Fitting and Comparison of Models of Radio Spectra

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R90

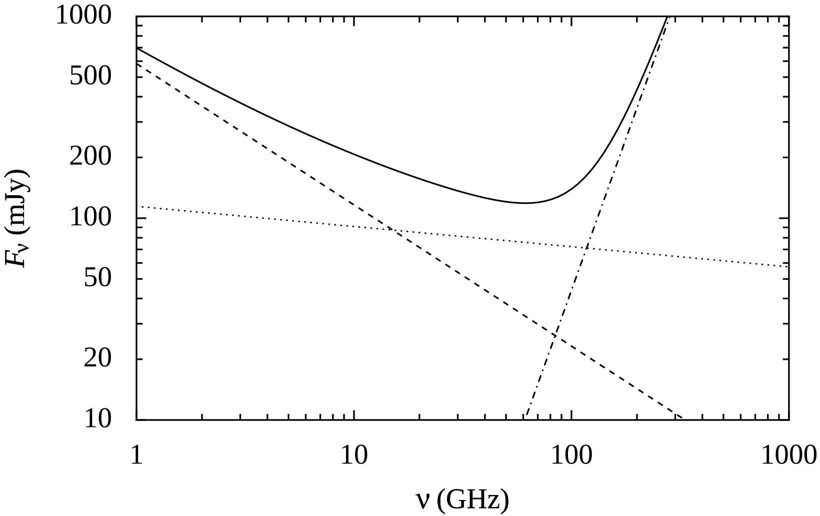
Outline

Introduction

- Motivation:
  - Started as preparation for analysis of some forthcoming observational data (GBT+MUSTANG)
  - Turned out useful in its own right
- Mostly a *spare-parts* project - reused various components I developed for other purposes
- Method paper: Nikolic (2009)
- All of the source code available under GPL license from: `http://www.mrao.cam.ac.uk/~bn204/galevol/speca/index.html`

Schematic radio spectrum of a star-forming galaxy

Maybe add low-frequency turn-over?

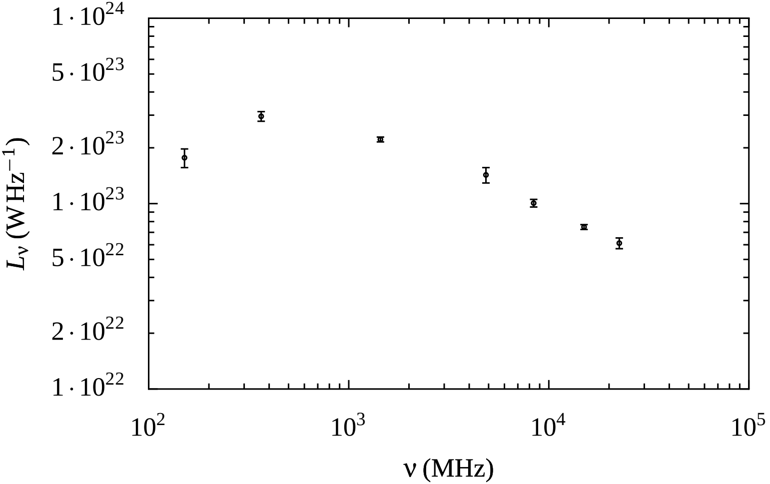


Schematic & **hypothetical** (continuum-only) spectrum of NGC 3627: the dashed line is the synchrotron the component; the

Why analyse radio spectra

- Energetics
  - Reconstruct the total energy balance from few/sparse measurements of the spectrum
  - What physical process is the source of the energy?
- Geometry:
  - Size/filling factor from the low-frequency turnover
- Dynamics:
  - Electron ageing
  - Are equilibrium models of star formation sufficient?
- Redshift determination $\equiv$ radio "photometric" redshifts
  - Currently mostly used for sub-millimetre selected ("SCUBA") sources
- Physics:
  - Free-free emission
  - Slope of the dust continuum – physics of interstellar dust

Example observed radio spectrum



Observed spectrum of Arp 220 [data from Clemens et al. (2010)].

Analysis strategy: model fitting

- Non-linear (free-free absorption, slope of synchrotron, etc.) so simple inversions not in general reliable
- In radio, sub-mm and far-IR, the physics is fairly well understood so can propose candidate models easily
- Computationally relatively easy:
  - Synchrotron radiation (analytic or 1-D integral)
  - Thermal free-free (analytic)
  - Modified black-body emission from dust (analytic or 1-D integral)
  - Spinning dust models (analytic)

Requirements for model fitting

1. An objective measure of how well the model fits the observed data
2. For all model parameters:
   1. Unbiased estimates
   2. Error on these estimates
   3. Correlations between the errors
   4. Full probability distributions if significantly non-Gaussian
3. An objective way of **comparing** how well *different* models fit the data
4. A mechanism to incorporate already known constraints on model parameters
5. Visualisation of the fit in comparison to observations

Outline

Bayesian analysis in a nutshell

1. Can handle "nuisance" parameters
   1. They do not bias estimates of other parameters
   2. They are correctly taken into account when calculating significance
2. Fully describes non-Gaussian distributions
   1. Even multi-modal distributions (although not very efficiently in the present implementation of my program)
3. Unbiased
   1. No fitting into the noise (e.g., "flux-boosting")
4. Objective model (or hypothesis) selection

Bayes equation & the evidence

$$p(\theta|D,H) = \frac{p(D|\theta,H)p(\theta|H)}{p(D|H)}$$

- D: Observed data → flux density at several frequencies
- H: Hypothesis → model for emission & priors for parameters
- $p(D|\theta,H)$: Likelihood → given a model *and* its parameters, how likely are the observed data?
- $p(\theta|D,H)$: Posterior → given a model, what we know about it's parameters
- $p(D|H)$: "Evidence", objective measure of how good the model is

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Calculating the evidence

Evidence is an integral over the likelihood over prior volume

$$p(D|H) = \int \mathrm{d}\theta p(D|\theta, H) p(\theta|H)$$

- Evidence is not available from standard Markov Chain Monte Carlo calculations
- I use a new implementation of the nested sampling algorithm by Skilling (2006). Compared to MCMC, this algorithm is:
  - Efficient (fewer likelihood function evaluations)
  - Reliable (less chance of getting stuck in local maxima)
  - The output is both the evidence and the posterior distribution
- The algorithm is available under GPL

Inputs/outputs

Inputs

1. The model (as a routine/class)
2. Priors: only flat, independent priors supported. I.e., a “prior box”
3. Observed data and errors (for the moment assumed Gaussian)

Outputs

1. The evidence value
2. Histograms of marginalised distributions of each model parameter
3. Two-dimensional histograms of partially marginalised distributions
4. Fan-diagram of flux vs frequency
5. Maximum likelihood plot of flux vs frequency

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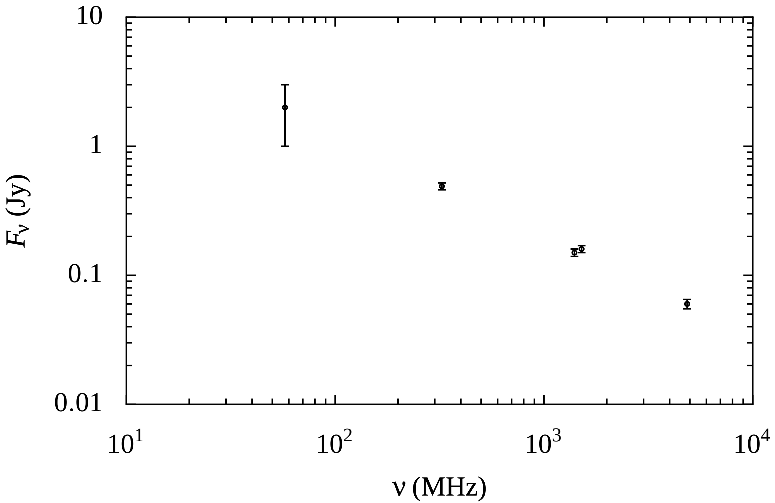
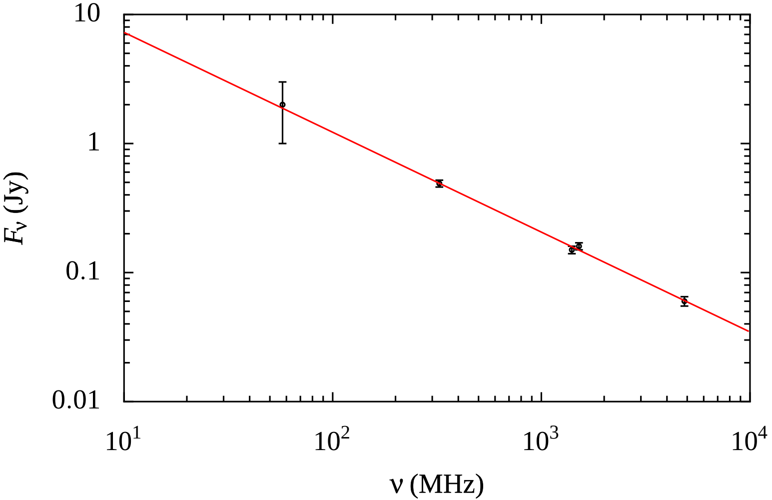
NGC 628 observations



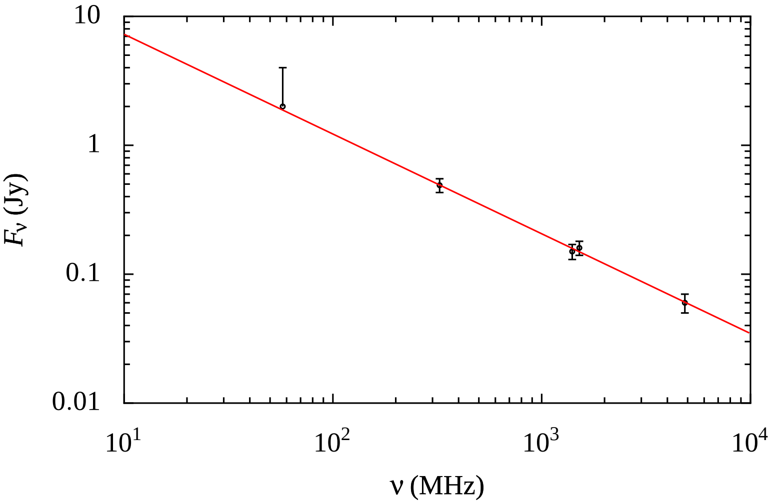
Observations at five frequencies of the near-by galaxy NGC 628 collected by Paladino et al. (2009)

NGC 628 – max. likelihood synchrotron fit



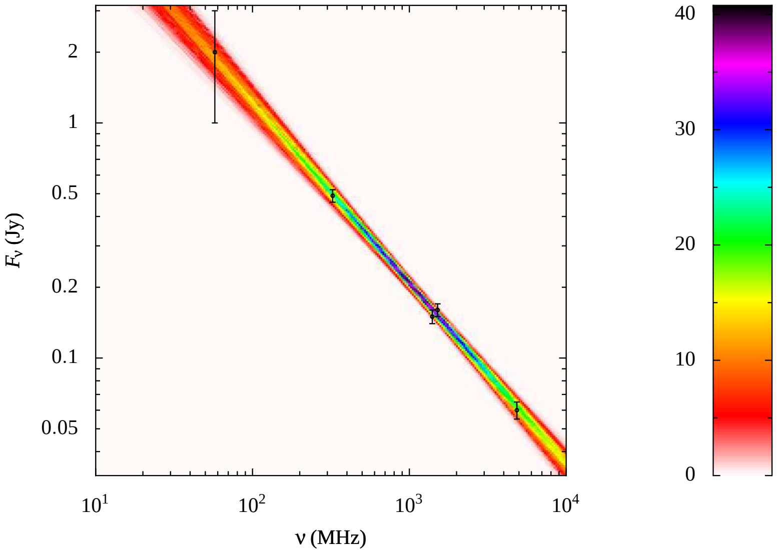
Observations at five frequencies of the near-by galaxy NGC 628 collected by Paladino et al. (2009)

NGC 628 – doubled errors & max. likelihood fit

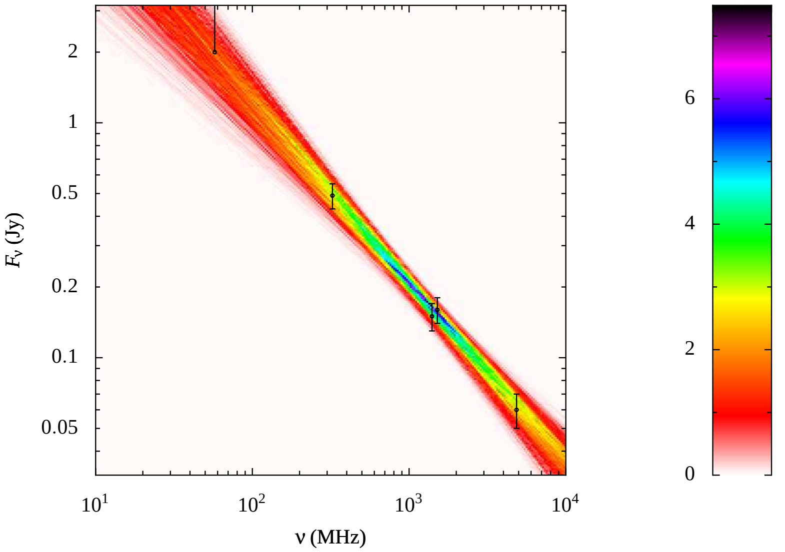


I have scaled up the error estimates by a factor of two

NGC 628 – original errors & fan-diagram



NGC 628 – doubled errors & fan-diagram



Outline

Introduction

Data courtesy of D. A. Green in Cambridge

- Analysis of spectrum of supernova remnant HB3
- Is there evidence for flattening of the spectrum?
  - Could be interpreted as a thermal free-free component due to interaction of a shock with the molecular cloud
- See Urošević et al. (2007), Green (2007), Onić & Urošević (2008)

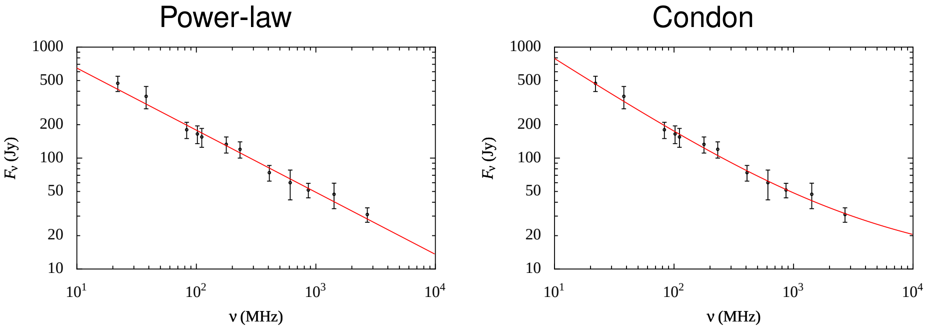
Condon model

- Single power-law synchrotron with slope (α) as a free parameter
- Free-free emission component (H is the thermal fraction at 1 GHz)
- Thermal free-free absorption at low frequencies (τ is the optical depth at 1 GHz)

$$A(\nu;\tau^*) = 1 - \exp\left[-10^{\tau^*}\left(\frac{\nu}{1\,\mathrm{GHz}}\right)^{-2.1}\right]$$

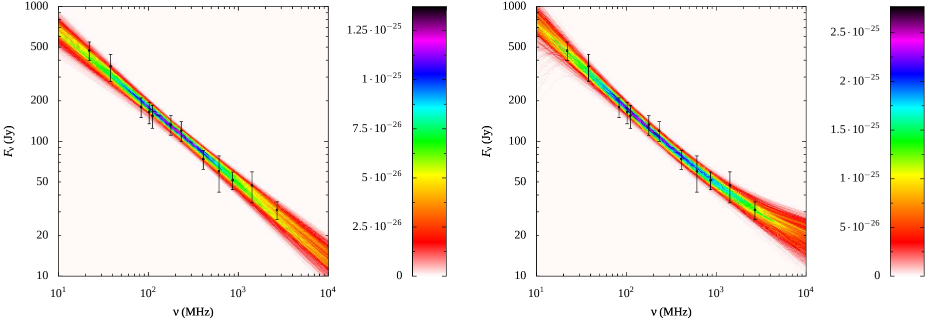
$$F_\nu(\nu;H,\alpha) = \frac{A(\nu)}{A(1\,\mathrm{GHz})}\left(\frac{\nu}{1\,\mathrm{GHz}}\right)^2\left[H + (1-H)\left(\frac{\nu}{1\,\mathrm{GHz}}\right)^{0.1+\alpha}\right]$$

Maximum likelihood model fits for HB3



Fan diagrams of the Bayesian analysis of HB3

Power-law $Z = 9 \times 10^{-25}$ | Condon $Z = 1.9 \times 10^{-24}$

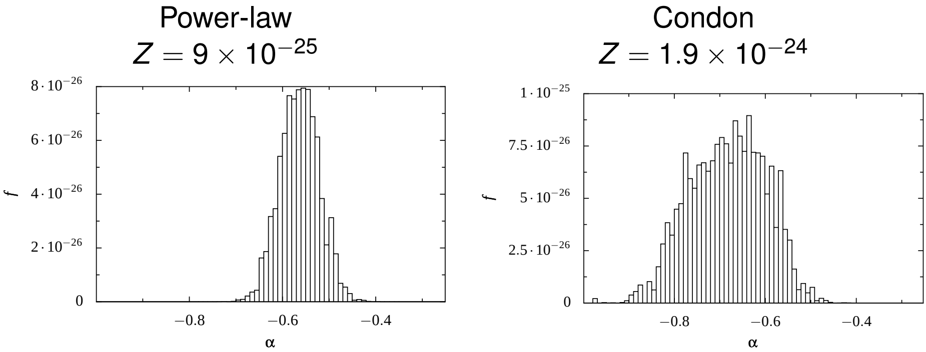


Priors: $-1 < \alpha < -0.25$

$0 < H < 0.6$

$-20 < \log \tau < -2$

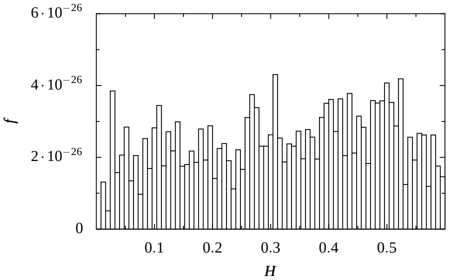
Marginalised distribution of α



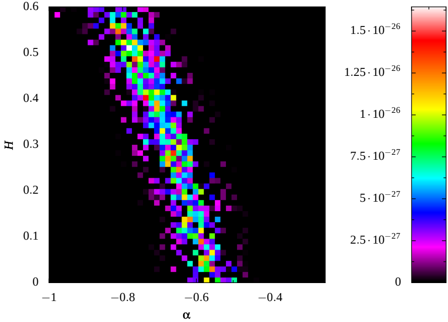
(Partially-)Marginalised distribution of H

Condon model, i.e., synchrotron + thermal emission

PD of thermal fraction at 1 GHz

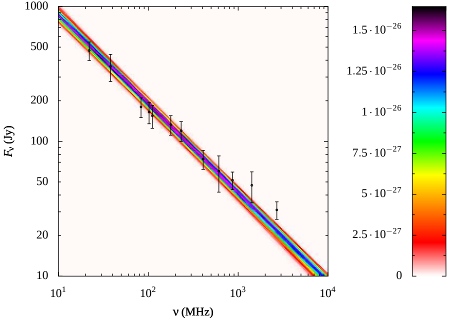


Correlation between thermal fraction and synchrotron slope

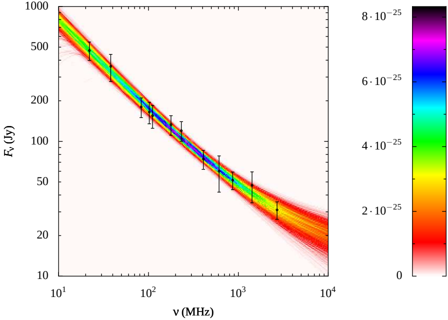


New prior: constrain α to 0.1 range around -0.7

Power-law
$Z = 1.2 \times 10^{-25}$



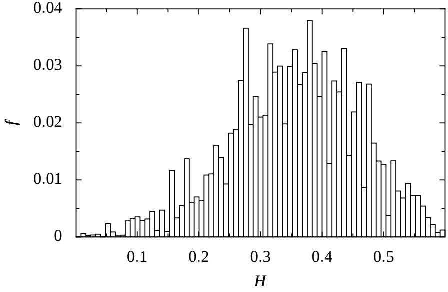
Condon
$Z = 5.4 \times 10^{-24}$



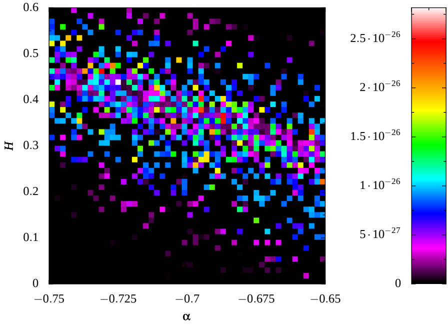
Priors:
$$-0.75 < \alpha < -0.65$$
$$0 < H < 0.6$$
$$-20 < \log \tau < -2$$

New prior: distribution of *H*

PD of thermal fraction at 1 GHz



Correlation between thermal fraction and synchrotron slope



Outline

Introduction I

See Scaife et al. (2010) for details

Original observation

- Centimetre-wave emission correlated with galactic dust foregrounds (e.g., de Oliveira-Costa et al., 1997) in excess to what is expected from simple models
- Draine & Lazarian (1998) suggest it could be due to spinning dust

Vigorous debate about the nature the emission...

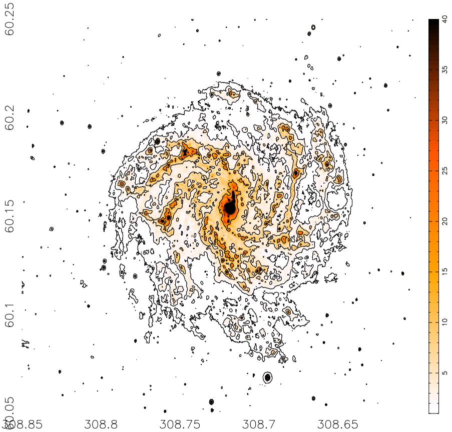
First detection in an external galaxy

Murphy et al. (2010) – but is it due to spinning dust or a very compact HII region?

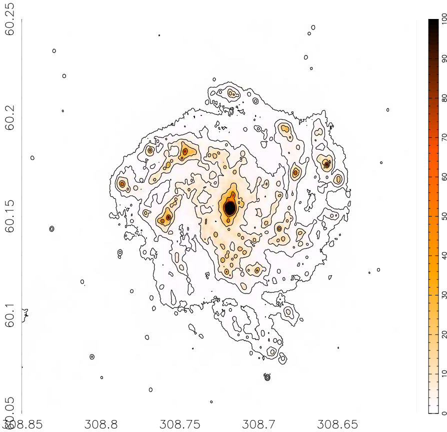
Introduction II

See Scaife et al. (2010) for details

8 micron



24 micron

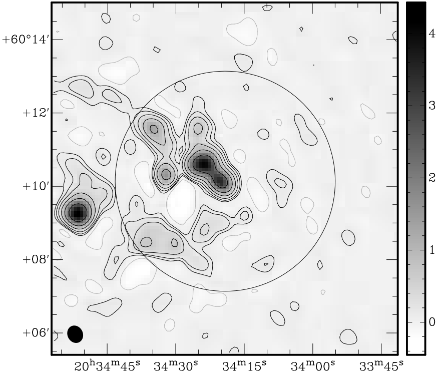


Spiter/SINGS images of NGC 6946

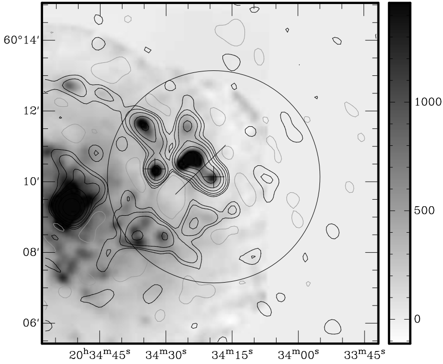
Introduction III

See Scaife et al. (2010) for details

15 GHz contour + grayscale



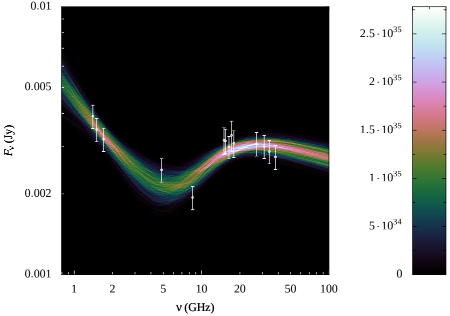
15 GHz countour + 8.5 GHz grayscale



Results

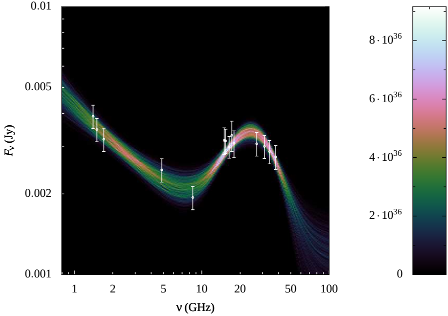
"Region 4" – the anomalous region

High-opacity HII region model



$Z=2.4 \times 10^{36}$

Spinning dust model



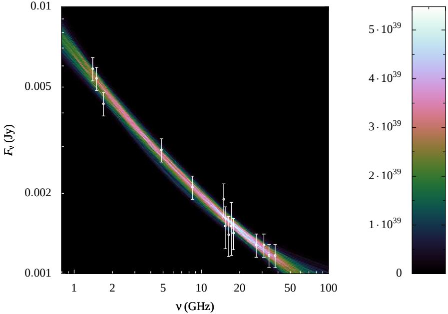
$Z=7.2 \times 10^{37}$

Clearly should prefer this model

Results

“Region 8” – a normal region

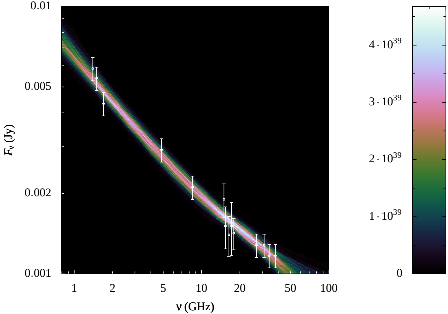
Simple thermal + synchrotron



$Z=4.8 \times 10^{40}$

Simpler model preferred!

Spinning dust model



$Z=4.3 \times 10^{40}$

Outline

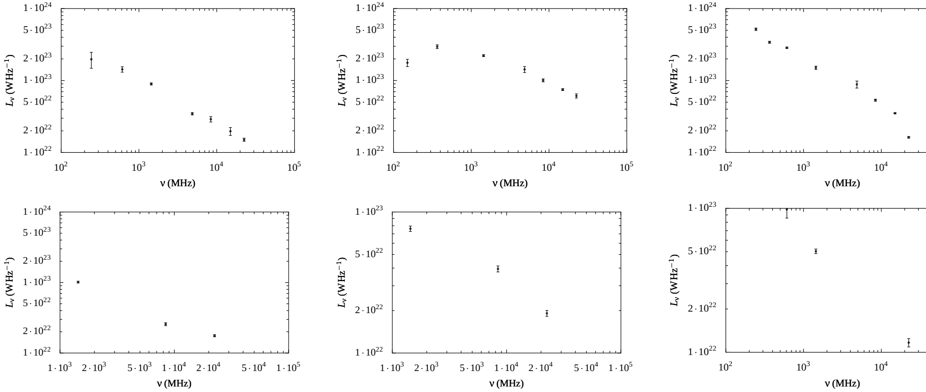
Spectra of (U)LIRGs

- Work in-progress – collaboration with Marcel Clemens from U. of Padua
- Data are based on Clemens et al. (2010)

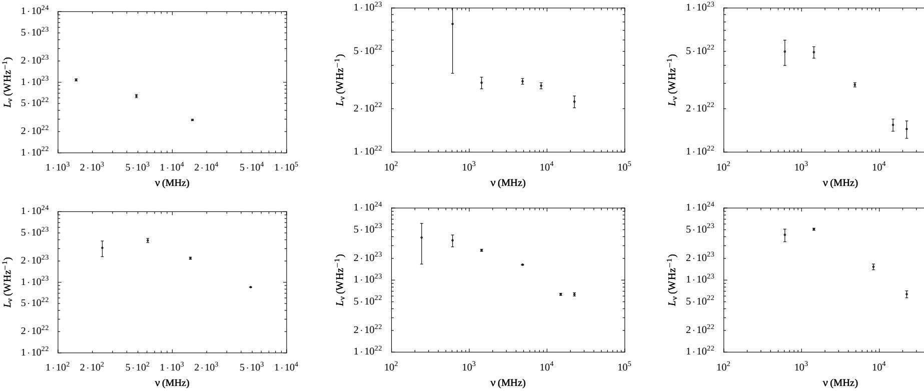
Goals:

- Constrain the geometry of star-formation from radio to far IR spectra
- Physics of synchrotron emission (ionisation losses, underlying causes for the far IR-radio correlation) and of free-free emission (why don't we see it?)
- Constrain AGN contribution?

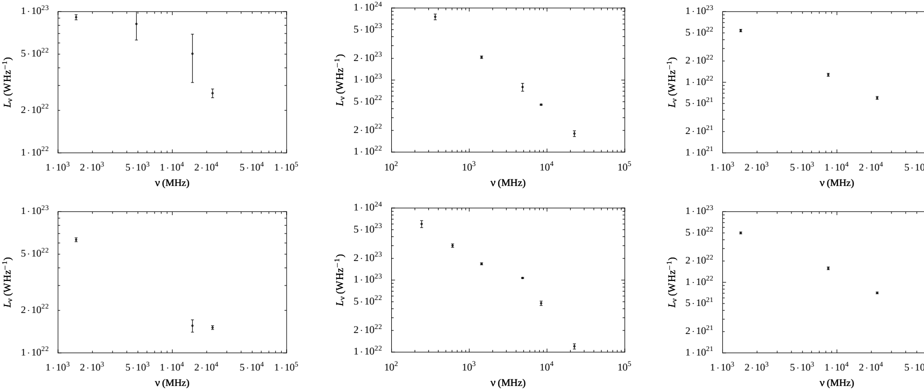
Simple one-component model I



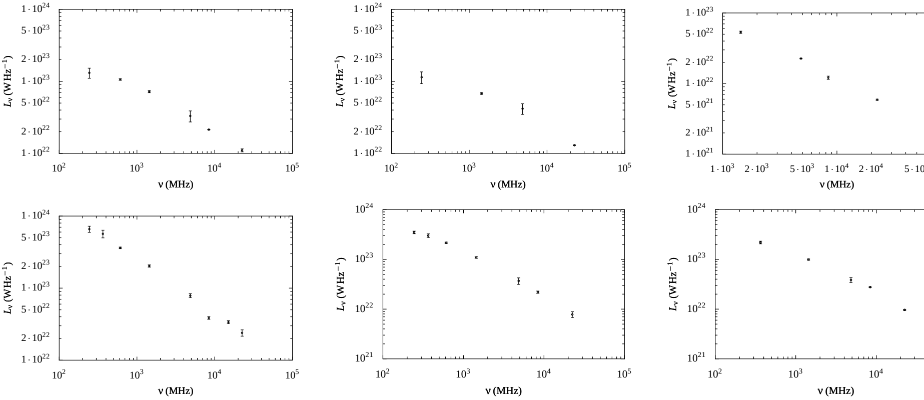
Simple one-component model II



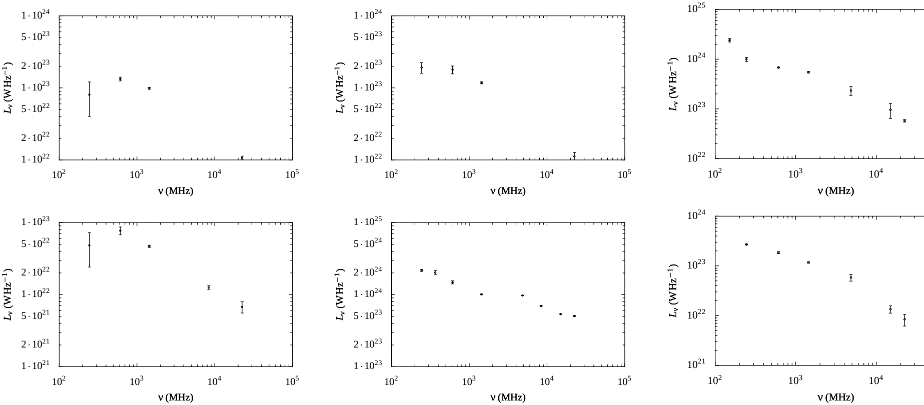
Simple one-component model III



Simple one-component model IV



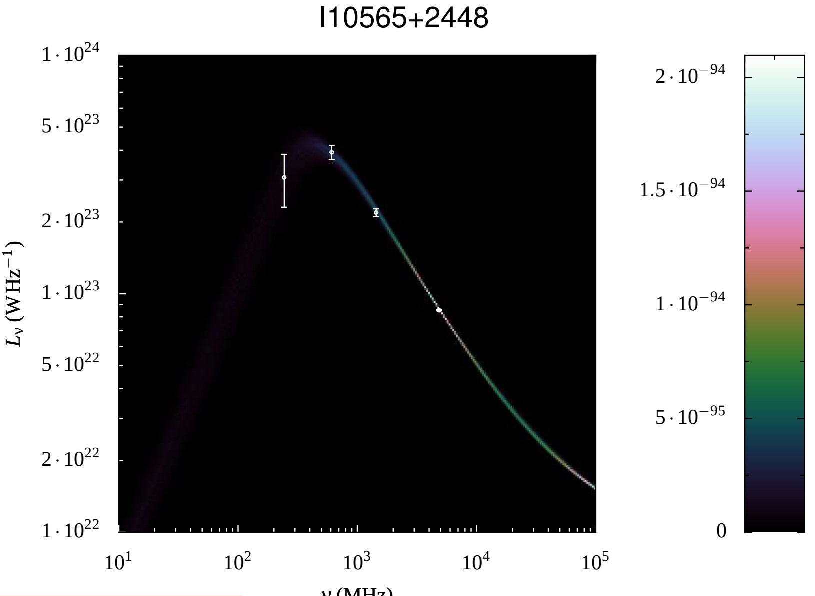
Simple one-component model V



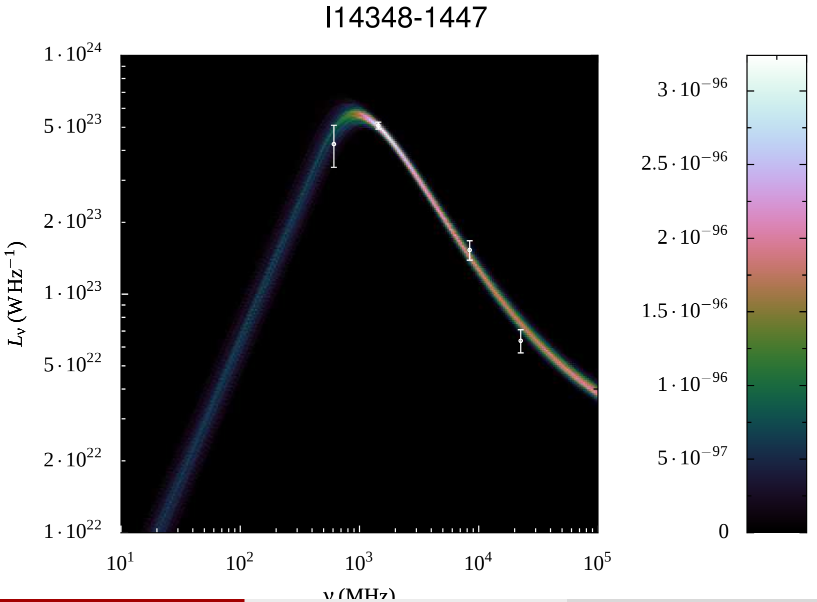
Simple one-component model

- Spherical, homogeneous, well-mixed ("giant") star-forming region
- Synchrotron emission based on the supernova rate
  α a free parameter from -1 to -0.5
- Free emission and absorption based on the ionising photon rate
  $\log 10 R_{\mathrm{g}}$ free parameter: radius of the region
- Far-infrared emission linearly dependent to SFR and based on the FIR-radio correlation

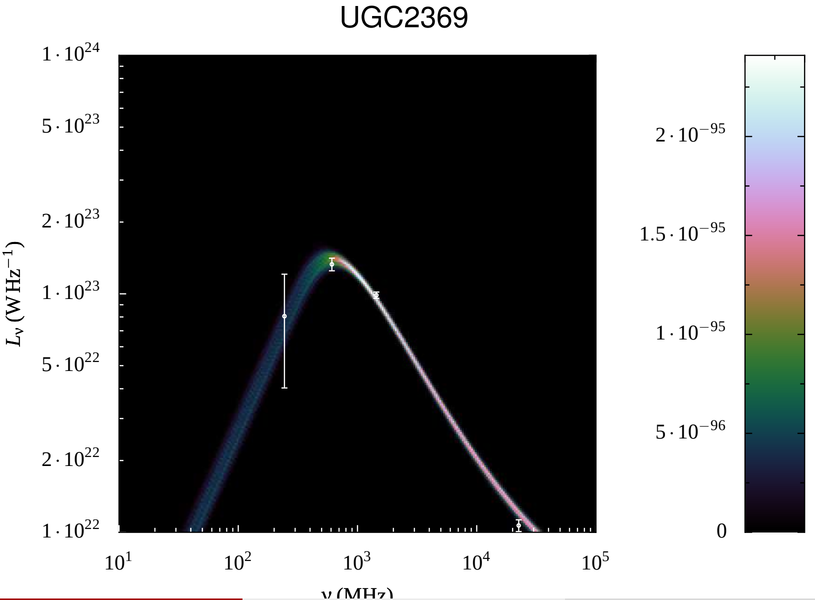
(U)LIRGs well fit by one-component models I



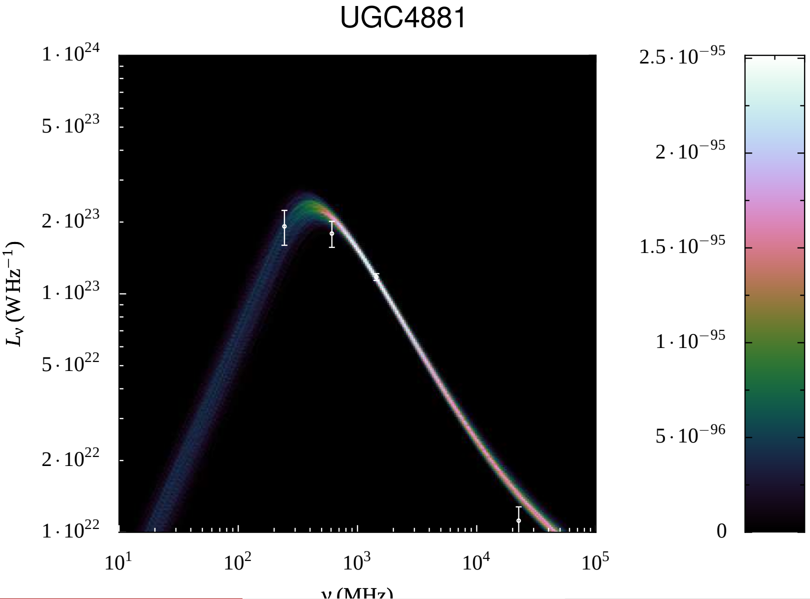
(U)LIRGs well fit by one-component models II



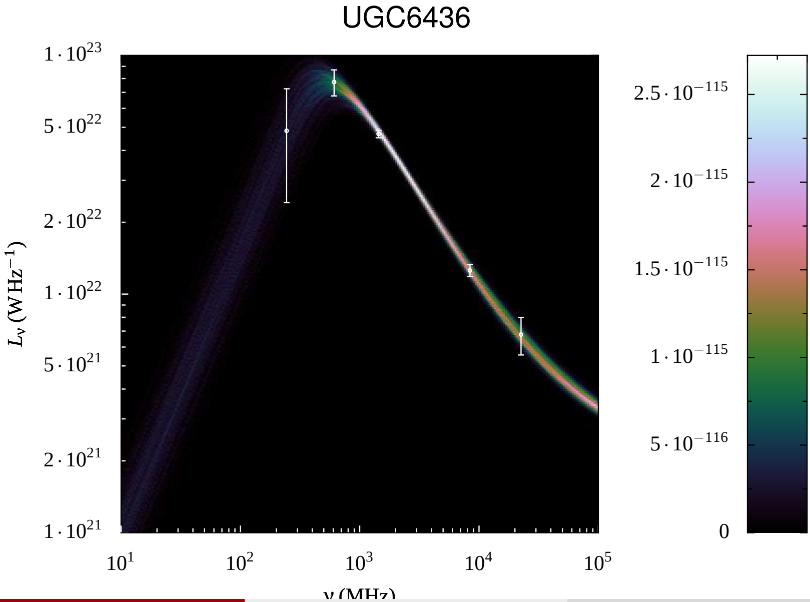
(U)LIRGs well fit by one-component models III



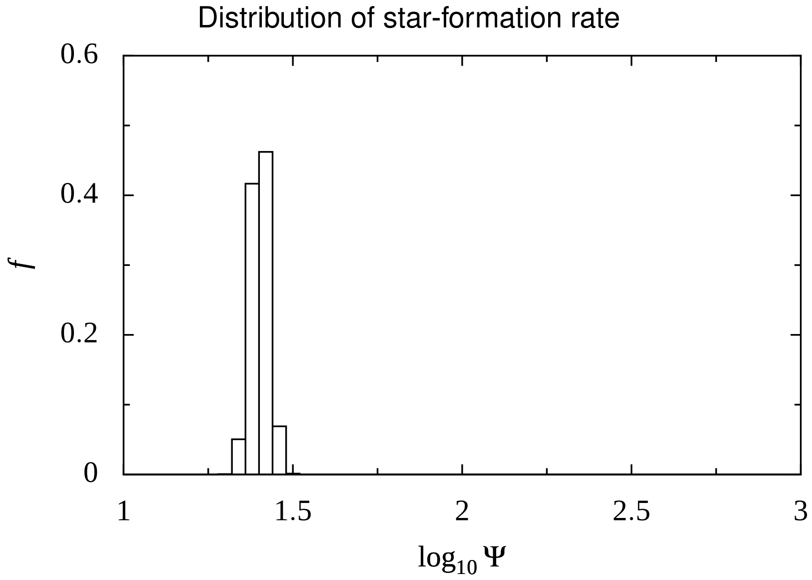
(U)LIRGs well fit by one-component models IV



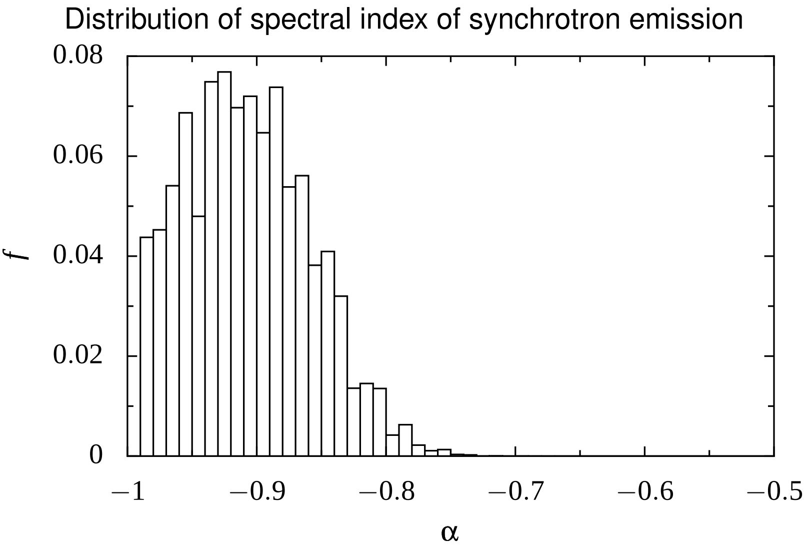
(U)LIRGs well fit by one-component models V



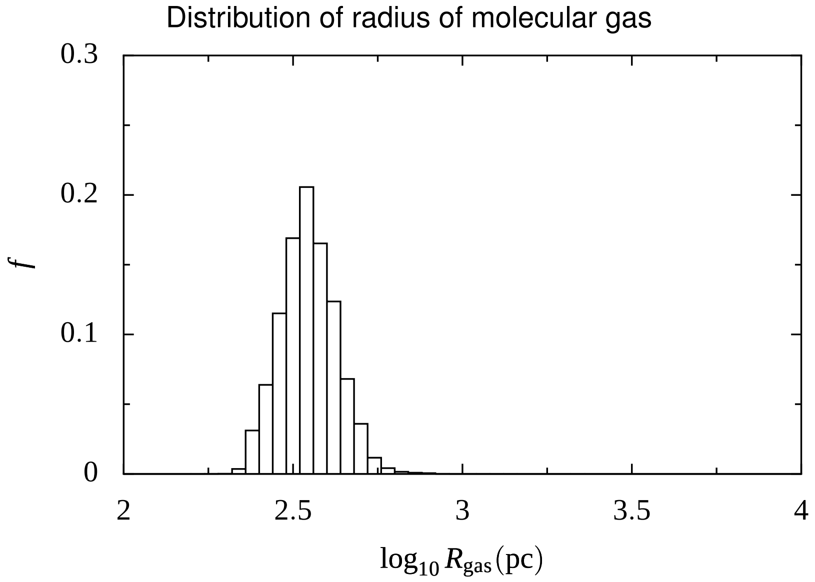
UGC 6436 – parameter distributions I



UGC 6436 – parameter distributions II

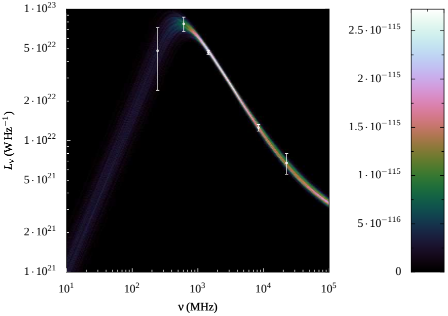


UGC 6436 – parameter distributions III

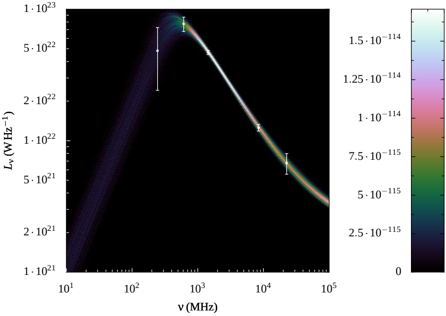


UGC 6436 – are radio and IR consistent? I

$Z=4.1 \times 10^{-7}$

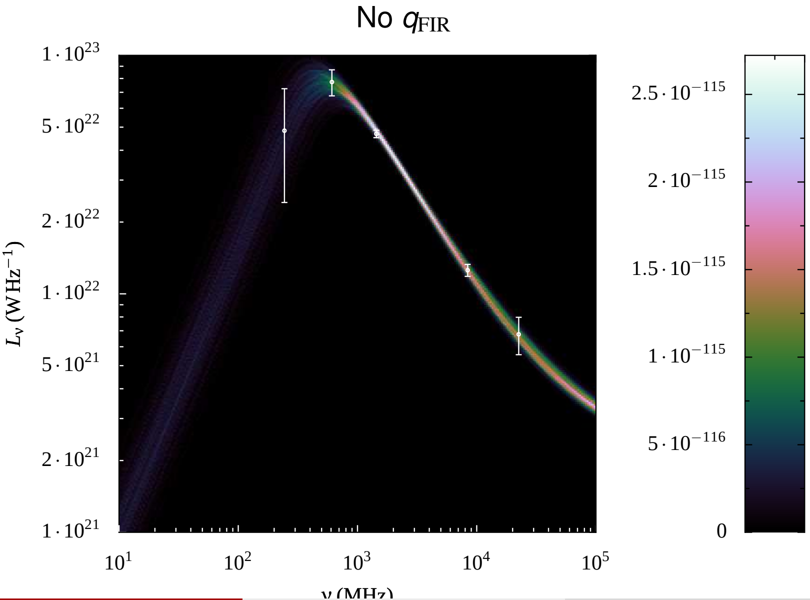


$Z=2.8 \times 10^{-6}$

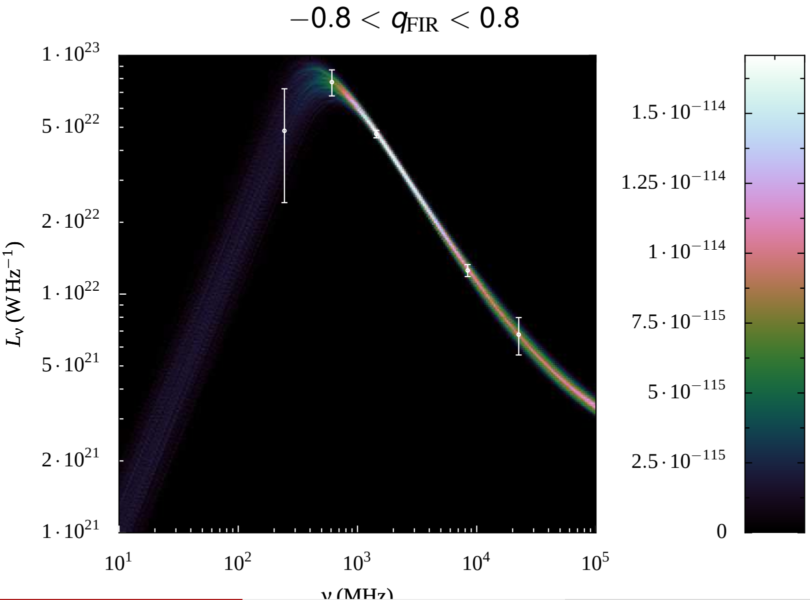


FIR emission per unit SFR
is a free parameter
[in fact: ratio to nominal ($q_{\rm FIR}$)]

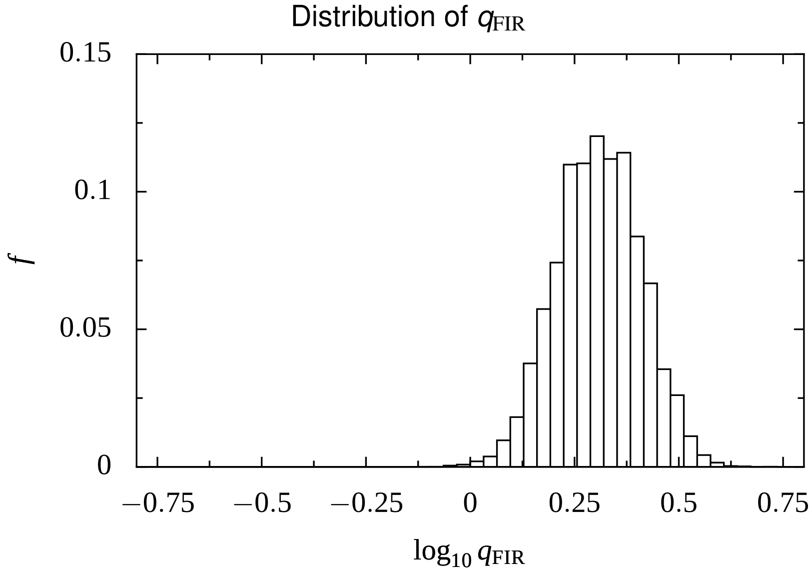
UGC 6436 – are radio and IR consistent? II



UGC 6436 – are radio and IR consistent? III



UGC 6436 – are radio and IR consistent? IV



Simple two-component model

Two of these!

- Spherical, homogeneous, well-mixed (“giant”) star-forming region
- Synchrotron emission based on the supernova rate
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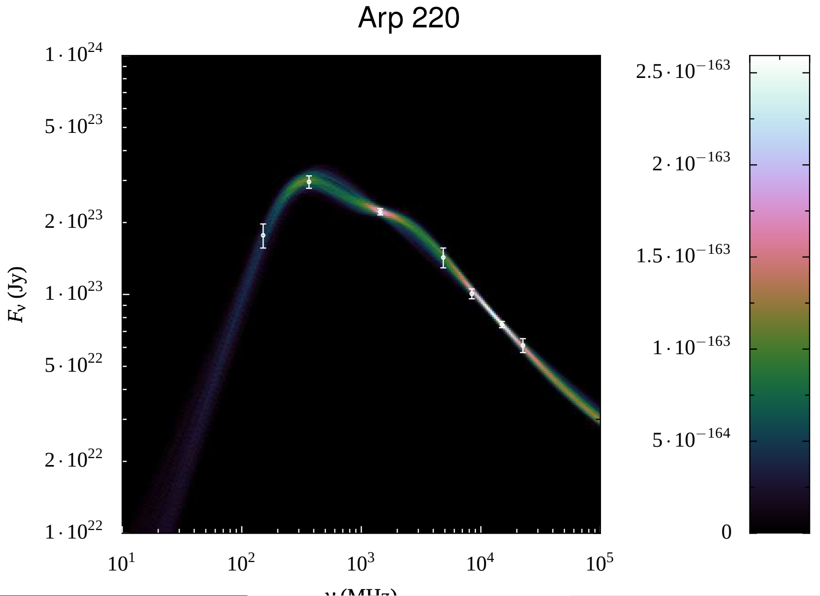
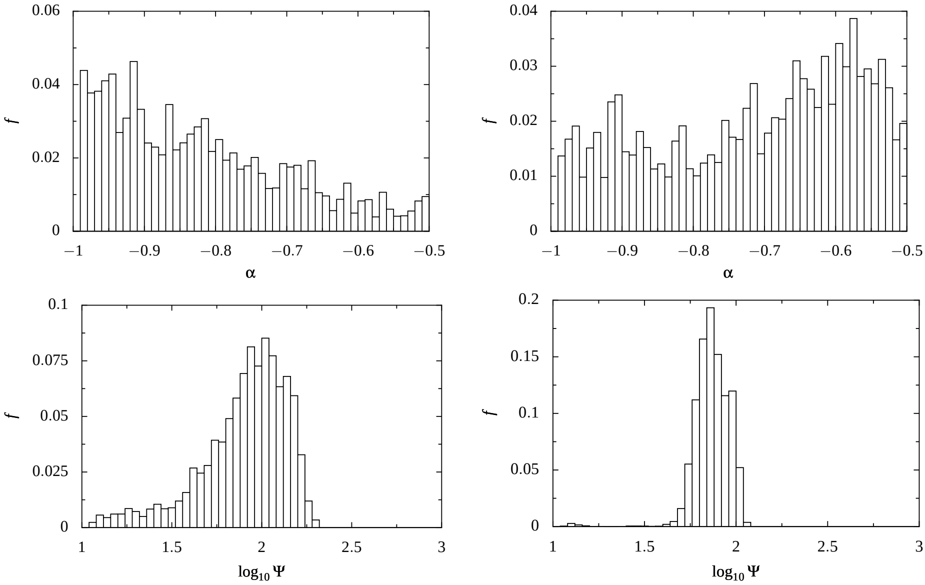
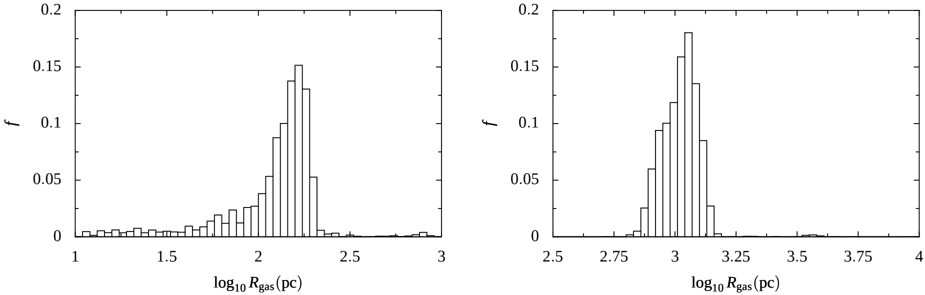
Arp 220 I


Arp 220
$1 \cdot 10^{24}$
$5 \cdot 10^{23}$
$2 \cdot 10^{23}$
$1 \cdot 10^{23}$
$5 \cdot 10^{22}$
$2 \cdot 10^{22}$
$1 \cdot 10^{22}$
F_ν (Jy)
10^1
10^2
10^3
10^4
10^5
$2.5 \cdot 10^{-163}$
$2 \cdot 10^{-163}$
$1.5 \cdot 10^{-163}$
$1 \cdot 10^{-163}$
$5 \cdot 10^{-164}$
0

Arp 220 – parameter distributions I



Arp 220 – parameter distributions II



Future directions

- ... complete the present analysis!
- More physical model:
  - Directly model synchrotron from the e^- spectrum
  - Kennicutt-Schmidt law for the star-formation rate
  - B by hydrostatic support? Minimum energy?
- Analysis across sample, i.e., all the galaxies together
- Volunteers?

Outline

Summary

Bayesian analysis of radio spectra:

- Objective model selection
- Full probability distributions, parameter correlations
- Natural way to introduce physical constraints through priors
- Easily handle nuisance parameters
- (Rigorous theoretical framework)

End result:

Fewer rules of thumb. Less fiddling with parameters.
Get the full picture and extract maximum information.
Let the data guide you to which experiments you should do next

References

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