

# Fitting and Comparison of Models of Radio Spectra

Bojan Nikolic

Astrophysics Group,  
Cavendish Laboratory/Kavli Institute for Cosmology  
University of Cambridge

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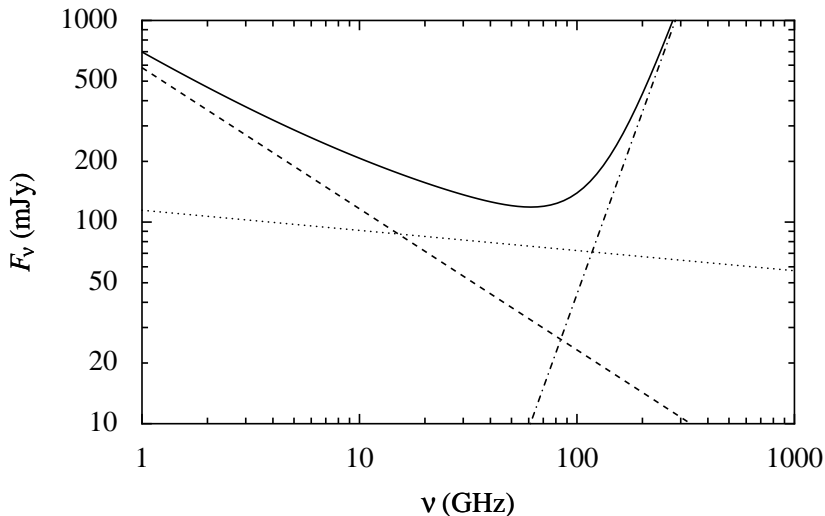
# Outline

- 1 Introduction
- 2 Method
  - Bayesian analysis
  - Implementation
  - Visualisation
- 3 Simple examples
  - NGC 628
  - NGC 3627
  - NGC 7331
- 4 Free-free component in a supernova remnant
- 5 (U)LIRGs
- 6 Summary/Further Directions/References

# Introduction

- Original motivation:  
Preparation for analysis of forthcoming data from GBT+MUSTANG
- A little bit of a “spare-parts” project in which I reused various software components I developed for other purposes
- Method paper: Nikolic (2009)
- All of the source code available under GPL license from:  
`http://www.mrao.cam.ac.uk/~bn204/galevol/specs/index.html`

# Schematic radio spectrum of a star-forming galaxy



Schematic & **hypothetical** (continuum-only) spectrum of NGC 3627: the dashed line is the synchrotron component; the dotted line is the free-free component; the dash-dot-dash line is the dust component; the solid line is the total emission.

# Why analyse radio spectra

- Energetics
  - Reconstruct the total energy balance from few/sparse measurements of the spectrum
  - What does the energy come from?
- Inference of properties of the source:
  - Geometry (e.g., filling factor from the low-frequency turnover)
  - Dynamics (e.g., through electron ageing)
- Redshift determination  $\equiv$  radio “photometric” redshifts
  - Currently mostly used for sub-millimetre selected (“SCUBA”) sources
- Physics:
  - Free-free emission
  - Slope of the dust continuum – physics of interstellar dust

# Analysis strategy

## Model fitting

In radio, sub-mm and far-IR, the physics is fairly well understood and candidate models are computationally easy. So analysis often consists of “fitting” a set of models to the observations.

- Synchrotron radiation (analytic or 1-D integral)
- Thermal free-free (analytic)
- Modified black-body emission from dust (analytic or 1-D integral)
- Spinning dust models (analytic)

# Requirements for model fitting

- Objective measure of how well the model fits the observed data
- For all model parameters:
  - Unbiased estimates
  - Error on these estimates
  - Correlations between the errors
  - Full probability distributions if significantly non-Gaussian
- Objective way of **comparing** how well *different* models fit the data
- A mechanism to incorporate already known constraints on model parameters
- Visualisation of the fit in comparison to observations

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# Bayesian analysis in a nutshell

- 1 Can handle “nuisance” parameters
- 2 Fully describes non-Gaussian distributions
- 3 Unbiased
- 4 Objective model (or hypothesis) selection

# Bayes equation & the evidence

$$p(\theta|D, H) = \frac{p(D|\theta, H)p(\theta|H)}{p(D|H)}$$

- $D$ : Observed data  $\rightarrow$  flux density at several frequencies
- $H$ : Hypothesis  $\rightarrow$  model for emission & priors for parameters
- $p(D|\theta, H)$ : Likelihood  $\rightarrow$  given a model *and* its parameters, how likely are the observed data?
- $p(\theta|D, H)$ : Posterior  $\rightarrow$  given a model, what we know about its parameters
- $p(D|H)$ : “Evidence”, objective measure of how good the model is

# Bayes equation & the evidence

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# Calculating the evidence

Evidence is an integral over the likelihood surface

$$p(D|H) = \int d\theta p(D|\theta, H)p(\theta|H)$$

- Evidence is not available from standard Markov Chain Monte Carlo calculations
- I use a new implementation of the **nested sampling** algorithm by Skilling (2006). Compared to MCMC, this algorithm is:
  - Efficient (fewer likelihood function evaluations)
  - Reliable (less chance of getting stuck in local maxima)
  - The output is both the evidence and the posterior distribution
- The algorithm is available under GPL

# Inputs/outputs

## Inputs

- 1 The model (as a routine/class)
- 2 Priors: only flat, independent priors supported. I.e., a “prior box”
- 3 Observed data and errors (for the moment assumed Gaussian)

## Outputs

- 1 The evidence value
- 2 Histograms of marginalised distributions of each model parameter
- 3 Two-dimensional histograms of partially marginalised distributions
- 4 Fan-diagram of flux vs frequency
- 5 Maximum likelihood plot of flux vs frequency

# Inputs/outputs

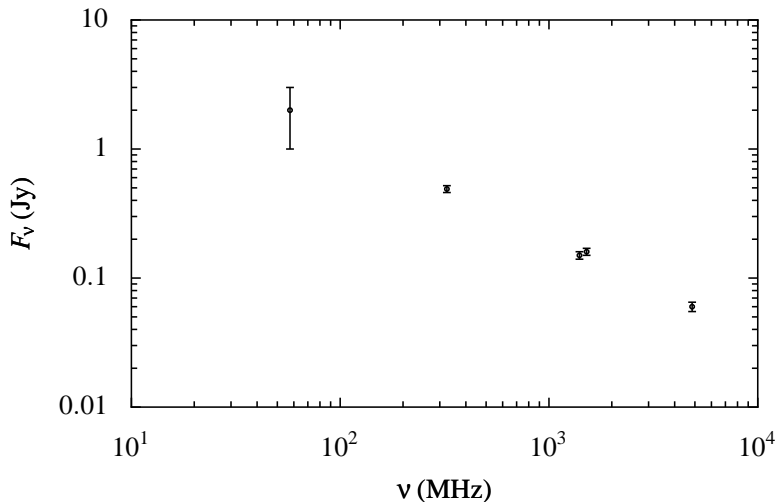
## Inputs

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## Outputs

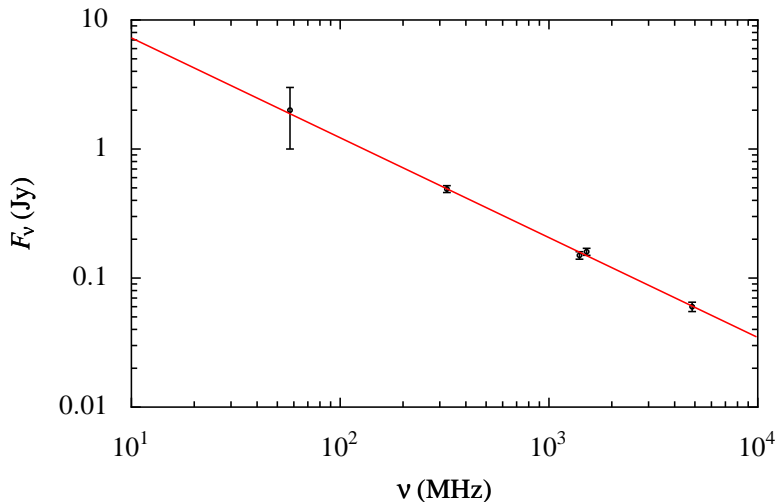
- 1 The evidence value
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- 5 Maximum likelihood plot of flux vs frequency

# NGC 628 observations



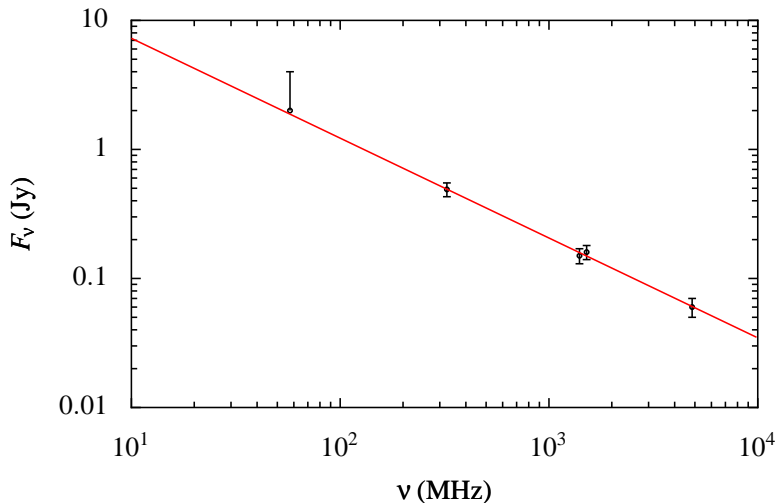
Observations at five frequencies of the near-by galaxy NGC 628 collected by Paladino et al. (2009)

# NGC 628 – max. likelihood line fit



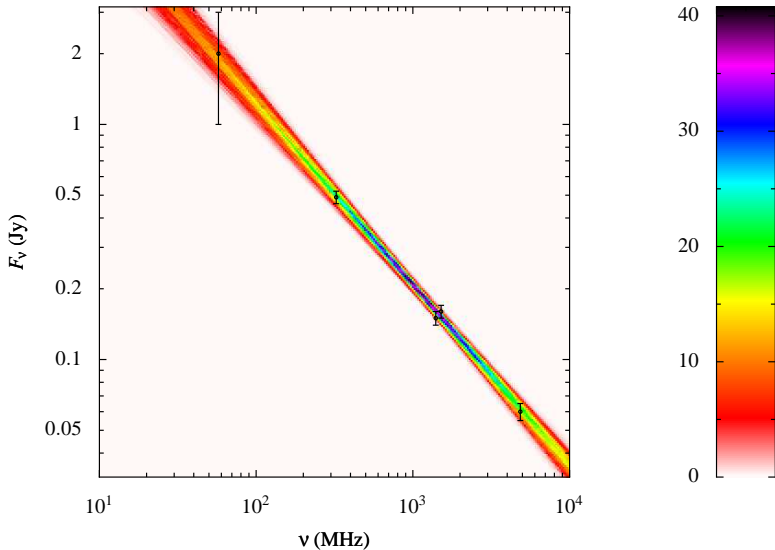
Observations at five frequencies of the near-by galaxy NGC 628 collected by Paladino et al. (2009)

## NGC 628 – doubled errors &amp; max. likelihood line fit

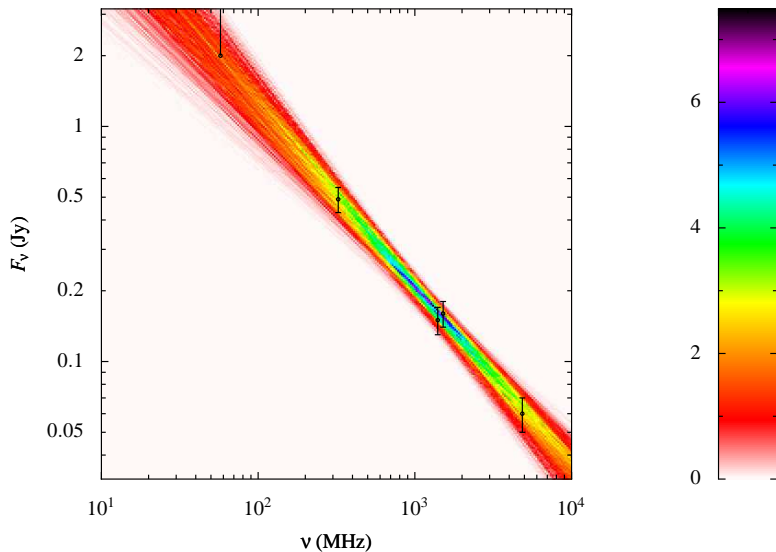


I have scaled up the error estimates by a factor of two

# NGC 628 – original errors & fan-diagram



# NGC 628 – doubled errors & fan-diagram



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# Four simple models for synchrotron emission

## Underlying synchrotron spectrum

- Power law

$$F_\nu(\nu) = F_\nu^0 \cdot \left(\frac{\nu}{1 \text{ GHz}}\right)^\alpha$$

- Continuous injection of electrons  $\rightarrow$  very approximately broken power law

$$\begin{aligned}
 &F_\nu^0 \cdot \left(\frac{\nu}{1 \text{ GHz}}\right)^\alpha && \nu \leq \nu_{\text{br}} \\
 &F_\nu^0 \cdot \left(\frac{\nu}{1 \text{ GHz}}\right)^\alpha \left(\frac{\nu}{\nu_{\text{br}}}\right)^{-1/2} && \nu > \nu_{\text{br}}.
 \end{aligned}
 \tag{1}$$

## Low-frequency optical depth effects

- None
- Synchrotron self-absorption

$$x = \nu / \nu_{\text{pk}}$$

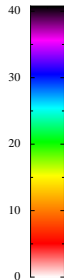
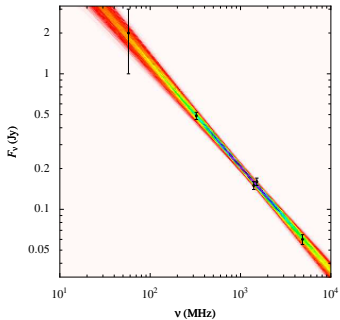
$$A_s = x^{-\alpha+5/2}$$

$$\times \left[ 1 - \exp\left(1 - x^{\alpha-5/2}\right) \right]$$

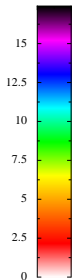
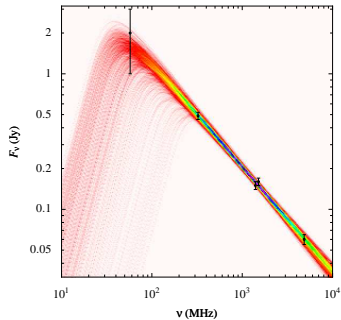
For the purposes of these examples, I've taken the models from Paladino et al. (2009) to go with their data – both more complex and more physical models could be used

# Model fits for NGC 628 I

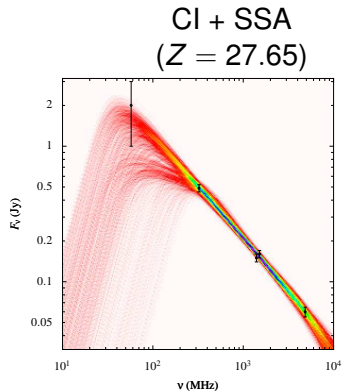
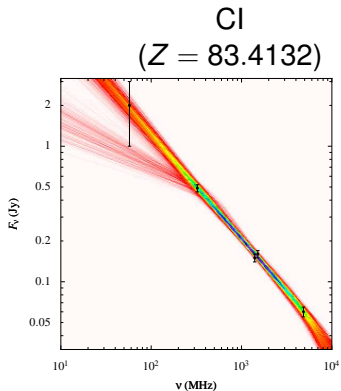
Power-law  
( $Z = 185.874$ )



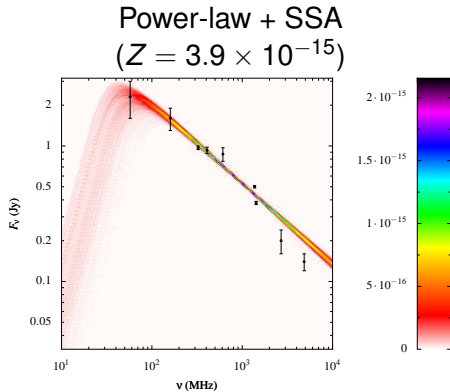
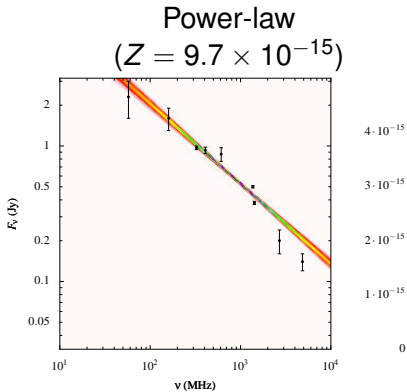
Power-law + SSA  
( $Z = 76.3034$ )



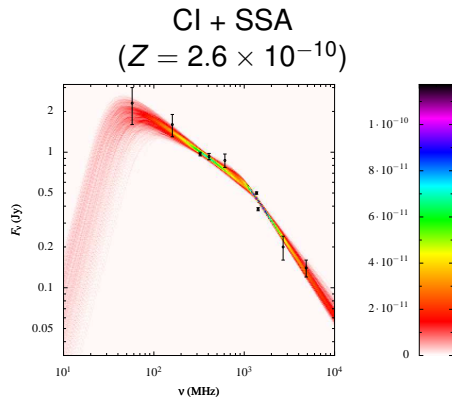
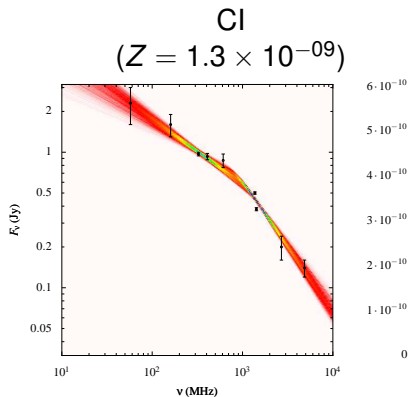
# Model fits for NGC 628 II



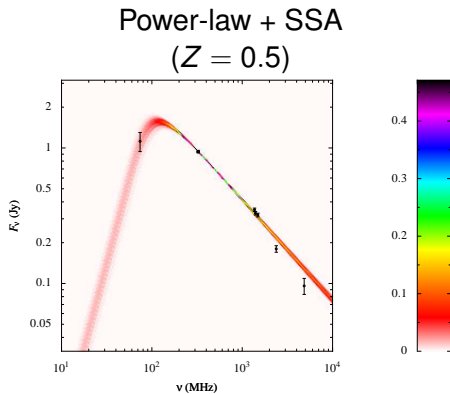
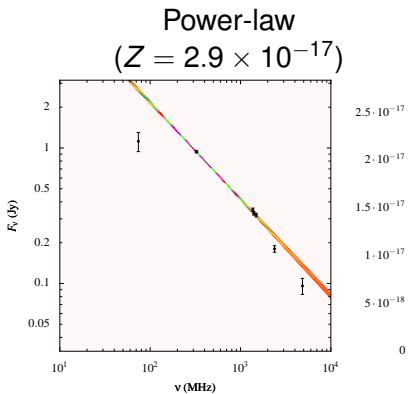
# Model fits for NGC 3627 I



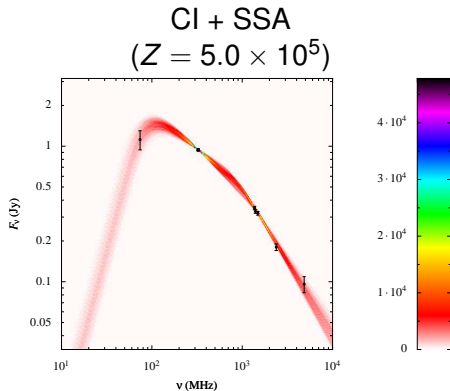
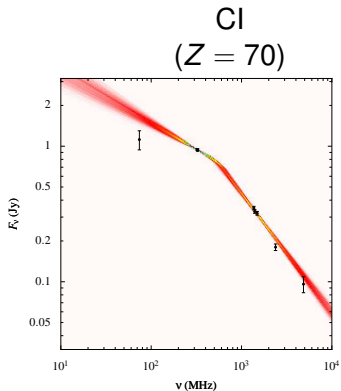
## Model fits for NGC 3627 II



# Model fits for NGC 7331 I



# Model fits for NGC 7331 II



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# Introduction

Data courtesy of D. A. Green in Cambridge

- Analysis of spectrum of supernova remnant HB3
- Is there evidence for flattening of the spectrum?
  - Could be interpreted as a thermal free-free component due to interaction of shock with the molecular cloud
- See Urošević et al. (2007), Green (2007), Onic & Urosevic (2008)

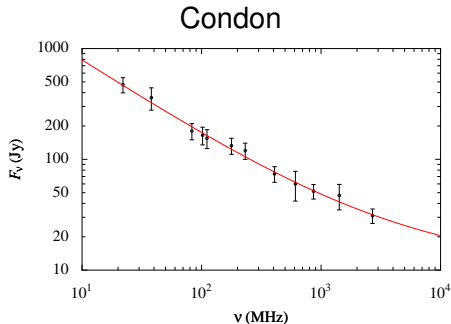
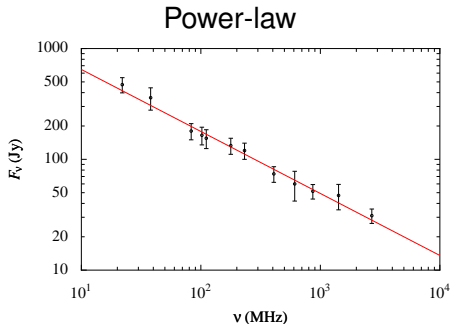
# Condon model

- Single power-law synchrotron with slope ( $\alpha$ ) as a free parameter
- Free-free emission component ( $H$  is the thermal fraction at 1 GHz)
- Thermal free-free absorption at low frequencies ( $\tau$  is the optical depth at 1 GHz)

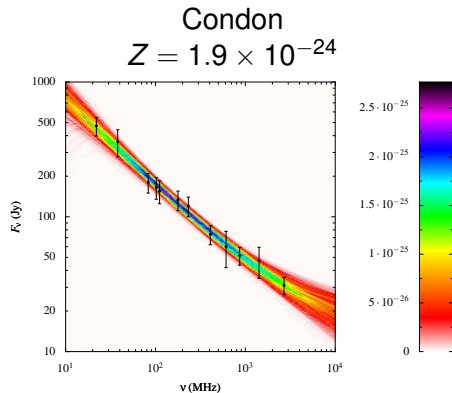
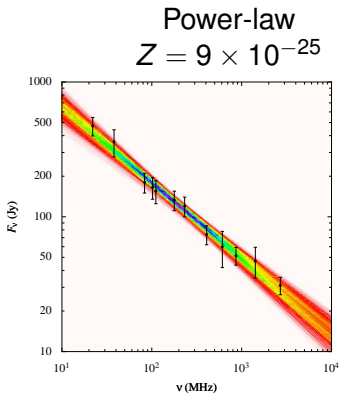
$$A(\nu; \tau^*) = 1 - \exp \left[ -10^{\tau^*} \left( \frac{\nu}{1 \text{ GHz}} \right)^{-2.1} \right]$$

$$F_\nu(\nu; H, \alpha) = \frac{A(\nu)}{A(1 \text{ GHz})} \left( \frac{\nu}{1 \text{ GHz}} \right)^2 \left[ H + (1 - H) \left( \frac{\nu}{1 \text{ GHz}} \right)^{0.1 + \alpha} \right]$$

## Maximum likelihood model fits for HB3

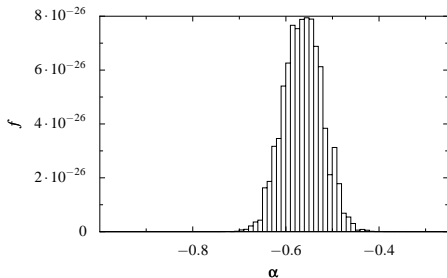


# Fan diagrams of the Bayesian analysis of HB3

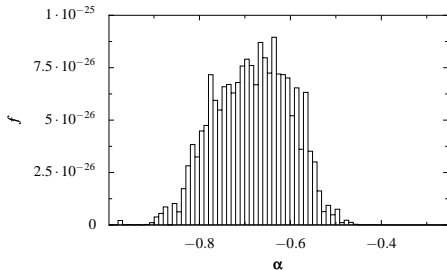


Marginalised distribution of  $\alpha$ 

Power-law  
 $Z = 9 \times 10^{-25}$

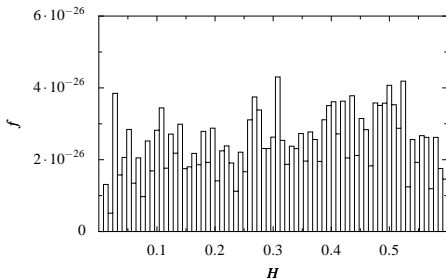


Condon  
 $Z = 1.9 \times 10^{-24}$

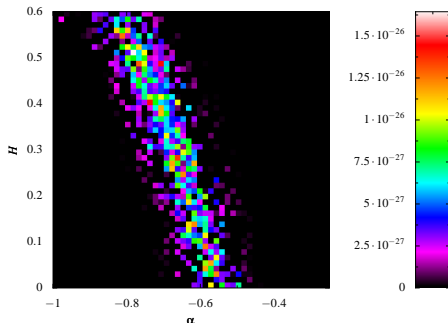


# (Partially-)Marginalised distribution of $H$

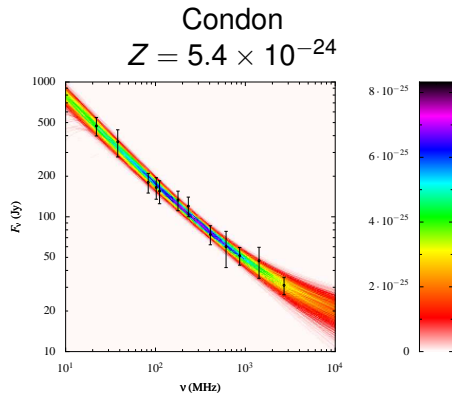
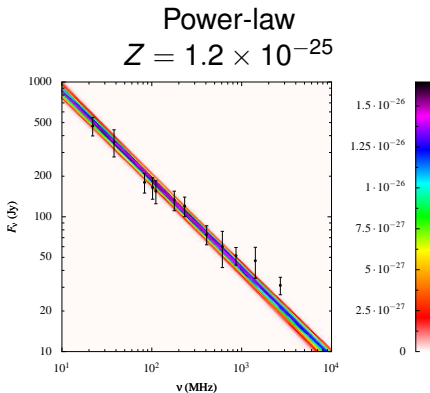
PD of thermal fraction at 1 GHz



Correlation between thermal fraction and synchrotron slope

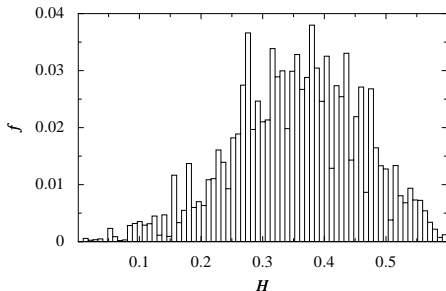


# New prior: constrain $\alpha$ to 0.1 range around -0.7

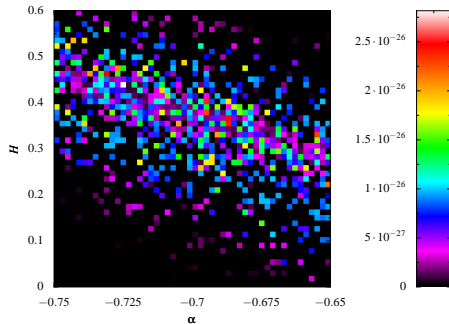


# New prior: distribution of $H$

PD of thermal fraction at 1 GHz



Correlation between thermal fraction and synchrotron slope



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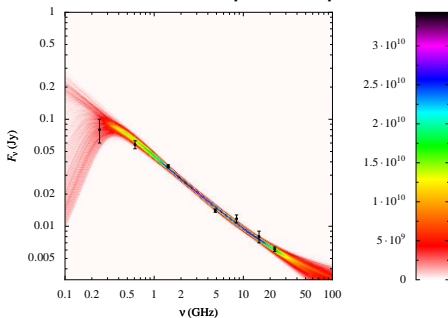
# A physically based analysis of (U)LIRG spectra

- In-progress work in collaboration with Marcel Clemens from U. of Padua
- Data are based on Clemens et al. (2010)
- Model:
  - Parametrise in terms of supernova rates, star-formation rates, filling factors  
*Not in terms of:* flux densities, opacities, electron densities, etc.
  - Two components: one with 0.5kpc scale, the other with 0.05kpc
  - Each component consists of star-formation driven thermal opacity and supernova-driven synchrotron component
  - Prior:
    - 0.5 kpc comp:  $-1 < \alpha < -1/2$ ,  $10^{-1} \text{ yr}^{-1} < \text{SN rate} < 10^1 \text{ yr}^{-1}$ ,  $1 \text{ M}_{\odot} \text{ yr}^{-1} < \text{SFR} < 10^3 \text{ M}_{\odot} \text{ yr}^{-1}$ ,  $10^{-3} < \text{areal filling factor} < 1$
    - 0.05 kpc comp:  $-1 < \alpha < -1/2$ ,  $10^{-1} \text{ yr}^{-1} < \text{SN rate} < 10^1 \text{ yr}^{-1}$ ,  $1 \text{ M}_{\odot} \text{ yr}^{-1} < \text{SFR} < 10^3 \text{ M}_{\odot} \text{ yr}^{-1}$ ,  $10^{-5} < \text{areal filling factor} < 10^{-2}$

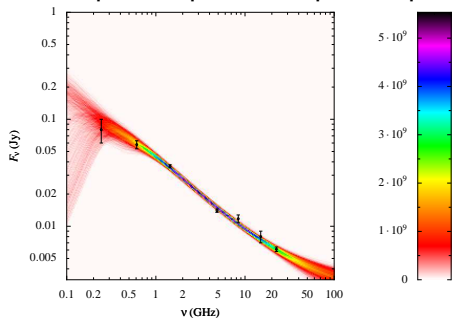
# Gallery of fan charts I

Arp 148

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp

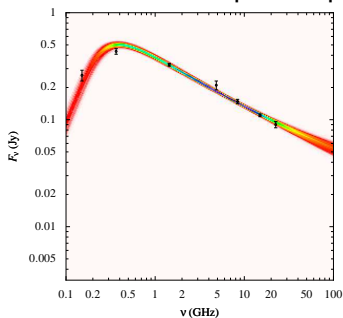


Z  $\rightarrow$  this model is preferred

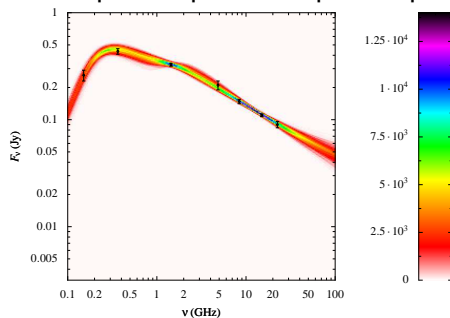
# Gallery of fan charts II

Arp 220

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp

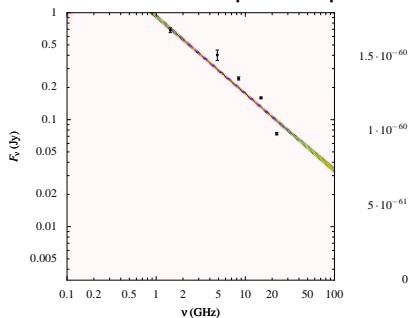


$Z \rightarrow$  this model is preferred

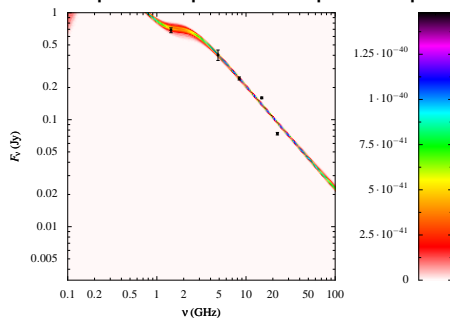
# Gallery of fan charts III

## Arp299

### One 0.5kpc comp



### 0.5 kpc comp + 0.05 kpc comp

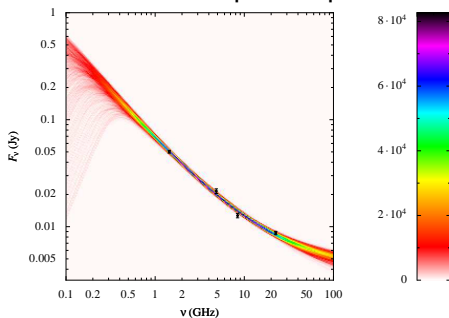


$Z \rightarrow$  this model is preferred

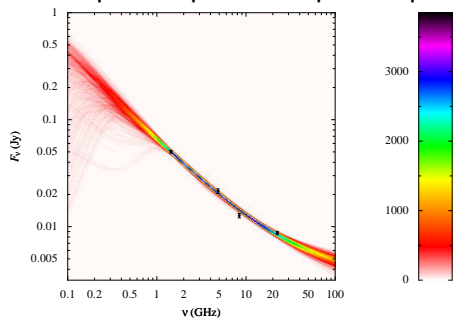
# Gallery of fan charts IV

## CGCG436-30

### One 0.5kpc comp



### 0.5 kpc comp + 0.05 kpc comp

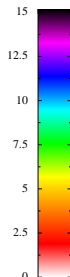
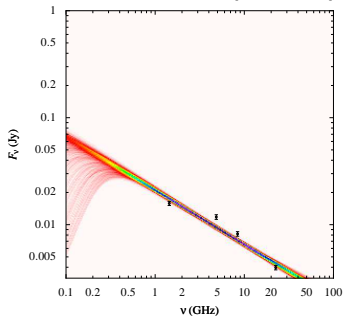


Z  $\rightarrow$  this model is preferred

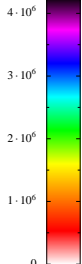
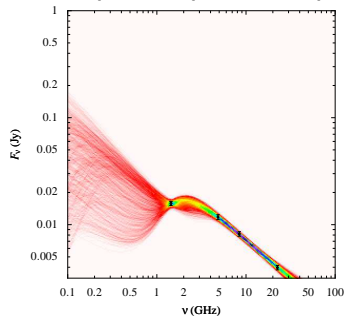
# Gallery of fan charts V

I0136-1042

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp

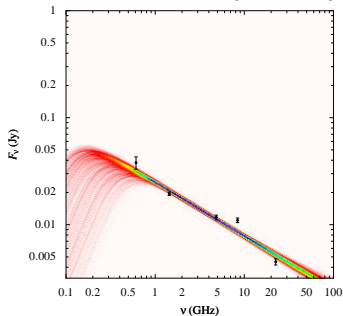


$Z \rightarrow$  this model is preferred

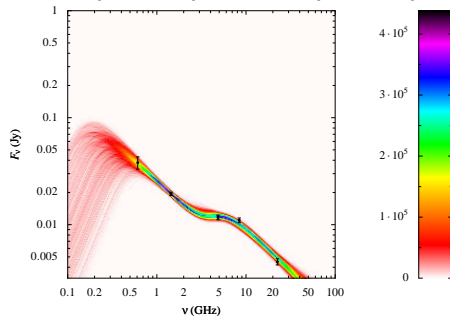
# Gallery of fan charts VI

I03359+1523

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp

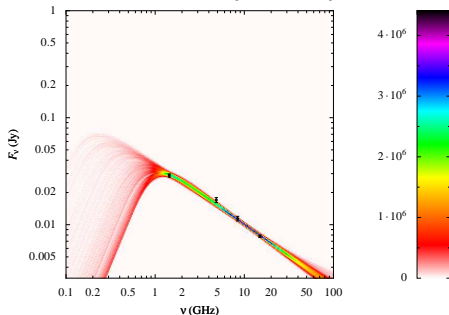


$Z \rightarrow$  this model is preferred

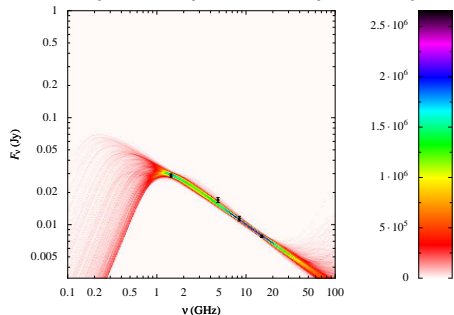
# Gallery of fan charts VII

## I05189-2524

### One 0.5kpc comp



### 0.5 kpc comp + 0.05 kpc comp

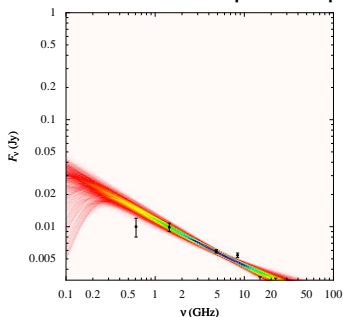


Evidence about the same

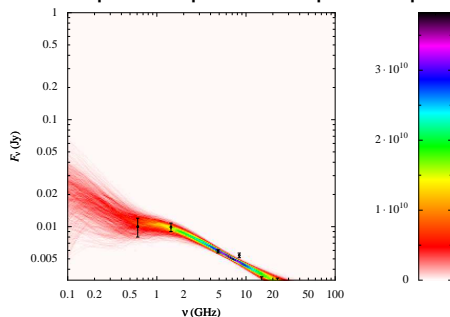
# Gallery of fan charts VIII

110173+0828

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp

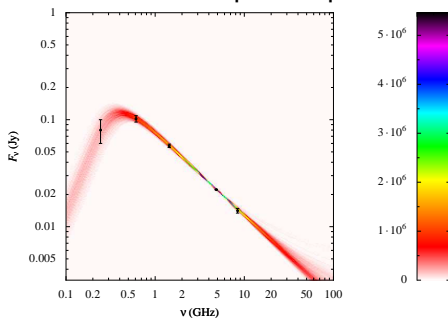


$Z \rightarrow$  this model is preferred

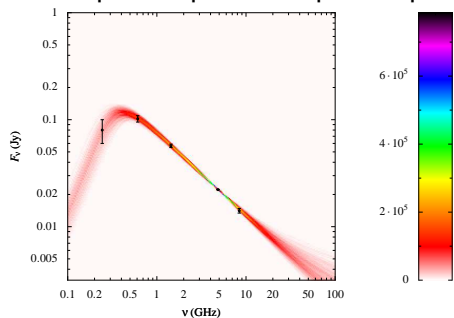
# Gallery of fan charts IX

110565+2448

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp

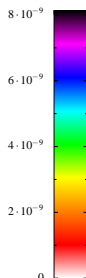
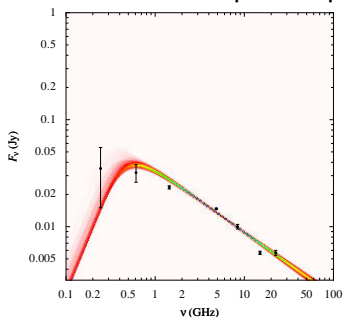


Z  $\rightarrow$  this model is preferred

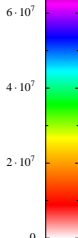
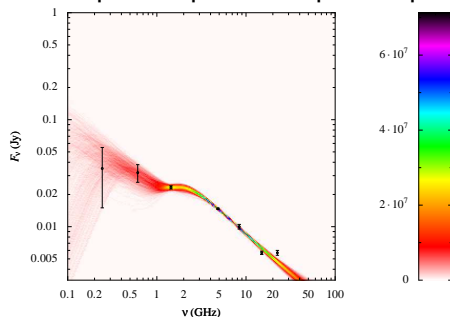
# Gallery of fan charts X

112112+0305

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp

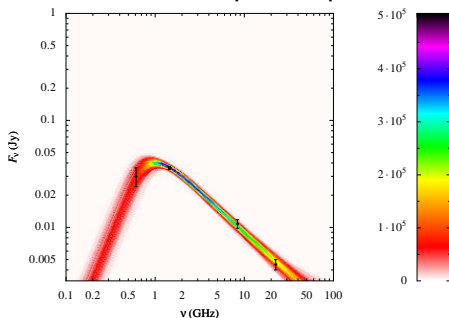


$Z \rightarrow$  this model is preferred

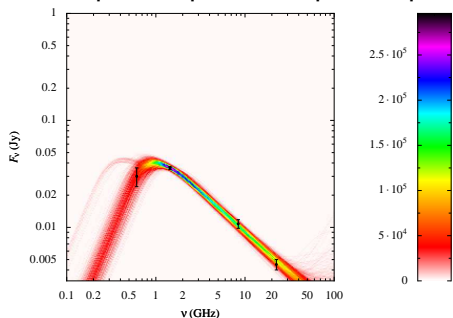
# Gallery of fan charts XI

## I14348-1447

### One 0.5kpc comp



### 0.5 kpc comp + 0.05 kpc comp

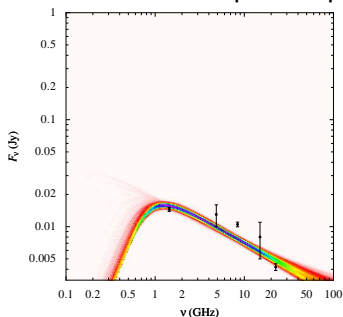


Z → this model is preferred

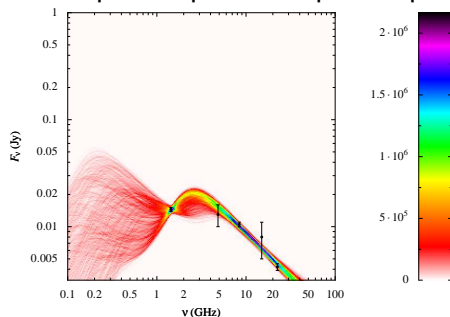
# Gallery of fan charts XII

115250+3609

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp

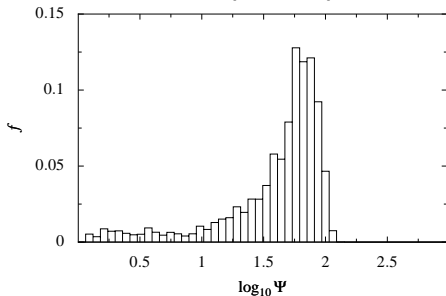


$Z \rightarrow$  this model is preferred

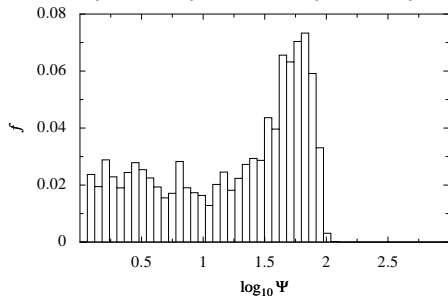
# Star-formation as evidenced by the free-free emission

Arp 148

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp

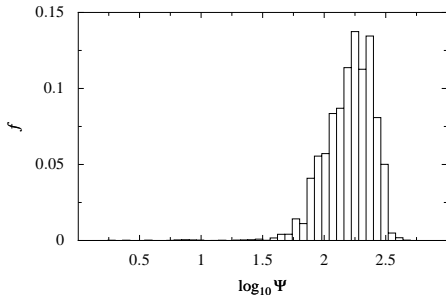


$Z \rightarrow$  this model is preferred

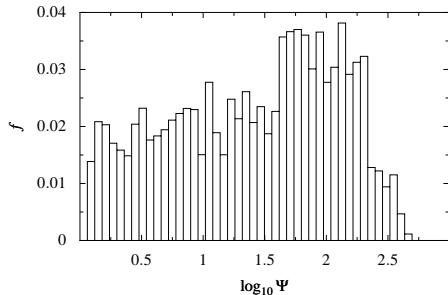
# Star-formation as evidenced by the free-free emission II

Arp 220

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp

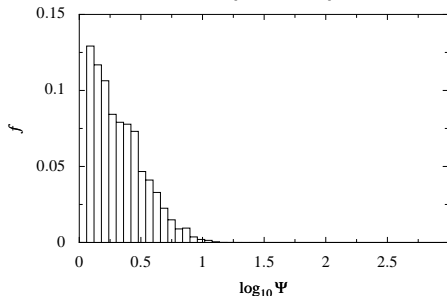


$Z \rightarrow$  this model is preferred

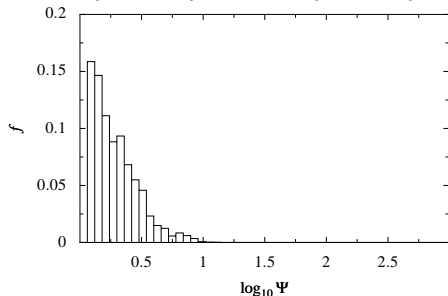
# Star-formation as evidenced by the free-free emission III

Arp 299 = IC 694 + NGC 3690

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp



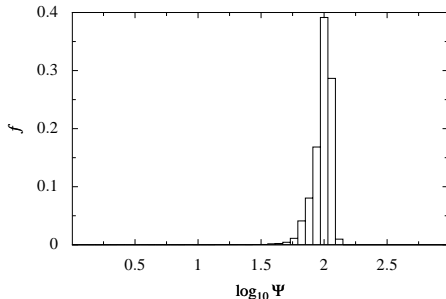
$Z \rightarrow$  this model is preferred

**Comment:** good upper limits on thermal emission. Normally classified as star-forming but does also have an AGN

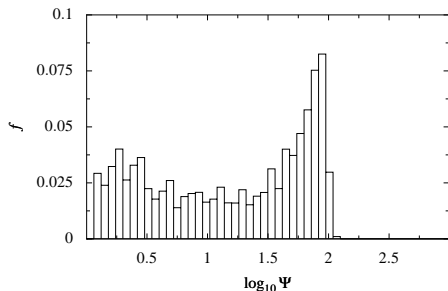
# Star-formation as evidenced by the free-free emission IV

CGCG436-30

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp



$Z \rightarrow$  this model is preferred

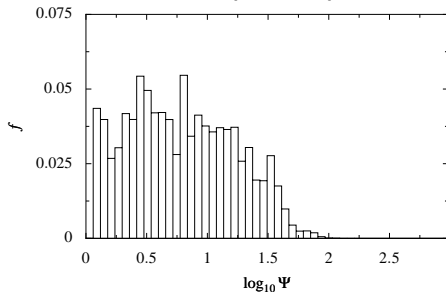
**Comment:** nice clear detection of the thermal emission, with SFR roughly consistent with the IR luminosity

# Star-formation as evidenced by the free-free emission

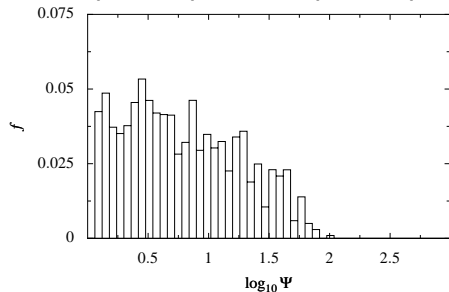
## V

I0136-1042

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp

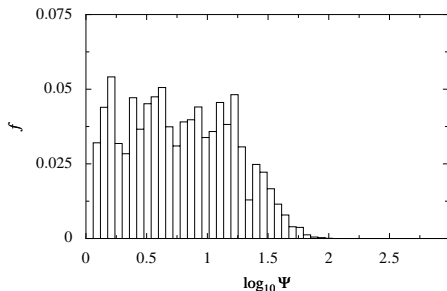


$Z \rightarrow$  this model is preferred

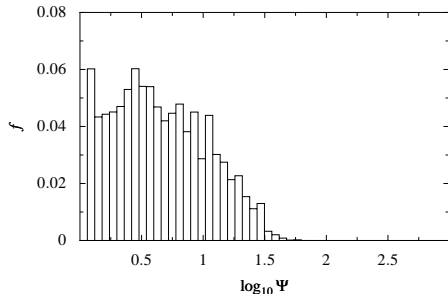
# Star-formation as evidenced by the free-free emission VI

103359+1523

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp

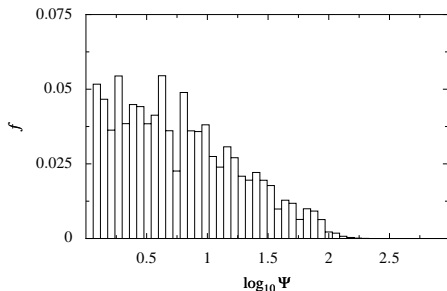


$Z \rightarrow$  this model is preferred

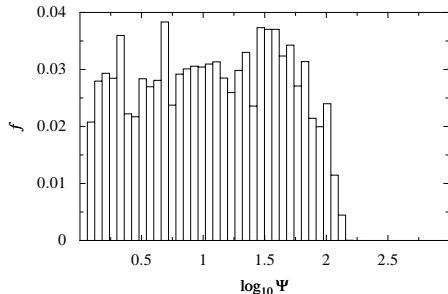
# Star-formation as evidenced by the free-free emission VII

I05189-2524

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp

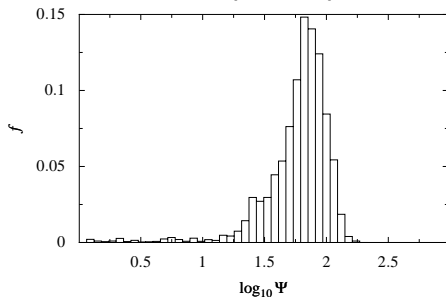


Evidence about the same

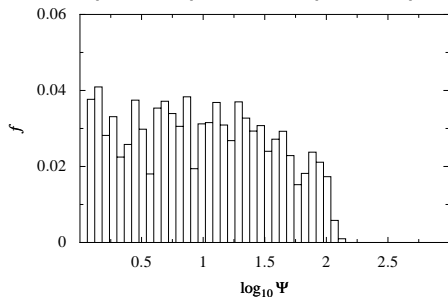
# Star-formation as evidenced by the free-free emission VIII

I10173+0828

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp

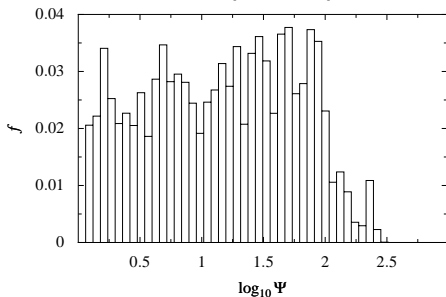


$Z \rightarrow$  this model is preferred

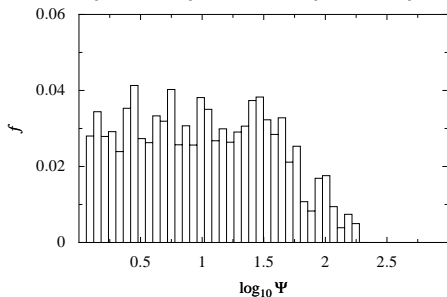
# Star-formation as evidenced by the free-free emission IX

I10565+2448

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp

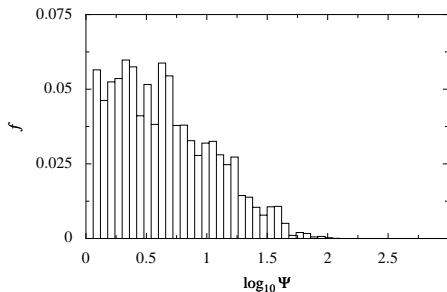


$Z \rightarrow$  this model is preferred

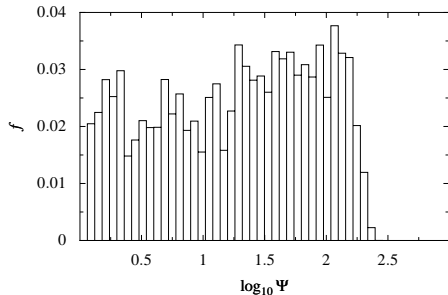
# Star-formation as evidenced by the free-free emission

112112+0305

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp

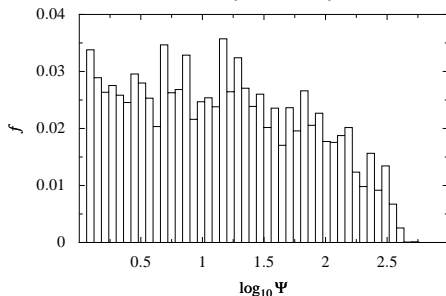


$Z \rightarrow$  this model is preferred

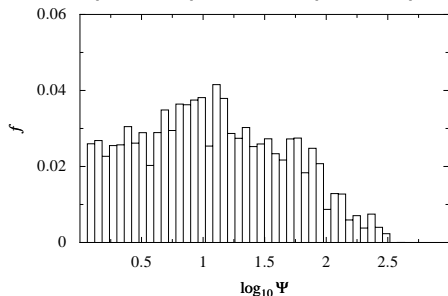
# Star-formation as evidenced by the free-free emission XI

I14348-1447

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp

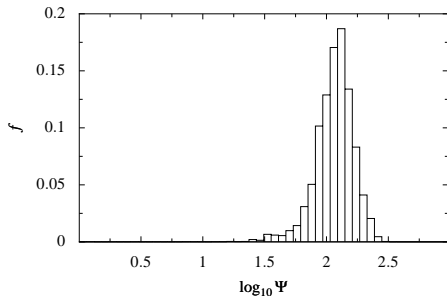


$Z \rightarrow$  this model is preferred

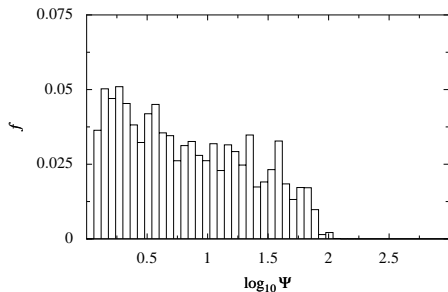
# Star-formation as evidenced by the free-free emission XII

I15250+3609

One 0.5kpc comp



0.5 kpc comp + 0.05 kpc comp



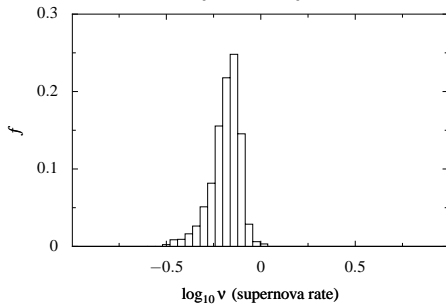
$Z \rightarrow$  this model is preferred

**Comment:** An example of where using the simpler one-component model would give a very wrong estimate of the thermal emission (and associated star-formation rate)

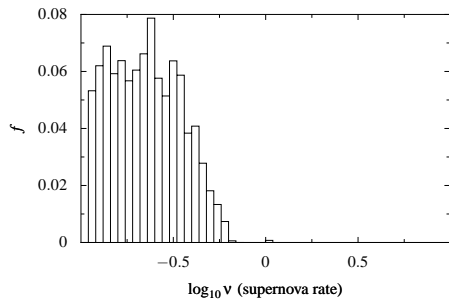
# Supernova rates I

Arp 148

0.5kpc comp



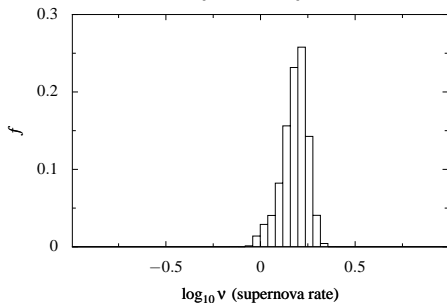
0.05kpc comp



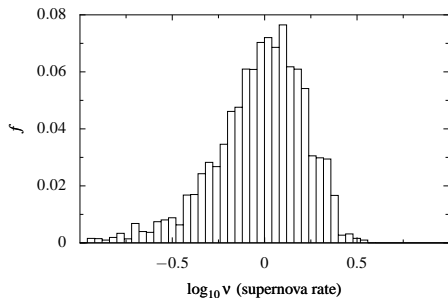
# Supernova rates II

Arp 220

0.5kpc comp



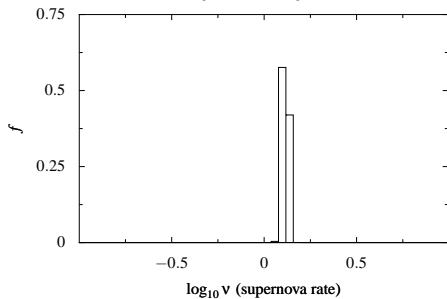
0.05kpc comp



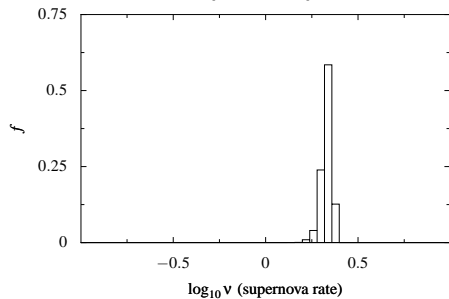
## Supernova rates III

Arp299

0.5kpc comp



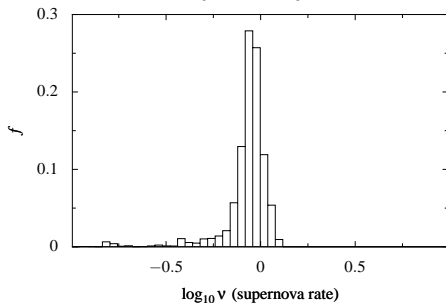
0.05kpc comp



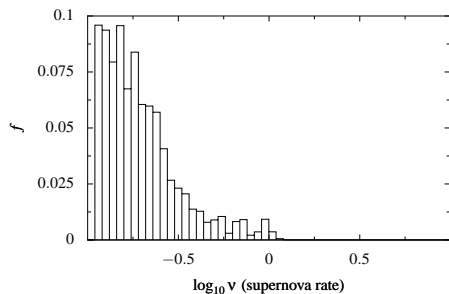
# Supernova rates IV

CGCG436-30

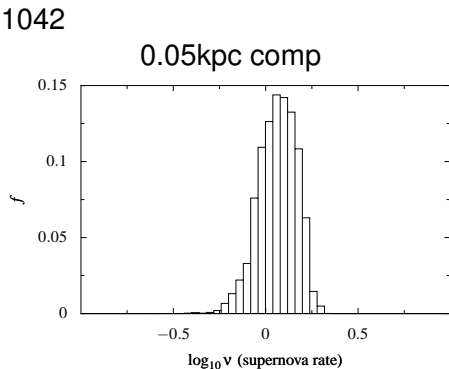
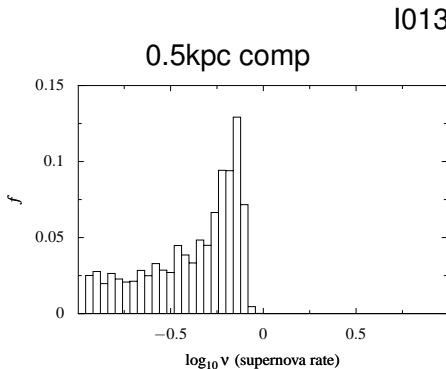
0.5kpc comp



0.05kpc comp



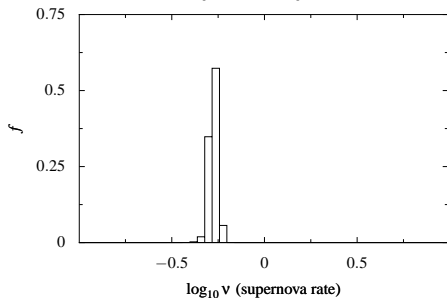
# Supernova rates $V$



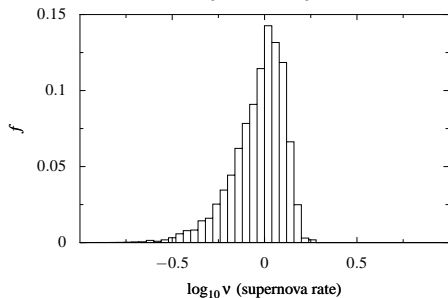
## Supernova rates VI

I03359+1523

0.5kpc comp



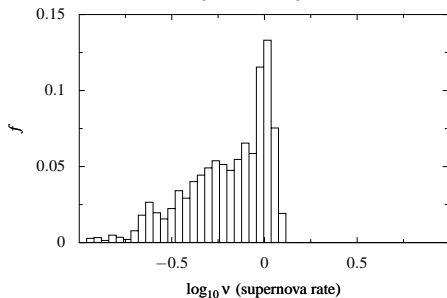
0.05kpc comp



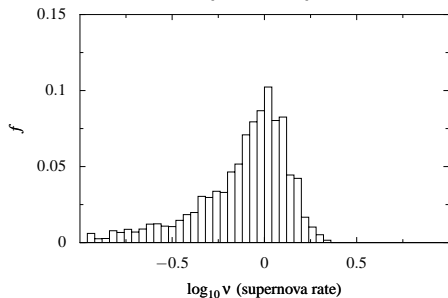
# Supernova rates VII

I05189-2524

0.5kpc comp



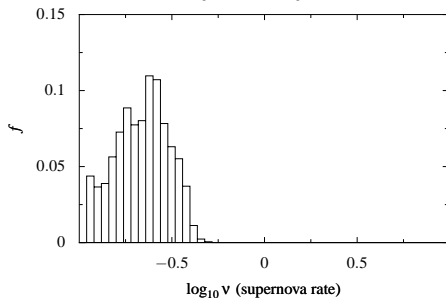
0.05kpc comp



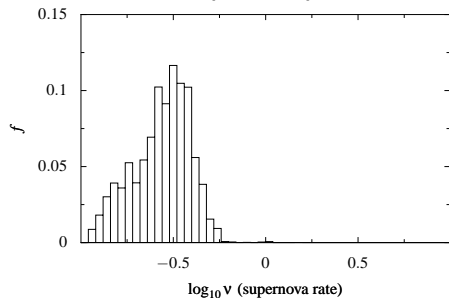
# Supernova rates VIII

I10173+0828

0.5kpc comp



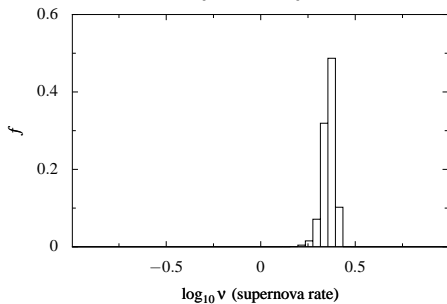
0.05kpc comp



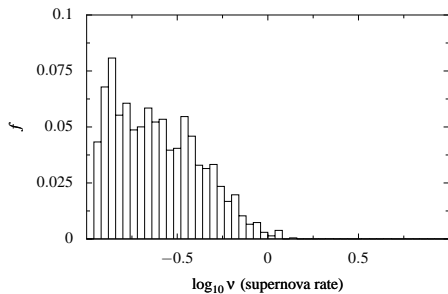
# Supernova rates IX

I10565+2448

0.5kpc comp



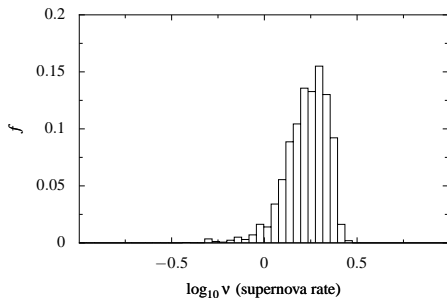
0.05kpc comp



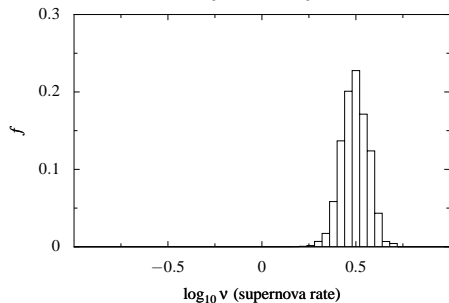
# Supernova rates X

I12112+0305

0.5kpc comp



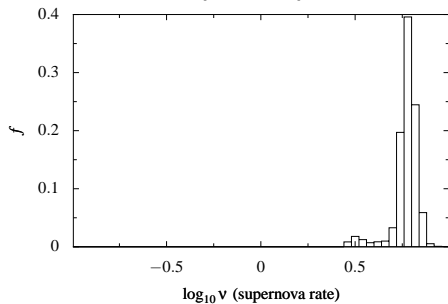
0.05kpc comp



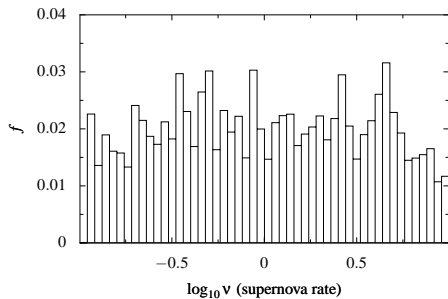
# Supernova rates XI

I14348-1447

0.5kpc comp



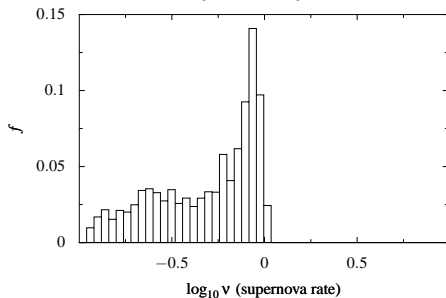
0.05kpc comp



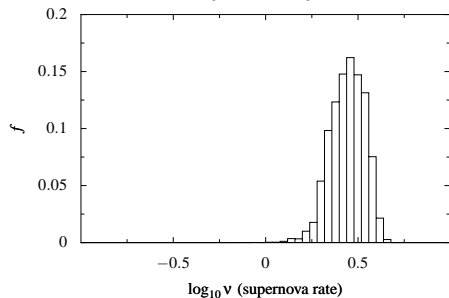
# Supernova rates XII

I15250+3609

0.5kpc comp



0.05kpc comp



# Outline

- 1 Introduction
- 2 Method
  - Bayesian analysis
  - Implementation
  - Visualisation
- 3 Simple examples
  - NGC 628
  - NGC 3627
  - NGC 7331
- 4 Free-free component in a supernova remnant
- 5 (U)LIRGs
- 6 **Summary/Further Directions/References**

# Summary/Further Directions

Advantages of this fitting approach:

- Full probability distributions
- Objective model selection
- Natural way to introduce physical constraints through priors
- (Rigorous theoretical framework)

End result:

Fewer rules of thumb, fiddling with parameters, etc.

You see the full picture and extract maximum information

# References

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