PARTICLE CREATION BY ELECTROSTATIC FIELDS

(AND GRAVITATIONAL BACKGROUNDS)

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- Novel state-space approach to QFT gives insight into mechanism of particle creation by electromagnetic and gravitational backgrounds.
- Based on single-particle Dirac equation in Schrödinger form

$$i\hbar\partial_t\Psi = \widehat{H}\Psi$$

- We make multiparticle state space as the Slater determinant of these single particle states.
- How can a multiparticle wavefunction Ψ represent varying numbers of particles without second quantisation?
- Dirac equation has equal number of positive and negative energy states and cannot represent single-electron states anyway!
- Our formulation of QFT makes a virtue of this "difficulty" — the conserved number of states contained in Ψ now represents the conservation of charge.

STATE SPACE QFT

- Essential ingredient is the identification of particle/antiparticle states for an observer (particle detector) at time t depending on the sign of E in

$$\widehat{H}\Psi = E\Psi$$

- We model the vacuum state as the state in which all the negative energy states are filled — the Dirac sea.
- An initial vacuum state can then evolve under electromagnetic and other influences so that some positive energy states become occupied.
- The resulting ripples on surface of the Dirac sea represent the creation of electron/positron pairs.
- Vacuum subtraction is equivalent to normal ordering, but Dirac equation is kept intact. There is also no need for the Feynman propagator.
- Limitation is that particle-particle interactions are not yet included.

PARTICLE CREATION BY ELECTROMAGNETIC FIELDS

- Gauge-invariant form of Dirac equation:

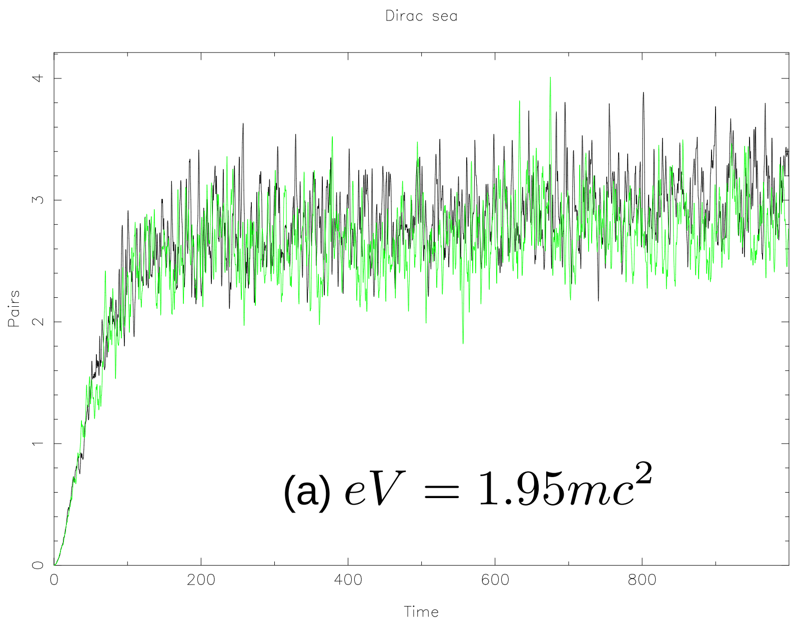
$$i\hbar\partial_t\Psi - eV\Psi = \widehat{H}\Psi$$

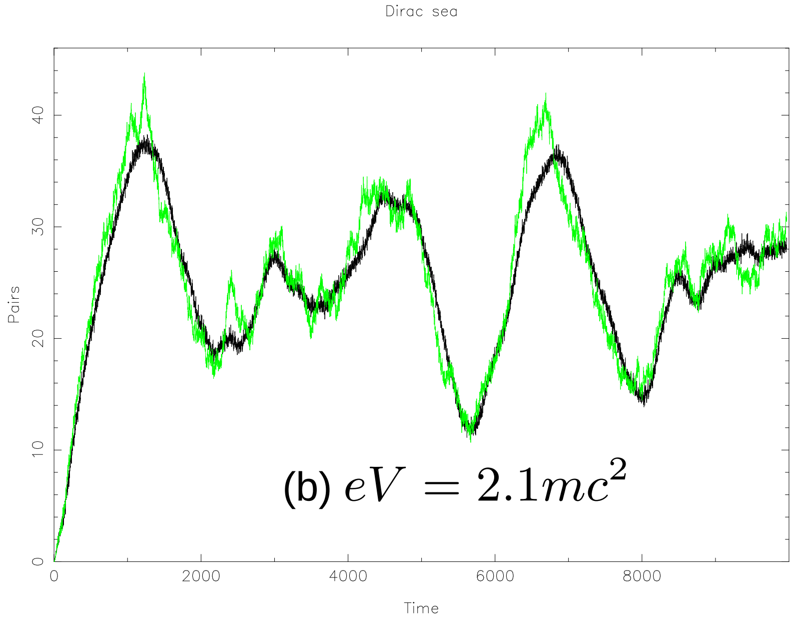
- The electrostatic potential does not contribute to the particle-defining Hamiltonian.
- An negative-energy state at $t = 0$ will evolve in an electrostatic potential and will contain positive-energy components for $t > 0$.
- We have illustrated this for potential barriers in one dimension analytically and using a Monte Carlo approach.
- For $eV < 2mc^2$ the particle pairs are confined to the regions of high electric field (within $\hbar/mc$).
- Evanescent stationary states are asymmetric and cause separation of electron/positrons (vacuum polarisation).
- For $eV > 2mc^2$ particle creation is continuous and pairs move away from the high-field regions (Klein paradox).
- The process is limited by Fermionic saturation.

Z>137 NUCLEI AND EXTENSIONS

- We have been able to simulate particle creation by potentials with radial symmetry.
- The behaviour of the radial square well is essentially the same as before.
- For Coulomb potentials $V = Ze/4\pi\epsilon_0 r$ there is no stable $1s_{\frac{1}{2}}$ state for $Z > 137$.
- Monte-Carlo simulations for this case show that the electrons created are captured and the positrons are expelled to infinity.
- The state-space approach also works for arbitrary-moving observers and gravitational fields. We have plotted the particle distribution seen by an accelerated observer.
- The approach is equivalent to ordinary QED (without particle-particle interactions) and it is easy to interpret.

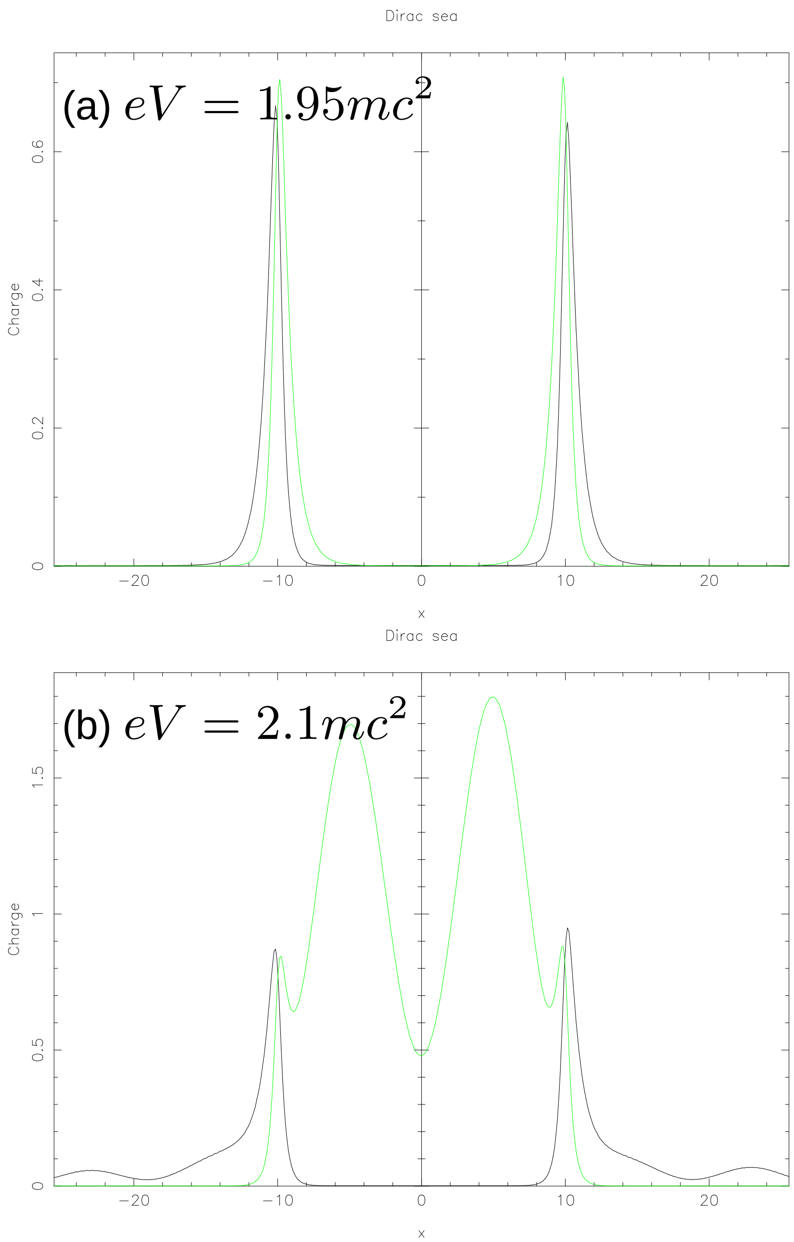
PARTICLE CONTENT OF POTENTIAL BARRIER





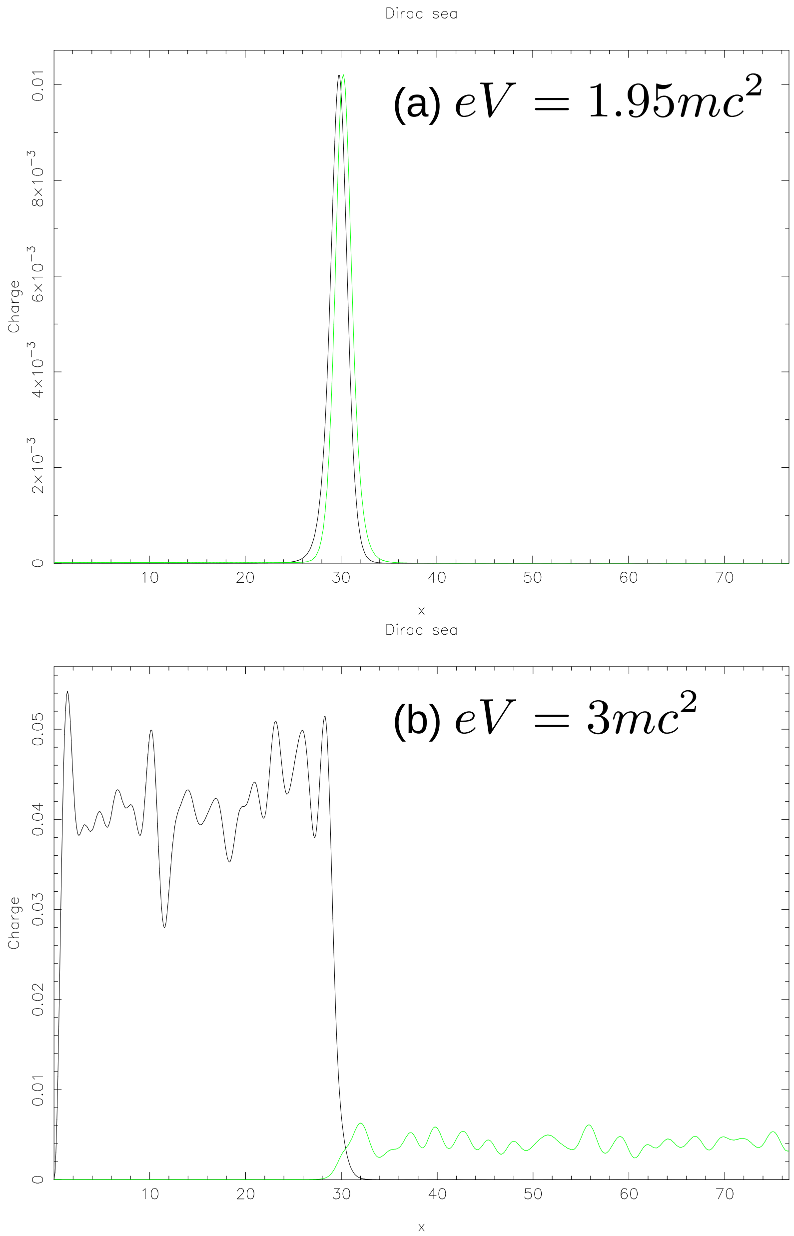
- The system is set up in the vacuum state at $t = 0$.
- For $eV < 2mc^2$ the particle content quickly stabilises.
- For $eV > 2mc^2$ waves propagate around the periodic grid.

PARTICLE DISTRIBUTIONS IN POTENTIAL BARRIER



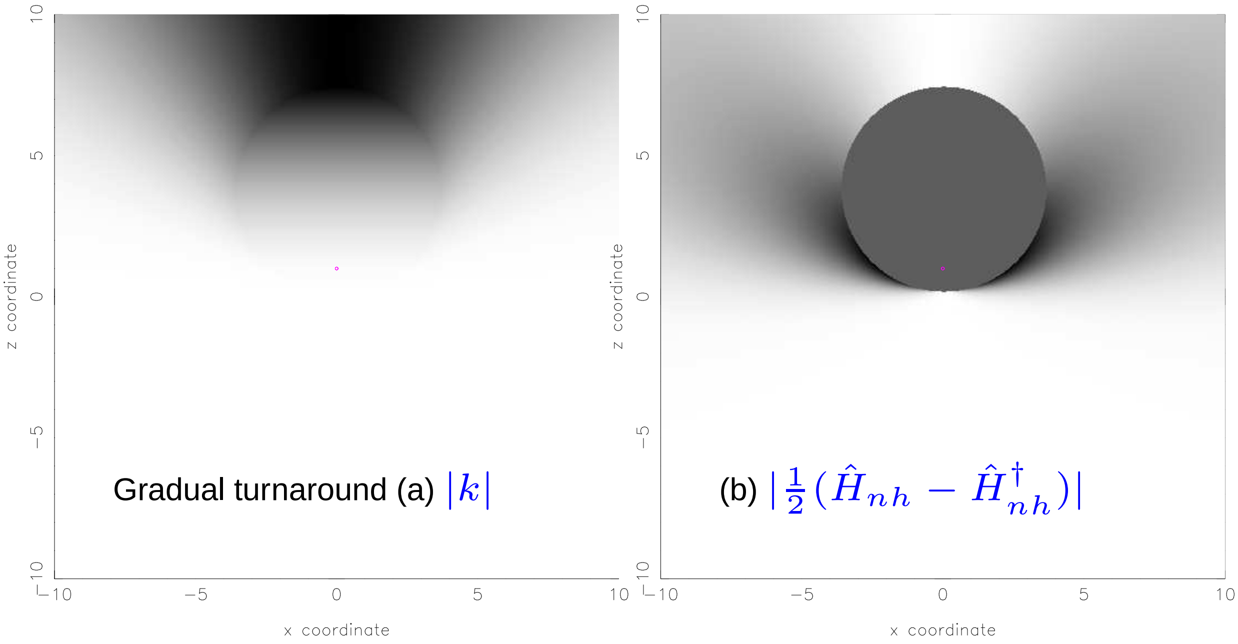
- For $eV < 2mc^2$ the particles are confined within a Compton wavelength of the potential steps.
- For $eV > 2mc^2$ the positrons fill the potential barrier and the electrons are expelled.

Particle distributions in spherical square well



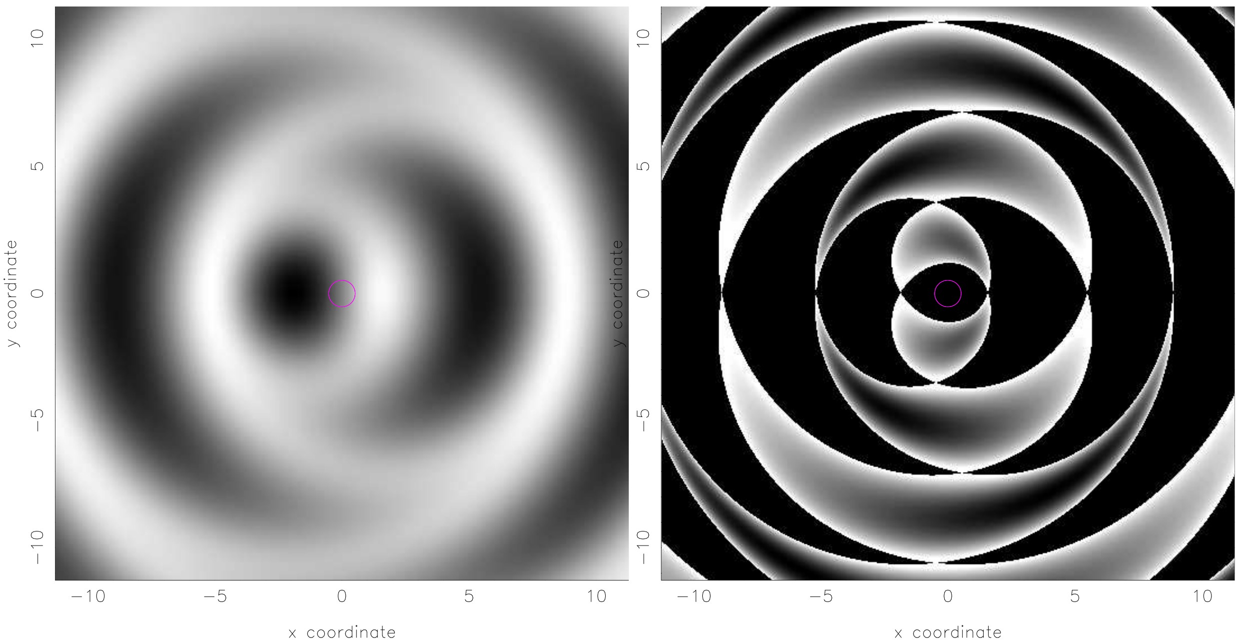
- MC simulation of overlap of $1s_{\frac{1}{2}}$ and $2p_{\frac{1}{2}}$ orbitals.
- For $eV < 2mc^2$ the particles are confined within a Compton wavelength of edge of the well.
- For $eV > 2mc^2$ the electrons fill the potential well and the positrons are expelled (eV is negative for this case).

Accelerated observers



Gradual turnaround (a) $|k|$ (b) $|\frac{1}{2}(\hat{H}_{nh} - \hat{H}^{\dagger}_{nh})|$

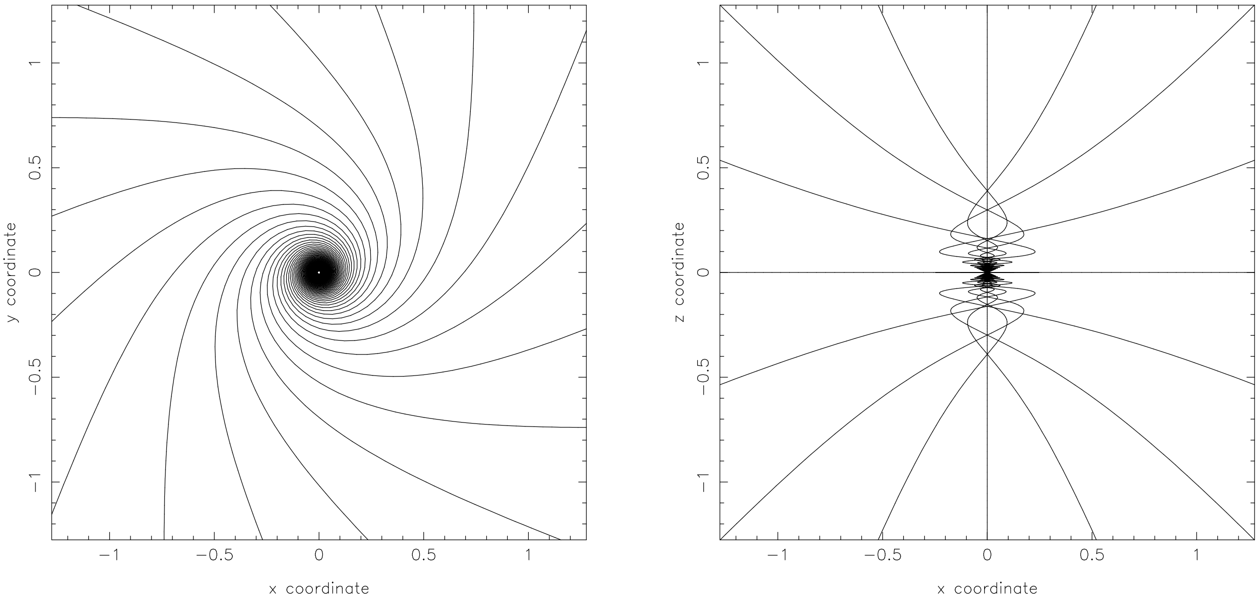
- Observer accelerating for a finite time.



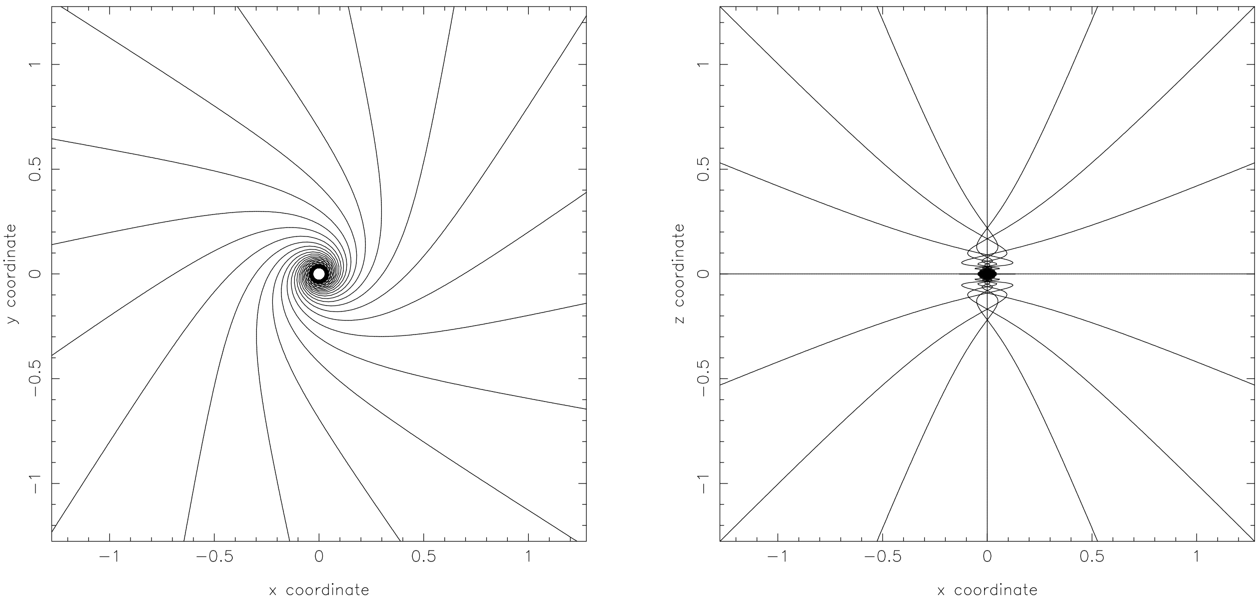
Rotating observer (a) $|k|$ (b) $|\frac{1}{2}(\hat{H}_{nh} - \hat{H}^{\dagger}_{nh})|$

- Observer rotating with rapidity $u = 1$.

SOURCES OF SPINNING PARTICLES

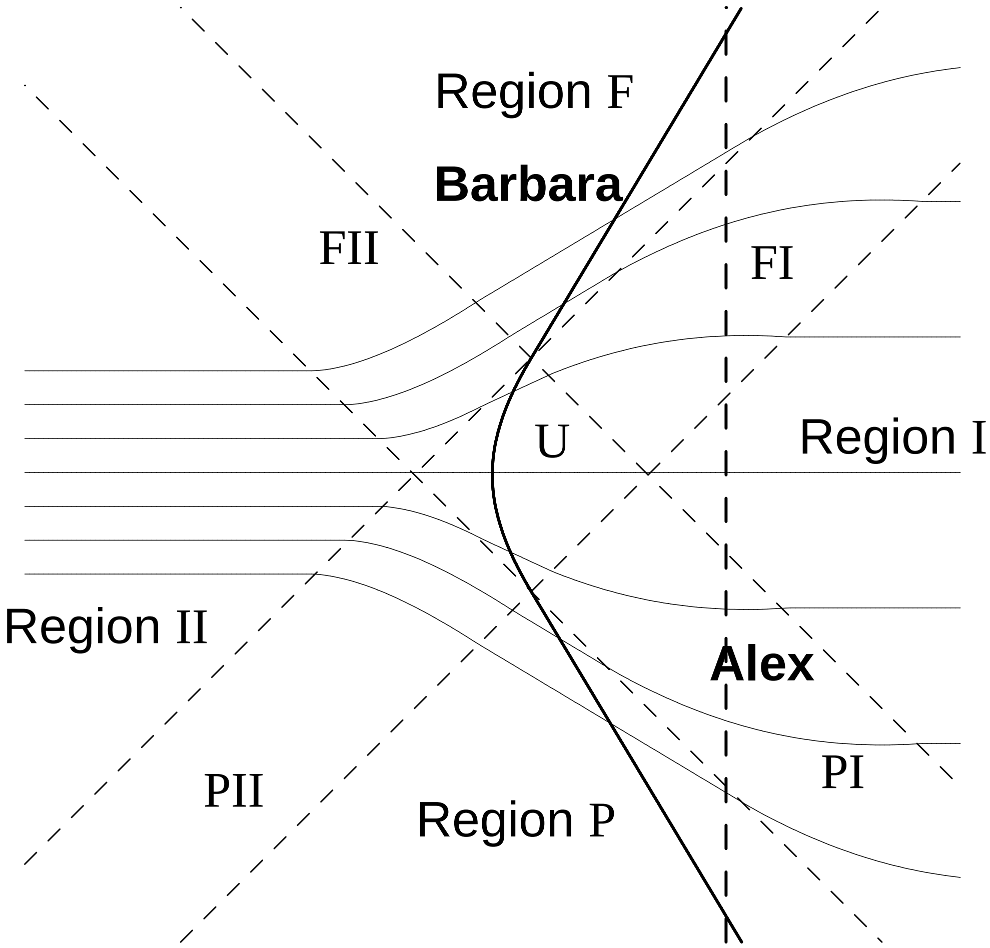


- Streamlines of the Dirac current for a source of spin-half particles.



- Streamlines of the current for source of a spin-one particles.

Radar time for gradual turnaround twin



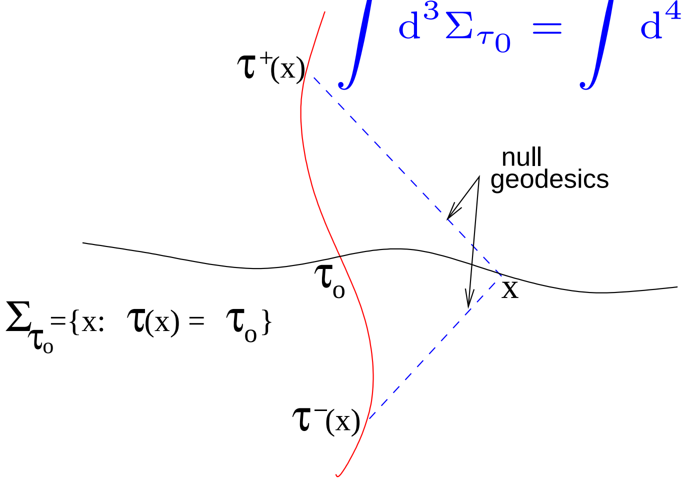
- Observer (Barbara) accelerating for finite time splits spacetime into 3 regions.
- Region U is equivalent to the Unruh observer.
- In regions I and II the dispersion relation is $E^2 = e^{\pm 2u}(m^2 + p^2)$, where u is the rapidity.

Formulation for arbitrary observers

- For moving observers and gravitational backgrounds we must define the particle-defining Hamiltonian over a covariant surface.

- We use null geodesics to define a surface at time τ_0 (Bondi's radar time) $\tau \equiv \frac{1}{2}(\tau_+ + \tau_-)$

$$\int \mathrm{d}^3\Sigma_{\tau_0} = \int \mathrm{d}^4x \det \underline{h}^{-1}\delta(\tau-\tau_0)\overline{h}(\boldsymbol{\nabla}\tau)$$



- We define a time-translation vector k satisfying $\underline{h}^{-1}(k) \propto \overline{h}(\boldsymbol{\nabla}\tau)$ and $k\cdot\boldsymbol{\nabla}\tau = 1$

- Schrödinger equation $\hat{j}\underline{h}^{-1}(k)\cdot D\Psi = \hat{H}_{\mathrm{nh}}\Psi$ satisfies

$$\Re\left\langle \Psi \hat{H}_{\mathrm{nh}}(\Psi)\right\rangle = \left\langle \int \mathrm{d}^3\Sigma_{\tau_0}\mathcal{T}(\underline{h}^{-1}(k))\right\rangle_0$$

FORMULATION FOR ARBITRARY OBSERVERS II

- Particle content defined through $\hat{H}_1\Psi = E\Psi$

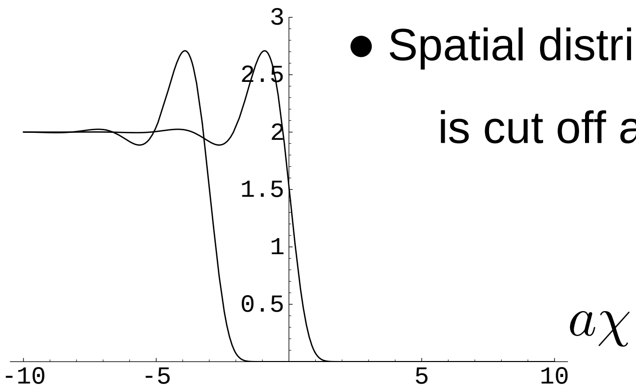
$$\hat{H}_1 \equiv \frac{1}{2}(\hat{H}_{\rm nh} + \hat{H}^\dagger_{\rm nh})$$

- Evolution Hamiltonian now contains several terms that can create particles

$$\hat{H}_{\rm ev} = \hat{H}_1 + ek\cdot A - \frac{1}{2}\hat{j}\Omega(k) + \frac{1}{2}(\hat{H}_{\rm nh} - \hat{H}^\dagger_{\rm nh})$$

- Example: Unruh effect. Observer with acceleration a observing the Minkowski vacuum sees a thermal spectrum with temperature $a\hbar/(2\pi k_{\rm B}c)$.

- Spatial distribution of massless particles is uniform.



- Spatial distribution of massive particles is cut off away from the horizon.