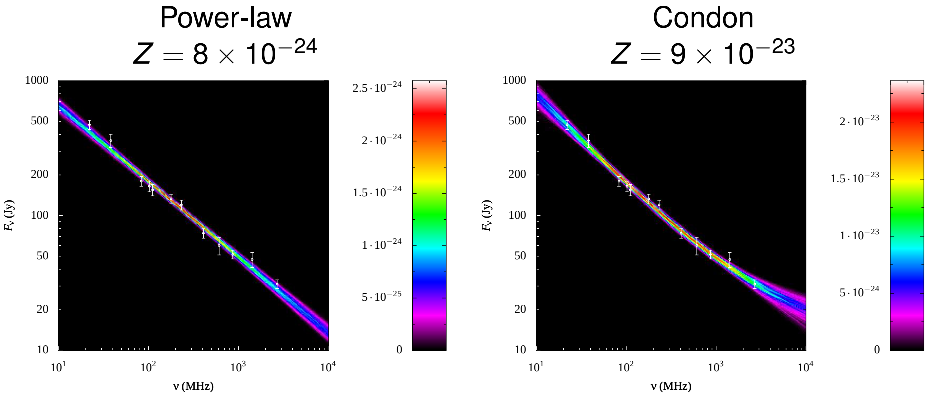
Marginalised distribution of α



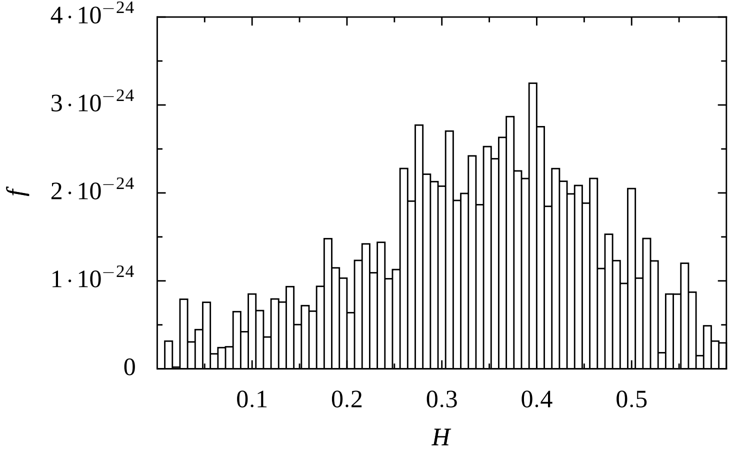
Marginalised distribution of *H*



Error estimates reduced by half



Marginalised distribution of *H*



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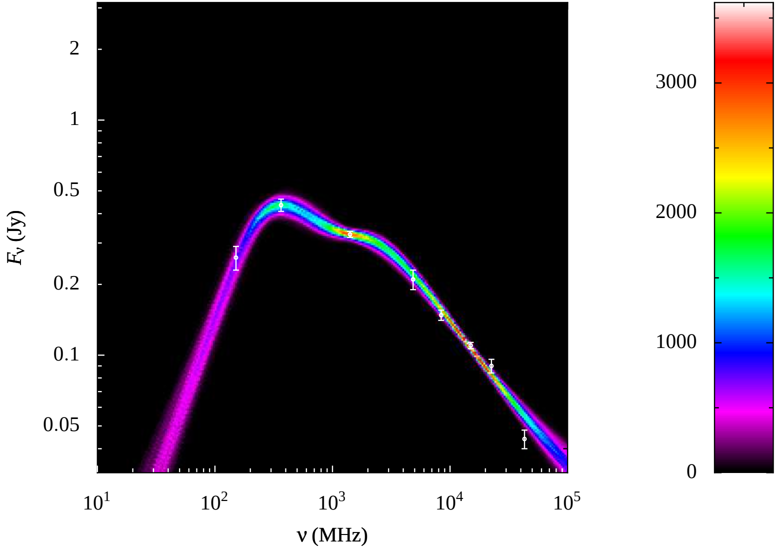
Future direction

Analysis of Luminous Infrared Galaxies (with Marcel Clemens from U. Padua)



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Urošević D., Pannuti T. G., Leahy D., 2007, ApJ, 655, L41. `arXiv:arXiv:astro-ph/0612691`