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All-sky MEM Component Separation

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Summary

- Simulations of Microwave Sky
- Simulated Planck Observations
- The Maximum Entropy Method (MEM) Component Separation Algorithm
- Separation Results
- Power Spectrum Estimators
- Power Spectrum Results

Simulations of Microwave Sky

- Simulate six components which contribute to observations at Planck wavelengths,
  - CMB,
  - Extragalactic components: Kinetic and thermal SZ effects,
  - Galactic components: dust, free-free and synchrotron.
- Assume spectral indices of components do not vary across sky. Simulate emission at $300\ \mathrm{GHz}$.
- Assume no point sources. May be removed in analysis of TOD or by using joint MEM and mexican hat wavelet technique.
- Maps in HEALPix with $N_{\mathrm{side}} = 2048$.

Extragalactic Components

- CMB anisotropy:
  - Gaussian realisation of spatially flat inflationary CDM model;
  - $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$.
- Thermal and Kinetic Sunyaev-Zel'dovich Effects:
  - 210 individual cluster templates simulated with gas dynamics code;
  - Distributed on sky according to Poisson distribution with random orientations;
  - Cluster radial velocities are Gaussian distributed with dispersion $400\ \mathrm{kms}^{-1}$ at $z = 0$;
  - Redshift distribution consistent with Press-Schechter model for $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$.

Galactic Components

- Dust:
  - Modelled using DIRBE-IRAS $100\ \mu\mathrm{m}$ dust map;
  - One-component dust model with $T_{\mathrm{dust}} = 18\ \mathrm{K}$ and emissivity $\beta = -2$.
- Synchrotron:
  - Based on $408\ \mathrm{MHz}$ Haslam survey;
  - Extrapolate structure to sub-degree scales using angular power spectrum with index -3;
  - Get spectral index from 408, 1420 and $2326\ \mathrm{MHz}$ surveys. Use to project emission to $300\ \mathrm{GHz}$.

Galactic Components II

- Free-free:
  - Reliable maps of Galactic free-free not currently available;
  - Based on DIRBE-IRAS dust map;
  - Assume 60% correlated with dust. 40% uncorrelated component is from dust map flipped North-South;
  - Assume spectral index of $\beta = -0.16$.

Simulated Planck Observations

- We observe the sky consisting of n_c components in n_f frequency channels.
- Beam convolution is diagonal in spherical harmonic space.
- Data $\mathsf{d}_{\ell m}$ are related to sources $\mathsf{a}_{\ell m}$ by

$$d_{\ell m}^{(\nu)} = B_\ell^{(\nu)} \sum_{p=1}^{n_c} F_{\nu p} a_{\ell m}^{(p)} + \epsilon_{\ell m}^{(\nu)},$$

  where $\epsilon_{\ell m}$ is the noise vector.

- Combine beams $B_\ell^{(\nu)}$ and frequency conversion matrix F into response matrix

$$R_\ell^{(\nu p)} = B_\ell^{(\nu)} F_{\nu p}.$$

Simulated Planck Observations II

- So at each mode independently

$$\mathsf{d}_{\ell m} = \mathsf{R}_{\ell}\mathsf{a}_{\ell m} + \epsilon_{\ell m}.$$

- In general, the covariance structure is

$$\begin{aligned}\left\langle \mathsf{a}_{\ell m}\mathsf{a}^{\dagger}_{\ell m}\right\rangle &= \mathsf{C}_{\ell m,\ell' m'},\\ \left\langle \epsilon_{\ell m}\epsilon^{\dagger}_{\ell m}\right\rangle &= \mathsf{N}_{\ell m,\ell' m'}.\end{aligned}$$

Component Separation

- Given the data $\mathsf{d}_{\ell m}$ we want to estimate the components $\hat{\mathsf{a}}_{\ell m}$.
- Use Bayes' theorem and find estimator that maximizes posterior probability

$$P(\mathsf{a}|\mathsf{d}) \propto P(\mathsf{d}|\mathsf{a})P(\mathsf{a}).$$

- Assume modes are independent,

$$\begin{aligned}\left\langle \mathsf{a}_{\ell m}\mathsf{a}^{\dagger}_{\ell m}\right\rangle &= \mathsf{C}_{\ell m}\delta_{\ell\ell'}\delta_{mm'}\,, \\ \left\langle \epsilon_{\ell m}\epsilon^{\dagger}_{\ell m}\right\rangle &= \mathsf{N}_{\ell m}\delta_{\ell\ell'}\delta_{mm'}\,.\end{aligned}$$

- Then posterior is independent at each mode,

$$P(\mathsf{a}_{\ell m}|\mathsf{d}_{\ell m}) \propto P(\mathsf{d}_{\ell m}|\mathsf{a}_{\ell m})P(\mathsf{a}_{\ell m}).$$

- Additionally, assume rotational invariance so $\mathsf{C}_{\ell m} = \mathsf{C}_{\ell}$, $\mathsf{N}_{\ell m} = \mathsf{N}_{\ell}$.

Harmonic space MEM

- Perform the reconstruction in terms of ‘hidden’ variables $\mathsf{h}_{\ell m}$,

$$\mathsf{a}_{\ell m} = \mathsf{L}_\ell \mathsf{h}_{\ell m},$$

where $\mathsf{C}_\ell = \mathsf{L}_\ell \mathsf{L}_\ell^\dagger$.

- Use the intrinsic correlation function L_ℓ to insert prior knowledge of correlation between components.

- If instrumental noise is Gaussian, the likelihood is

$$P(\mathsf{d}_{\ell m}|\mathsf{a}_{\ell m}) \propto \exp\left[-\chi^2(\mathsf{h}_{\ell m})\right],$$

where χ^2 is the misfit statistic

$$\chi^2(\mathsf{h}_{\ell m}) = (\mathsf{d}_{\ell m} - \mathsf{R}_\ell \mathsf{L}_\ell \mathsf{h}_{\ell m})^\dagger \, \mathsf{N}_\ell^{-1} \, (\mathsf{d}_{\ell m} - \mathsf{R}_\ell \mathsf{L}_\ell \mathsf{h}_{\ell m}).$$

The Entropic Prior

- The prior is written as the exponential of an entropy term

$$P(\mathsf{h}_{\ell m}) \propto \exp\left[\alpha S(\mathsf{h}_{\ell m}, \mathsf{m}, \mathsf{n})\right].$$

- For positive-only images the entropy term would be

$$S(\mathsf{h}, \mathsf{m}) = \sum_{i=1}^{n_c} \left[h_i - m_i - h_i \ln \left(\frac{h_i}{m_i} \right) \right].$$

- For positive-negative images the entropy becomes

$$S(\mathsf{h}, \mathsf{m}, \mathsf{n}) = \sum_{i=1}^{n_c} \left[\psi_i - m_i - n_i - h_i \ln \left(\frac{\psi_i + h_i}{2m_i} \right) \right],$$

where $\psi_i = \sqrt{{h_i}^2 + 4m_i n_i}$.

The Solution

- Maximising the posterior probability is equivalent to minimising the function

$$\Phi(\mathsf{h}_{\ell m}) = \chi^2(\mathsf{h}_{\ell m}) - \alpha S(\mathsf{h}_{\ell m}, \mathsf{m}).$$

- The minimisation can be performed using a variable metric minimiser.
- The regularising parameter α is fixed using Bayesian methods. It requires about 10 iterations of full algorithm.
- The regularisation is optimal at each scale.

Reconstruction Errors

- The covariance matrix of reconstruction errors can be estimated.
- Use a Gaussian approximation

$$\langle (\hat{\mathsf{a}}_{\ell m} - \mathsf{a}_{\ell m})(\hat{\mathsf{a}}_{\ell m} - \mathsf{a}_{\ell m})^\dagger \rangle = \mathsf{L}_\ell \mathsf{H}_{\ell m}^{-1} \mathsf{L}_\ell^\dagger$$

where $\mathsf{H}_{\ell m} = \nabla\nabla\Phi(\mathsf{h}_{\ell m})$ is Hessian matrix of the posterior distribution evaluated at its peak $\hat{\mathsf{h}}_{\ell m}$.

- The pth diagonal entry of the covariance matrix gives the errors in the pth component

$$\left(\mathsf{L}_\ell \mathsf{H}_{\ell m}^{-1} \mathsf{L}_\ell^\dagger\right)_{pp} = \left|\Delta a_{\ell m}^{(p)}\right|^2 .$$

Results

- Easy to parallelise the code - independent at each mode.
- Runs in around 6 hours on 30 processors of SGI Origin 2000.
- Requires 14GB of memory.

Power Spectrum Estimators

- At each mode $a_{\ell m}^{(p)} = \hat{a}_{\ell m}^{(p)} + \delta a_{\ell m}^{(p)}$.
- The power spectrum of a reconstructed map is

$$\hat{C}_\ell^{(p)} = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} \left| \hat{a}_{\ell m}^{(p)} \right|^2 ,$$

  which underestimates the true spectrum since it is biassed.
- The residuals power spectrum can be estimated from the errors:

$$\widehat{\delta C}_\ell^{(p)} = \frac{1}{2\ell+1} \sum_{m=-\ell}^{\ell} \left| \Delta a_{\ell m}^{(p)} \right|^2 .$$

- So we can construct an unbiassed estimator of true power spectrum, given by

$$\hat{\mathcal{C}}_\ell^{(p)} = \hat{C}_\ell^{(p)} + \widehat{\delta C}_\ell^{(p)} .$$

Conclusions

- Spherical Harmonic-based MEM is a fast and accurate component separation algorithm.
- We can obtain a good separation without applying a Galactic cut.
- The algorithm can accommodate spatially-varying spectral indices by introducing extra channels.
- It can also accommodate extragalactic point sources by generalising the ‘noise’ covariance matrix and combining MEM with a mexican hat wavelet technique.
- The algorithm is our current baseline for HFI Level 3 analysis.