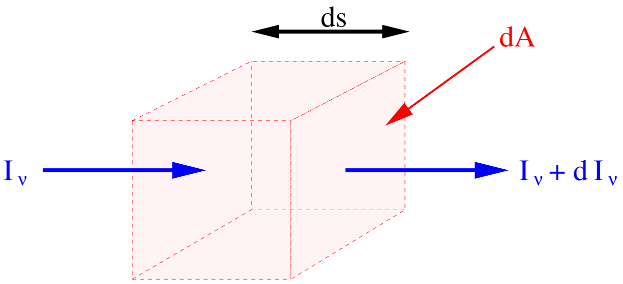
The Transfer of Radiation

When observing an astronomical source we may be looking through a cloud of matter which lies along the line of sight. This matter may *absorb* the radiation from the source, *scatter* it or in fact *emit* further radiation. Each of these will change the source's apparent intensity.

Absorption



Consider light shining into a sparse cloud of perfectly absorbing spheres of cross section σ_ν and number density n. As the beam of area dA propagates a distance ds into the cloud it encounters a total absorbing cross-section of $\sigma_\nu\, n\, ds,\ dA$, so we expect a fraction $\sigma_\nu\, n\, ds$ of the beam to be absorbed.

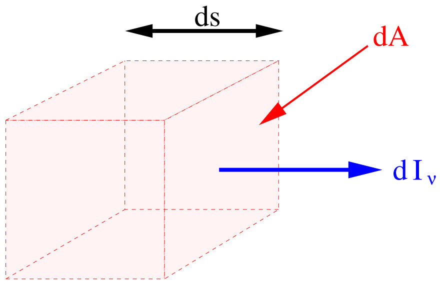
$$\Rightarrow dI_\nu = -I_\nu n \sigma_\nu ds \equiv -\alpha_\nu I_\nu ds$$

Thus the *absorption coefficient* $\alpha_\nu = n\sigma_\nu$ is defined as the fractional loss of intensity per unit length, with dimensions m^{-1}. It follows that the photon mean free path $l_\nu = 1/\alpha_\nu$.

Emission

An excited atom can return to its ground state through two distinct mechanisms: (i) the atom emits energy spontaneously; (ii) it is stimulated into emission by the presence of electromagnetic radiation.

- The amount of stimulated emission is proportional to I_ν, as was the amount of absorption. Therefore for simplicity it can be considered to be negative absorption and its effect included in α_ν .



- We define a *spontaneous emission coefficient* j_ν which is the energy emitted per unit *time* per unit *volume* per unit *solid angle* per unit *frequency*.

This has units $\mathrm{W\,m^{-3}sr^{-1}Hz^{-1}}$

$$dE = j_\nu \, dV \, d\Omega \, d\nu \, dt$$

Thus on crossing a length ds a beam's specific intensity is increased by spontaneous emission by

$$dI_\nu = j_\nu ds$$

The optical depth

A medium is said to be opaque or *optically thick* if on average a photon cannot pass through the medium without absorption. Conversely, a transparent medium is said to be *optically thin*. Both these properties are functions of wavelength; for example, a pane of glass is optically thin in the optical, but optically thick in the infrared.

We define the optical depth τ_ν

$$\tau_\nu = \int \alpha_\nu ds$$

- A medium is optically thick at a particular frequency if $\tau_\nu > 1$.

- The mean optical depth travelled by a photon before absorption is 1.

- In an optically thick homogenous medium the number of steps taken for a photon to diffuse out $\sim \tau^2$.

Example

- τ through 20m of water ~ 1.

→ τ through 2m of water ~ 0.1 $\Rightarrow$ You can see to the bottom of a swimming pool.

→ τ through 200m of water ~ 10 $\Rightarrow$ You cannot see to the bottom of the sea.

The Radiative Transfer Equation

Combining emission and absorbtion gives the Radiative Transfer Equation

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

This can be rewritten as

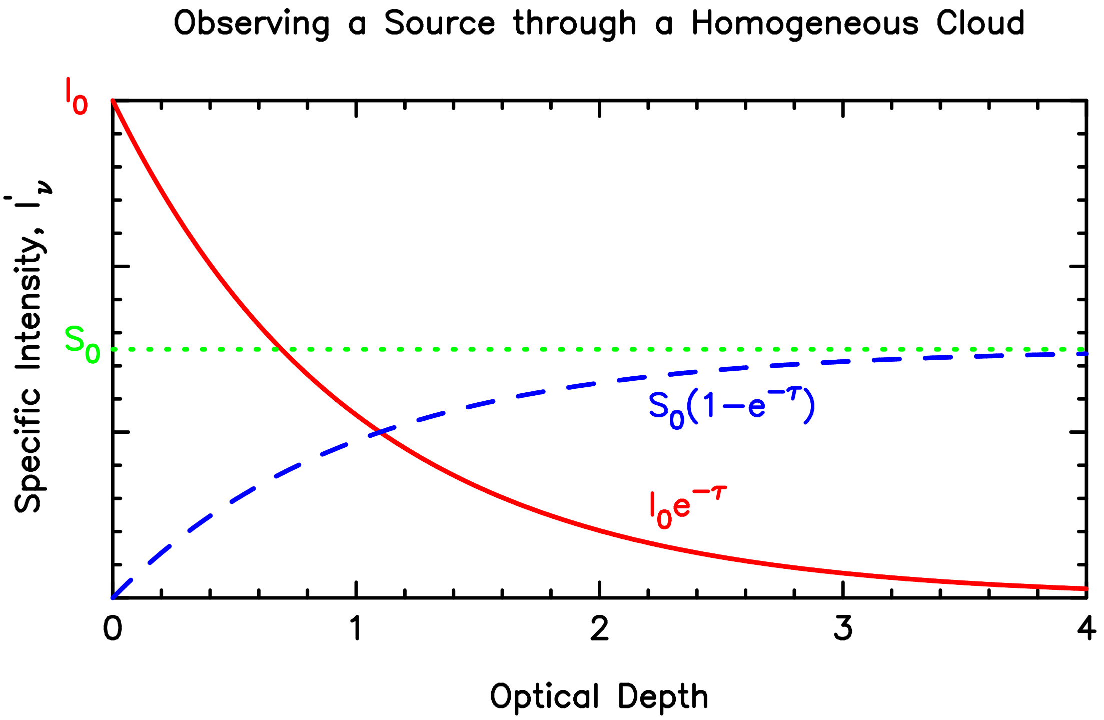
$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

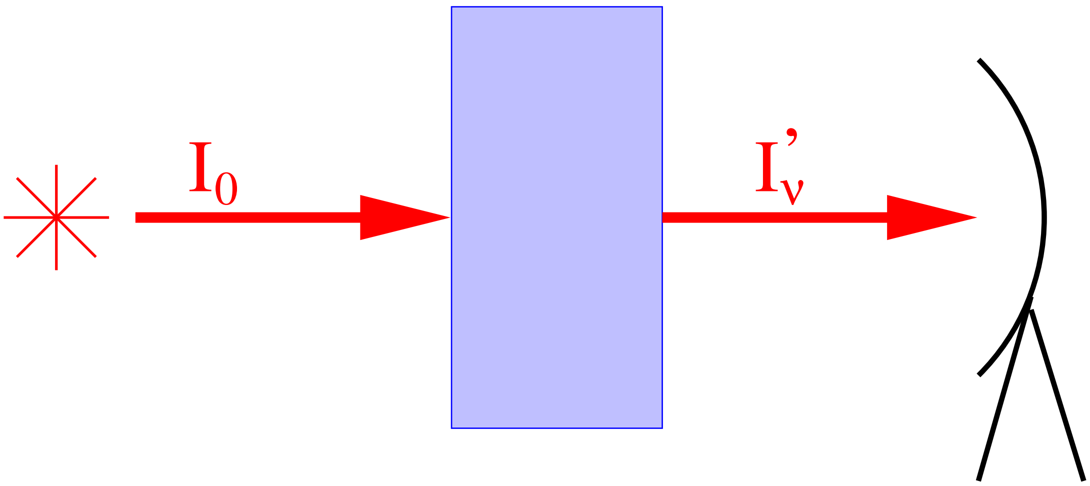
where we have defined the *source function* $S_\nu = j_\nu / \alpha_\nu$. This is the value approached by I_ν given sufficient optical depth.

For the case of a homogeneous cloud with a constant source function and optical depth τ_ν with initial incident intensity I_0, the emergent intensity is

$$I_\nu = I_0 e^{-\tau_\nu} + (1 - e^{-\tau_\nu}) S_\nu$$

This makes a lot of sense: the emergent radiation is the sum of the incident intensity attenuated by the total optical depth plus the sum of each section of cloud emission attenuated by the optical depth from that point to the receiver.



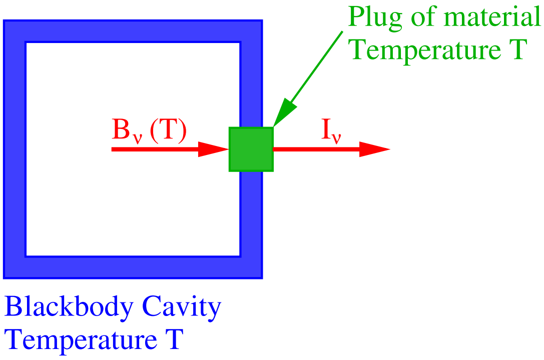


$$I'_\nu = I_0\ e^{-\tau} + S_0(1-e^{-\tau})$$

Kirchhoff's Law, $S_\nu = B_\nu$

Kirchhoff's law calculates the source function S_ν for a body in thermal equilibrium. S_ν depends only upon the material and the temperature, so we can find its general form with a particular example.

Consider a blackbody cavity at temperature T with a hole which is plugged by material also at temperature T (i.e. it is in thermal equilibrium with the cavity).



- The incident intensity from the blackbody is B_ν.

- The emergent intensity $I_\nu = B_\nu$ otherwise the material has gained energy from something at the same temperature.

$$\Rightarrow \frac{dI_\nu}{d\tau_\nu} = 0$$

- $$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu \Rightarrow S_\nu = I_\nu = B_\nu$$

Hence in general the source function for a body in thermal equilibrium is equal to the Planck function.

Example

- From Teide Observatory, Tenerife, the optical depth of the sky is approximately $\tau = 0.02$ at 30 GHz (mostly due to absorbtion by water vapour). Assume that the entire atmosphere is in thermal equilibrium at a temperature of 300K.

$$I_\nu = I_0 e^{-\tau_\nu} + (1 - e^{-\tau_\nu}) S_\nu$$

- Consider emission just from the atmosphere i.e. $I_0 = 0$. Since the atmosphere is in thermal equilibrium $S_\nu = B_\nu(300\mathrm{K}) \approx 2\mathrm{kT}/\lambda^2$.

$\Rightarrow I_\nu \approx \tau_\nu \frac{2kT}{\lambda^2} = 1.7 \times 10^{-18} \mathrm{W\,m^2\,Hz^{-1}\,strad^{-1}}$

- Consider observation of a source surface brightness $I_0 = 10^{-17} \mathrm{W\,m^2\,Hz^{-1}\,strad^{-1}}$

$\Rightarrow I_\nu \approx (1 - \tau_\nu) I_0 + \tau_\nu \frac{2kT}{\lambda^2} = 1.15 \times 10^{-17} \mathrm{W\,m^2\,Hz^{-1}\,strad^{-1}}$

- Subtracting off the contribution from the atmosphere, the source appears at 98% of its actual surface brightness.

The Effects of Scattering

We have until now considered pure absorption and emission. There are many applications when this approach is sufficient, but also many where this is not. For example, the transfer of optical photons in stellar interiors, or the propagation of infrared photons from a protostar through a dusty cloud, both involve scattering.

Scattering can be described by a new emission coefficient, but this is dependent on the incident radiation field; this makes it impossible to integrate the equation of transfer directly.

In general, scattering is very complicated — it is not isotropic and may involve energy loss of the photons.

Summary of Key Points

- α_ν the absorption coefficient, is the fractional loss of intensity per unit length. It includes the effect of stimuated emission.

- τ_ν the optical depth, determines whether a medium is opaque ($\tau_\nu > 1$) or transparent ($\tau_\nu < 1$). $\tau_\nu = \int \alpha_\nu ds$.

- j_ν the spontaneous emission coefficient, is the energy emitted per unit *time* per unit *volume* per unit *solid angle* per unit *frequency*.

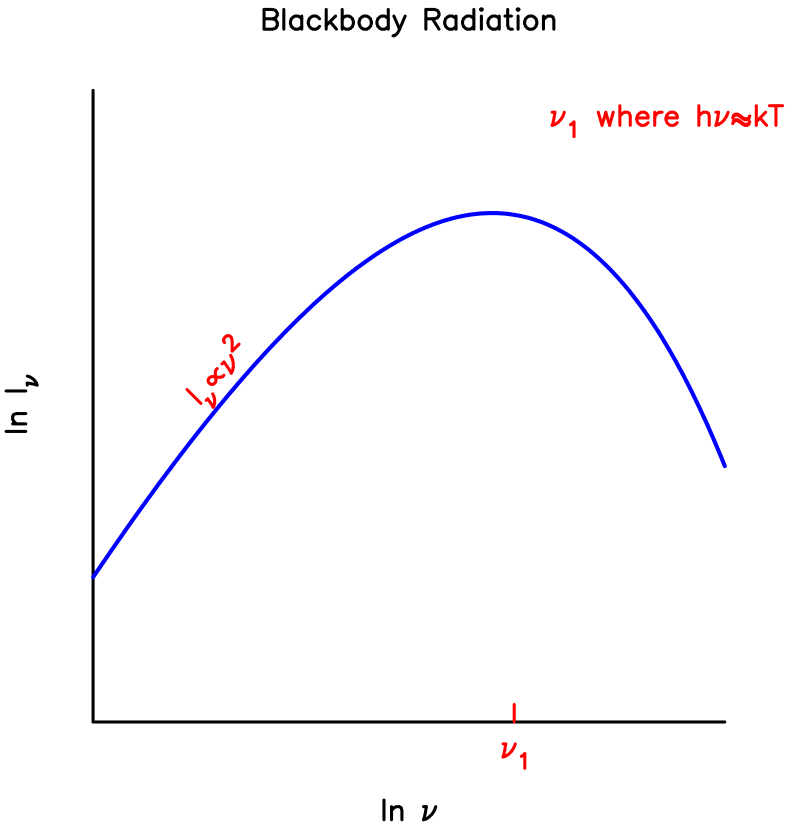
- S_ν the source function, is the value approached by I_ν given sufficient optical depth. $S_\nu = j_\nu/\alpha_\nu$.

- Kirchhoff's Law states that for emitters in thermal equilibrium $S_\nu = B_\nu$.

- the radiative transfer equation for a homogeneous cloud in thermal equilibrium is

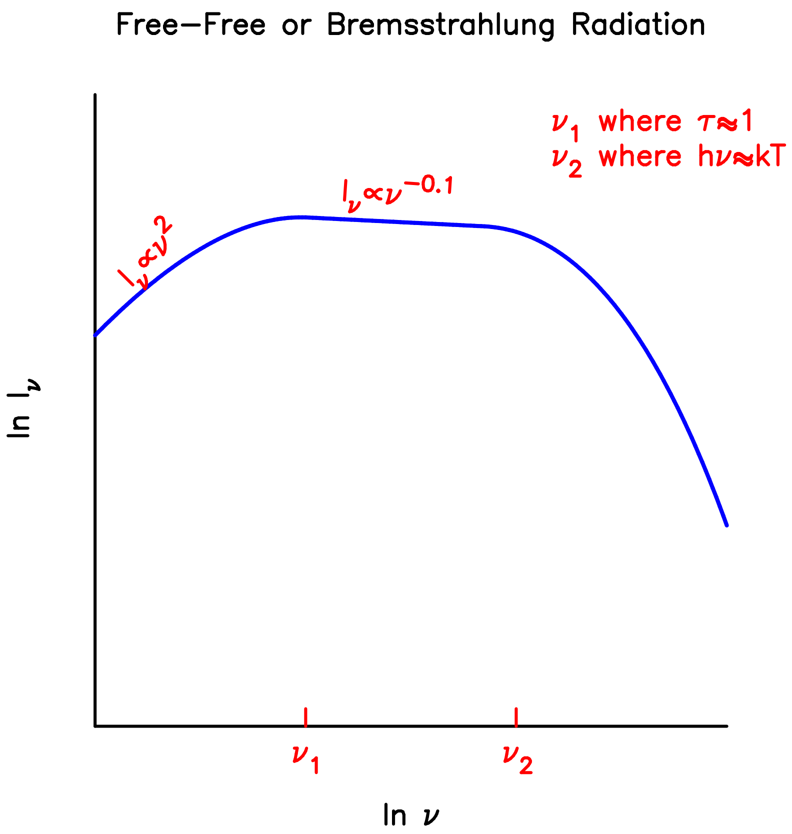
$$I_\nu = I_0 e^{-\tau_\nu} + (1 - e^{-\tau_\nu})B_\nu$$

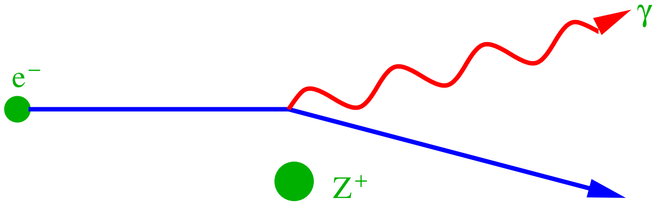
Blackbody Radiation



- Emitters *and radiation* in thermal equilibrium
- $\tau_\nu \to \infty$
- Unpolarised
- $I_\nu = B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/k_B T) - 1}$
- e.g. — Cosmic Microwave Background

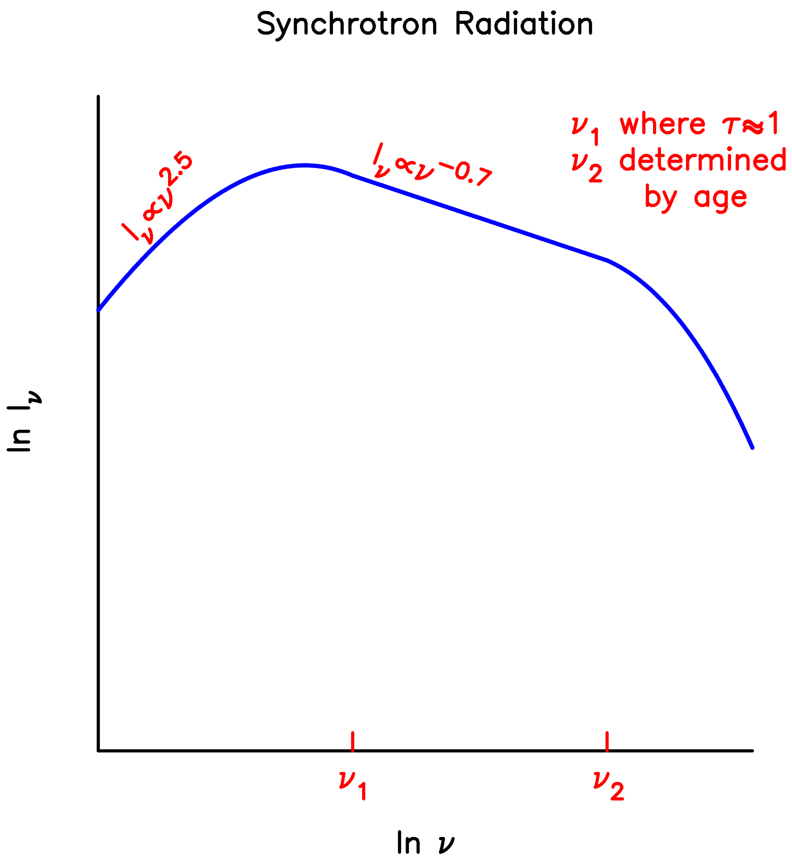
Free-Free or Bremsstrahlung Radiation

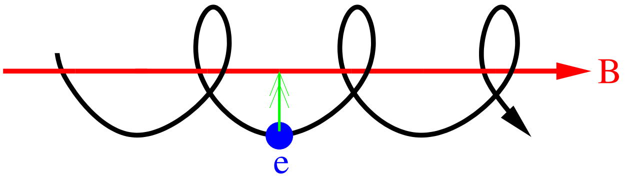




- Emission as result of collisions between charged particles, usually electrons and ions.
- Emitters in thermal equilibrium
- Unpolarised
- e.g. — HII regions
  — X-ray emission from clusters of galaxies
  — Ionised winds from stars

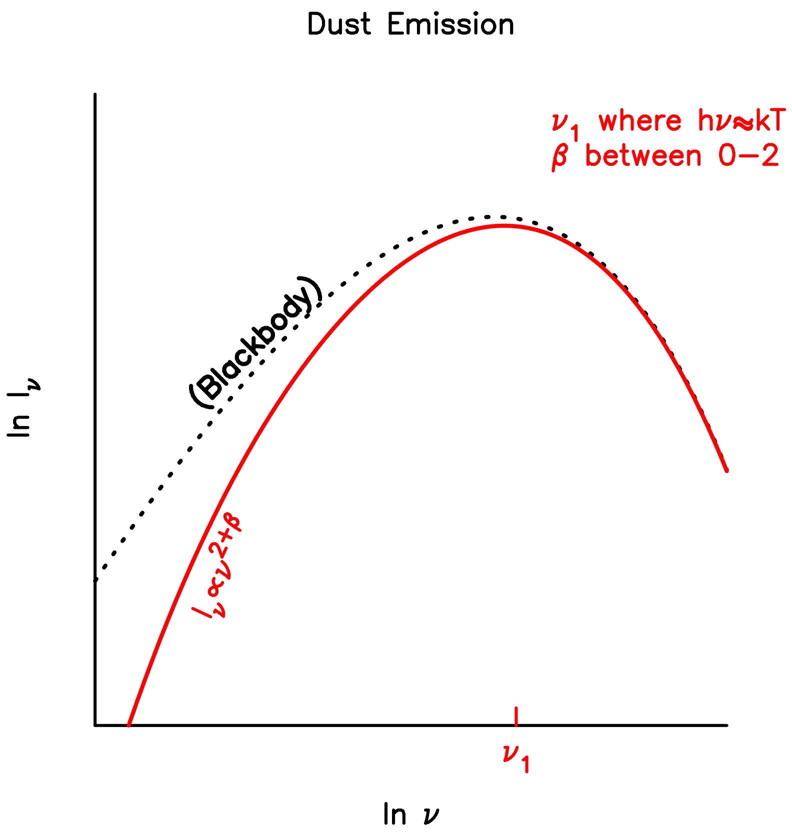
Sychrotron Radiation





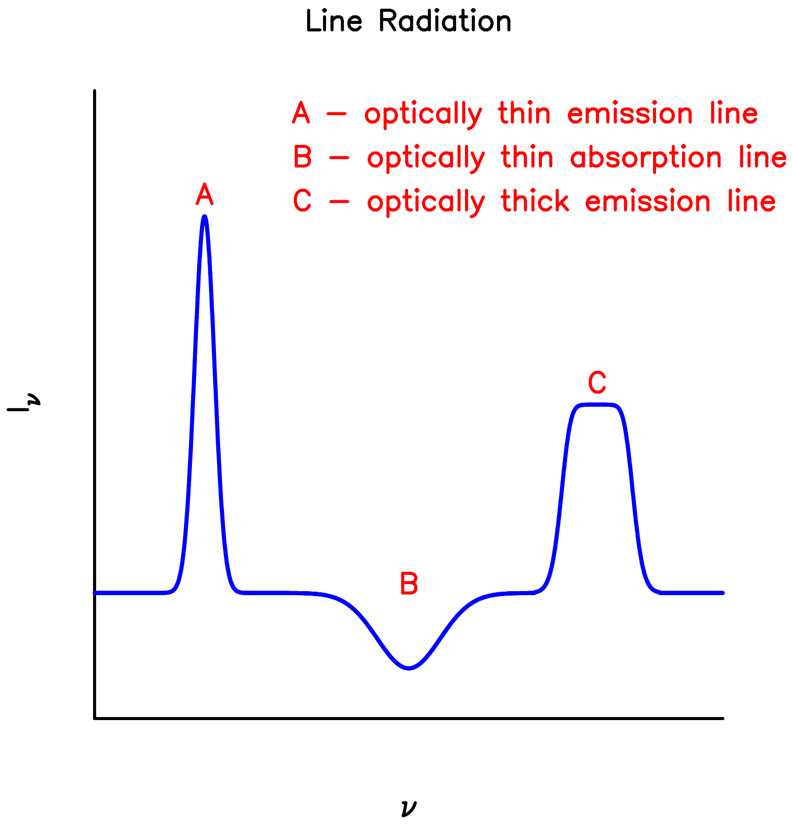
- Emission as result of relativistic electrons gyrating round magnetic field lines
- Emitters *not* in thermal equilibrium
- *Polarised*
- e.g. — quasars
  — radio galaxies
  — supernovae

Dust



- Emission as result of thermal excitation.
- Emitters in thermal equilibrium
- *Polarised*
- e.g. — star forming regions
  — optical extinction and polarisation through the galaxy
  — diffuse scattered light from the Inter-Stellar Medium

Line Emission



- Emission as result of electronic transitions; molecular vibrations and rotations.
- Emitters sometimes in thermal equilibrium.
- Sometimes polarised
- e.g. — 1.4 GHz neutral hydrogen line
  - — CO emission
  - — Lyman series
  - — Masers