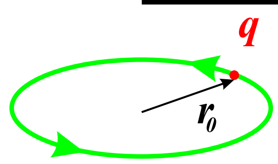
Preamble to Diamagnetism



- Orbiting charge q, mass m_q, radius r_0, angular velocity ω_0

$$\left.\begin{array}{lll}\text{Current} & = & \dfrac{q\omega_0}{2\pi} \\ \text{Area} & = & \pi r_0^2\end{array}\right\} \Rightarrow \text{moment } |\boldsymbol{m}| = \frac{q\omega_0 r_0^2}{2}$$

- Angular momentum $|\boldsymbol{J}| = m_q \omega_0 r_0^2$

$$\Rightarrow \boldsymbol{m} = \frac{q}{2m_q}\boldsymbol{J}$$

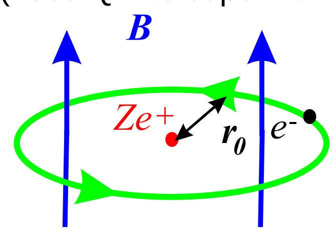
- Quite general result, but elementary particles have different *gyromagnetic ratios*: g

$$\boldsymbol{m} \equiv g\frac{q}{2m_q}\boldsymbol{J}$$

- $g = 1$ for case where momentum $\boldsymbol{p} \propto \boldsymbol{v}$
  $g = 2$ for electron spin!
- $\Rightarrow$ Momentum of electron not collinear with current.

Magnetic Materials: Diamagnetism

- All substances show a magnetisation in opposition to the applied magnetic field — caused by perturbation of the electron orbits.
- General idea can be seen from simple model (need QM to cope with electron spin (next year)).



- Suppose electron orbits at radius r_0, angular velocity ω_0
- If $B = 0$, balance of forces $\Rightarrow$

$$\frac{Ze^2}{4\pi\epsilon_0 r_0^2} = m_e\omega_0^2 r_0 \Rightarrow \omega_0^2 = \frac{Ze^2}{4\pi\epsilon_0 m_e r_0^3}$$

- If $B \neq 0$ there is an additional $-e\boldsymbol{v}\times\boldsymbol{B}$ force (inwards).
- New balance of forces at radius $r_0 + \Delta r$, $\omega_0 + \Delta\omega$

$$(\omega_0 + \Delta\omega)^2 = \frac{Ze^2}{4\pi\epsilon_0 m_e}\frac{1}{(r_0 + \Delta r)^3} + \frac{eB}{m_e}(\omega_0 + \Delta\omega)$$

Simple model of Diamagnetism

- Expand to first order in Δr, $\Delta\omega$

$$\omega_0^2\left(1+2\frac{\Delta\omega}{\omega_0}\right) \approx \frac{Ze^2}{4\pi\epsilon_0 m_e r_0^3}\left(1-3\frac{\Delta r}{r_0}\right) + \frac{eB\omega_0}{m_e}$$

- Leading terms cancel

$$\frac{2\Delta\omega}{\omega_0} + \frac{3\Delta r}{r_0} \approx \frac{eB}{m_e\omega_0}$$

- In fact $\Delta r/r_0$ is very small compared to $\Delta\omega/\omega_0$ (not obvious!)

$$\Rightarrow \Delta\omega \approx \frac{eB}{2m_e} = \tfrac{1}{2}\omega_L$$

where $\omega_L = eB/m_e$ is the Larmor frequency.

- This gives an additional dipole moment opposed to applied field (Lenz' law)

$$d|\boldsymbol{m}| = dIS = \frac{e\omega_L}{2\times 2\pi}\pi r_0^2 = \frac{e^2 r_0^2 B}{4m_e}$$

- Average (factor of 2/3) over orientations of orbit:

$$\langle d|\boldsymbol{m}|\rangle = \frac{e^2\left\langle r_0^2\right\rangle B}{6m_e}$$

Diamagnetism — non-examinable

- Q. Why is $\Delta r/r_0 \ll \Delta\omega/\omega_0$?
- A. Because $\boldsymbol{J}$ is quantised, and doesn't change when $\boldsymbol{B}$ is applied.
- Q. But isn't $J = m_e\omega r^2$, so $\Delta\omega/\omega_0 = -2\Delta r/r_0$?
- A. No, it isn't *that* angular momentum that is quantised. The correct momentum to use is the *canonical momentum* $\boldsymbol{p} = m_q\boldsymbol{v} + q\boldsymbol{A}$ ($\boldsymbol{A}$ = vector potential).
- Q. But what is $\boldsymbol{A}$?
- A. $\boldsymbol{A} = \frac{1}{2}Br\hat{\boldsymbol{u}}_\phi$ will generate $\boldsymbol{B}$ in the z-direction, so for electron ($q=-e$) $|\boldsymbol{p}| = m_e v - \frac{1}{2}eBr$.

$$\Rightarrow (m_e\omega_0 - \tfrac{1}{2}eB)r^2 = \text{constant} \Rightarrow r = \text{constant}$$

- Q./ A. Alternative: velocity increases due to induced $\boldsymbol{E}$ field (next section).

Diamagnetism — Induction Effects

- Suppose magnetic field is increasing at rate $\dot{B}$
- There is an electromotive force induced around the path of the orbit, given by Faraday's law (coming soon. . .).

$$\mathcal{E} = \oint \boldsymbol{dl}\cdot\boldsymbol{E} = 2\pi r_0 E = -\frac{\partial}{\partial t}\int \boldsymbol{dS}\cdot\boldsymbol{B} = -\pi r_0^2 \dot{B}$$

- If B is increasing, this electric fields acts to *accelerate* the charge so that

$$\dot{p} = -eE = \frac{er\dot{B}}{2}$$

$$\Rightarrow \Delta(p) = m_e \Delta(r\omega) \approx m_e(r_0\Delta\omega + \omega_0\Delta r) = \frac{er_0 B}{2}$$

$$\Rightarrow \frac{\Delta\omega}{\omega_0} + \frac{\Delta r}{r_0} \approx \frac{eB}{2m_e\omega_0}$$

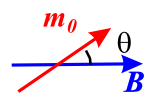
- But we have already show that

$$\frac{2\Delta\omega}{\omega_0} + \frac{3\Delta r}{r_0} \approx \frac{eB}{m_e\omega_0}$$

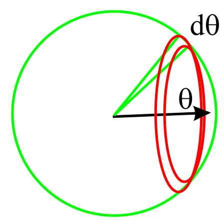
- $\Rightarrow \Delta r = 0$.

Paramagnetism

- Due to permanent magnetic dipoles in medium.
- Suppose each dipole has moment m_0.
- If $\boldsymbol{B} = 0$ dipoles have random orientation.
- If $\boldsymbol{B} \neq 0$ field tends to align dipoles.



- Dipole energy $U = -Bm_0\cos\theta$,



- Probability of alignment in $(\theta, \theta + d\theta)$ $= \Pr(\theta)\, d\theta)$

$$\Pr(\theta)d\theta \quad \propto \quad \tfrac{1}{2}\sin\theta d\theta \quad \exp\left(\frac{m_0 B\cos\theta}{kT}\right)$$

solid angle — Boltzmann factor

- Set $\mu \equiv \cos\theta$ and $x \equiv m_0 B/kT$

$$\Rightarrow \Pr(\theta)d\theta = \Pr(\mu)d\mu \propto e^{\mu x} d\mu$$

Paramagnetism II

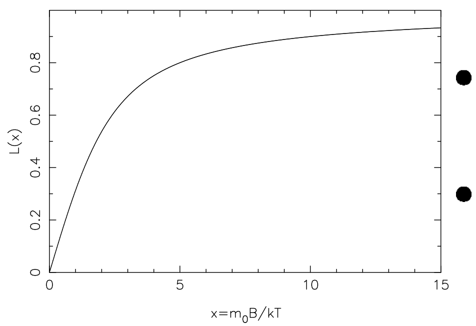
- Average alignment along $\boldsymbol{B}$ is

$$\langle m_0 \cos\theta \rangle = \langle m_0 \mu \rangle = \frac{m_0 \int_{-1}^{1} d\mu\, \mu\, e^{\mu x}}{\int_{-1}^{1} d\mu\, e^{\mu x}}$$

- Denominator = $\frac{2}{x}\sinh(x)$
- Numerator = $\frac{2}{x}\cosh(x) - \frac{2}{x^2}\sinh(x)$

$$\Rightarrow \frac{\langle m_\parallel \rangle}{m_0} = \left(\coth(x) - \frac{1}{x}\right) \equiv L(x)$$

- $L(x)$ is the Langevin function



- High $\boldsymbol{B}$ or low T — saturates $L(x) \to 1$
- Low field strength $L(x) \propto x$

$$L(x) = \frac{x}{3} - \frac{x^3}{45} + O(x^5)$$

Curie-Weiss Theory

- At low field strengths $L(x) \approx x/3$

$$\Rightarrow \frac{\langle m_\parallel \rangle}{m_0} \approx \frac{m_0 B}{3kT} \qquad \textit{(Curie's Law)}$$

- Curie's Law: $\chi = \frac{C}{T}$
  C is the Curie constant.
- Approximate theory of paramagnetism. We have so far ignored the effect of the dipoles on each other.
- Cooperative effects: Microscopic moments tend to align with each other.
- Macroscopic fields satisfy $\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$ but field $\boldsymbol{B}_{\text{local}}$ at the atom can can be very different from the macroscopic $\boldsymbol{B}$.
- Cooperative effects are *very strong* for *ferromagnetic* media.

WEISS THEORY

- Weiss (1907) suggested model

$$\boldsymbol{B}_{\text{local}} = \mu_0 \left(\boldsymbol{H} + \lambda \boldsymbol{M}\right)$$

  λ is the Weiss constant $(\lambda > 1)$

- Expresses cooperative effect as increased contribution from local $\boldsymbol{m}$.

- Use this $\boldsymbol{B}_{\text{local}}$ in Langevin expression for $\langle m_{\parallel} \rangle$.

$$\frac{m_{\parallel}}{m_0} = \frac{M}{n m_0} =$$

$$\coth\left(\frac{m_0 \mu_0 (H + \lambda M)}{kT}\right) - \frac{kT}{m_0 \mu_0 (H + \lambda M)}$$

- Changes weak field limit

- Predicts spontaneous magnetisation below a critical Curie temperature.

CURIE-WEISS THEORY

- Weiss theory: $B = \mu_0 (H + \lambda M)$

- Consider weak field limit $\dfrac{m_0 B}{kT} \ll 1$

$$\frac{M}{m_0 n} \approx \frac{m_0 \mu_0 (H + \lambda M)}{3kT}$$

- Define $\chi = M/H$, rearrange to get

$$\chi = \frac{m_0 \mu_0 / 3kT}{\frac{1}{m_0 n} - \frac{\lambda m_0 \mu_0}{3kT}} = \frac{n m_0^2 \mu_0}{3k} \frac{1}{T - \lambda m_0^2 \mu_0 n / 3k}$$

- Curie-Weiss law: $\chi = \dfrac{C}{T - T_{\text{c}}}$

- C is the Curie constant: $C = n m_0^2 \mu_0 / 3k$

  T_{c} is the critical, or Curie temperature: $T_{\text{c}} = \lambda C$.

- $T > T_{\text{c}} \Rightarrow$ paramagnetism, with Curie-Weiss law.

  $T < T_{\text{c}} \Rightarrow$ spontaneous magnetisation.

Ferromagnetism — Example

- Iron has $m_0 = 2.2\,\mu_B$, where μ_B is the Bohr magneton

$$\mu_B = \frac{e\hbar}{2m_e} = 9.27 \times 10^{-24}\ \text{J T}^{-1}$$

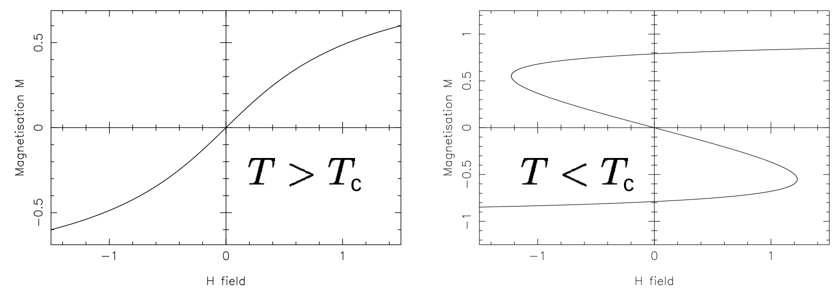
- Density of iron (7800 kg m^{-3}) $\Rightarrow n \approx 8.5 \times 10^{28}$ m^{-3}
- Field if all dipoles aligned $= \mu_0 m_0 n \approx 2$ T.
  (This is approximately correct...)
- Curie constant $C = \mu_0 m_0^2 n / 3k \approx 1.0$ K.
- But T_c=1043 K, so $\lambda \approx 1000$.
- Very strong cooperative quantum interactions of spins, much stronger than classical magnetic field due to dipole m_0 at atom site.

Weiss Theory

- At constant T, plot M against H.
- Can make plots as follows:

$$\boldsymbol{B}_{\text{local}} \rightarrow \quad M \rightarrow H \quad \rightarrow B = \mu_0(M + H)$$

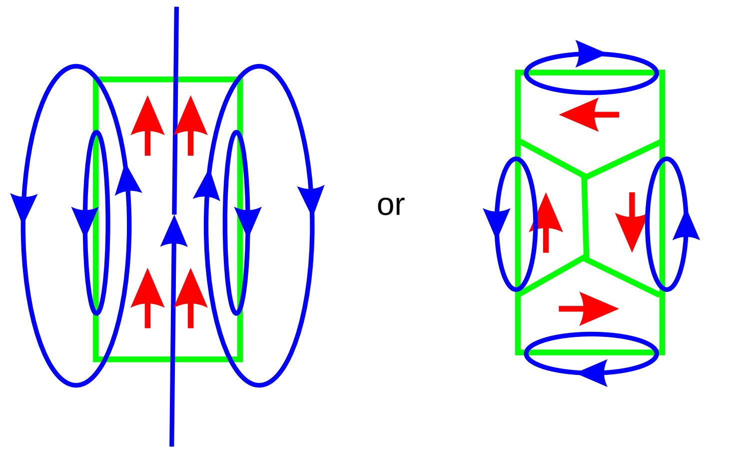
Langevin Weiss Macroscopic B



- Weiss theory predicts qualitatively different behaviour for $T > T_c$ and $T < T_c$.
- For $T < T_c$ the (M, H) graph is multivalued
  $\Rightarrow M \neq 0$ at $H = 0$.
- Part of curve with $\chi < 0$ is *unstable*
  $\Rightarrow$ qualitative explanation of phenomenon of *magnetic hysteresis*.

Magnetic Domains

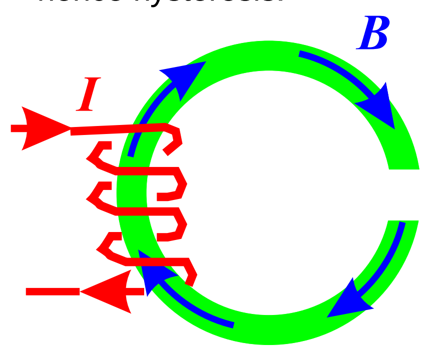
- Curie-Weiss Law $\Rightarrow$ Iron highly magnetised for $T < T_c$.
- But can easily get unmagnetised iron bars.
- Magnetic structure breaks up into *domains* which reduce the total energy of the magnetic field.



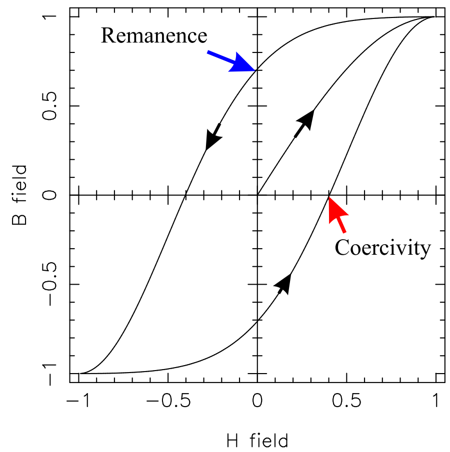
- Domains reduce external field energy, but *domain boundaries* take energy to form.

Permanent Magnets

- Can make permanent magnets using materials which have high potential barriers to formation of domain walls.
- Even non-permanent magnetic materials present some potential barrier — hence hysteresis.

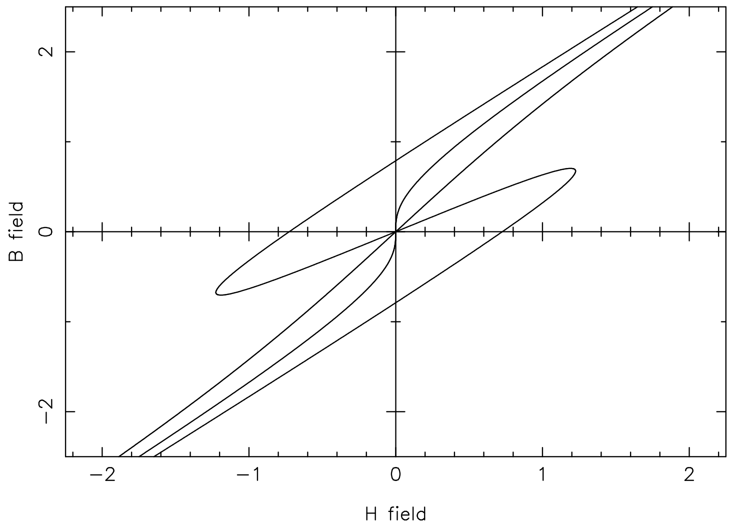


- Apply current I $\Rightarrow$ magnetic field H
- Measure resulting B field in small gap.



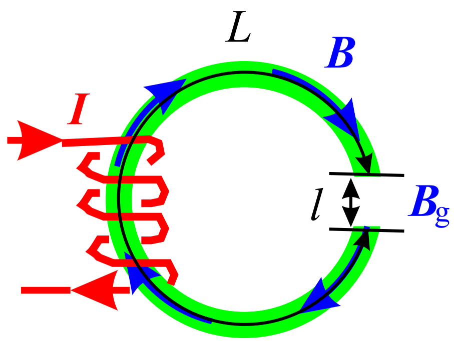
- Some B remains when $H = 0$ called *Remanence* B_r
- Finite H required to flip sign of B. Called *Coercivity* H_c.

Weiss Theory and Magnetic Hysteresis



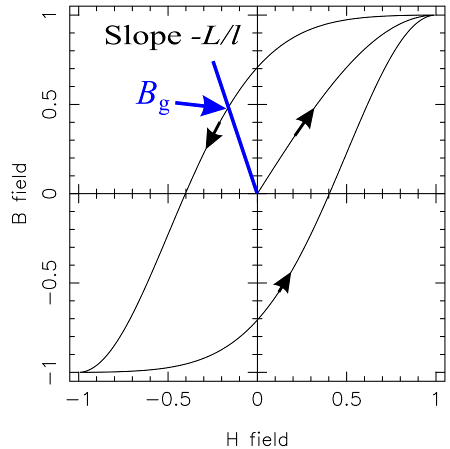
- Weiss theory B versus H graph for $T < T_c$ provides qualitative explanation of hysteresis.
- Scale of B is approximately correct.
- Scale of H is much too large (by factor of about 10^4) for most ferromagnets.
- Coercivity is very variable between different materials.
- M changes by movement of domain walls, rather than by rotation of dipoles.

Hysteresis and Permanent Magnets



- Magnetic ring length L
- Saturate with current I with gap l shorted out.
- Reduce current to zero and unshort the gap.

- What is the field in the gap?



- No current
  $\Rightarrow \oint \boldsymbol{H}\cdot d\boldsymbol{l} = 0$
  $\Rightarrow H_i L + H_g l = 0$
- If gap small $B_g = B_i$

$$\Rightarrow H_i = -\frac{l}{L\mu_0} \quad \text{(sloping up to left)}$$

- $\Rightarrow H_i$ is small, opposing B_i.
- $\Rightarrow B_g$ is a little less than B_r.

Magnetisation Process in Iron

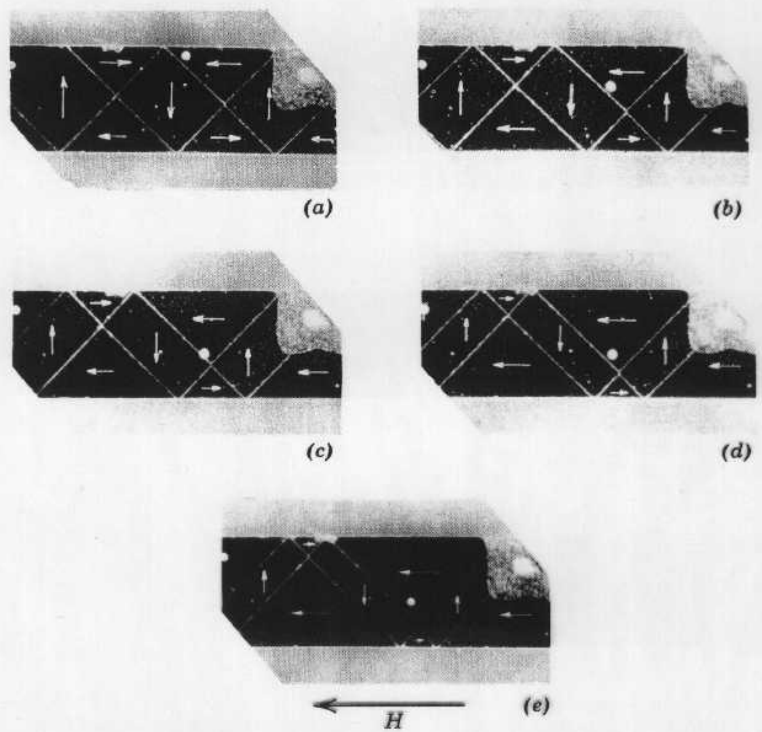


Fig. 18.21. Magnetization process of iron whisker (De Blois and Graham[52]).

- Illustration of movement of domain walls as magnetic field is increased.
- The dipoles of the atoms are ‘flipped’, rather than being continuously rotated.

Coercivity of Magnetic Materials

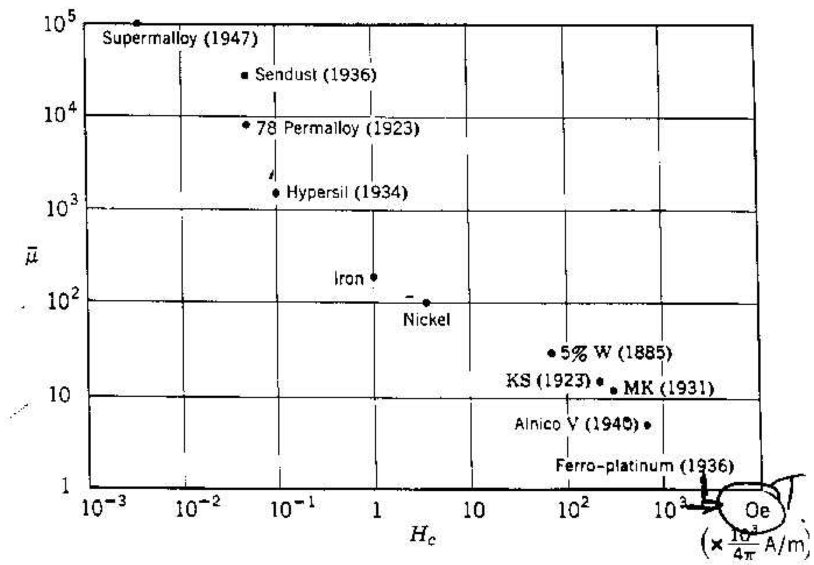


Fig. 2.5. Relative permeability and coercive force of various magnetic m (after C. Kittel[9.6])*.

- Illustration of wide range of coercivity found in magnetic materials.
- Some materials have very strong domain walls — scale of H axis is about 1 T.
- Other materials developed for high permeability and low hysteresis.