NATURAL SCIENCES TRIPOS Part IB
NATURAL SCIENCES TRIPOS Part II (General)

Friday 27 May 1994 9 to 12

ADVANCED PHYSICS (3)

Attempt the **whole** *of Section A and* **three** *questions from Section B. Section A will carry approximately a quarter of the total marks. Answers to Section A and* **each question of Section B** *must be tied up separately, with the letter of the Section and the question number written clearly on* **each** *cover sheet.*

SECTION A

Answers should be concise, and relevant formulae may be assumed without proof.

1 The maximum height h for a vertical cylindrical rod of radius a and density ρ to be stable against bending under its own weight is given by

$$h = 1.99\left(\frac{Ya^2}{4\rho g}\right)^{\frac{1}{3}},$$

where Y is the Young modulus and g the gravitational acceleration.

Use this result to determine Y for a steel rod, given the following experimental values:

$$\begin{aligned} a &= 5.0 \pm 0.1 \text{ mm}; \\ \rho &= (7.8 \pm 0.1)\times 10^3 \text{ kg m}^{-3}; \\ h &= 4.80 \pm 0.08 \text{ m}. \end{aligned}$$

What is the fractional error in this value of Y?

2 Explain the essential features of the technique of phase-sensitive detection and describe how it can be implemented. What are its particular advantages for the measurement of small physical effects?

3 Outline the methods you would choose to measure **each** of the following quantities, indicating what factors influence the choice of devices to use, estimating the accuracy of measurements you expect to achieve, and illustrating your proposed experimental arrangements by a diagram:

(a) the amplitude of vibration of the rim of an empty glass when it is tapped;

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(b) the fluctuations of air temperature in a gentle breeze (temperature variations of ≈ 0.1 K on scales of several centimetres);

(c) the time of contact of a steel ball, 3 cm in diameter, with a thick steel plate on to which it is dropped from a height of 1 m.

SECTION B

4 Write down Maxwell's equations for electromagnetic fields in a homogeneous medium of magnetic permeability μ and relative permittivity ϵ. Hence show that, if the medium has an electrical conductivity σ and the displacement current can be neglected, the magnetic flux density $\boldsymbol{B}$ obeys the differential equation

$$\nabla^2 \boldsymbol{B} = \sigma\mu\mu_0 \frac{\partial \boldsymbol{B}}{\partial t}.$$

A long solenoid of radius b is wound with n turns per unit length of resistanceless wire and carries an alternating current of angular frequency ω. A long cylinder made from material of permeability μ and conductivity σ and of radius a ($a \leq b$) is placed coaxially inside the solenoid. If $a \gg \delta$ [$\delta \equiv \sqrt{2/(\omega\sigma\mu\mu_0)}$], use the previous result to describe how the magnetic field decays with distance in from the surface of the cylinder. Hence show that the cylinder produces an effective resistance of the solenoid R per unit length given by

$$R \approx \frac{2\pi a n^2}{\sigma\delta}.$$

Explain qualitatively why the introduction of the cylinder produces a change ΔL in the self-inductance per unit length of the solenoid, given by

$$\Delta L \approx -\mu_0 n^2 \pi (a^2 - \mu a \delta).$$

5 Discuss the factors that determine the widths of optical spectral lines.

The emission spectrum of hydrogen has a pair of closely-spaced spectral lines at wavelengths of 656.272 nm and 656.285 nm. The ratio of intensities of these two lines is 8:5.

If these two lines are to be resolvable in the light given out by a discharge tube lamp, what limits must be set on the temperature and pressure of the gas in the lamp?

A photodiode capable of responding to changes of light intensity at frequencies up to 20 GHz is illuminated by the lamp. Draw a calibrated sketch of the autocorrelation function of the output of the photodiode.

[Assume that the cross-section for collisional de-excitation of the hydrogen atoms is 10^{-19} m^2.]

6 Discuss the relationship between strain and stress for a uniform elastic isotropic medium having Young modulus Y and Poisson ratio σ.

A long pipe of internal radius a and external radius b is made of an elastic material having Young modulus Y and Poisson ratio σ. The pipe connects two vessels containing gas at high pressure p_1, the external pressure being p_0. There are no longitudinal stresses in the pipe.

(a) Write down the radial strain e_r and the tangential strain e_θ in terms of the radial displacement $R(r)$ at radius r.

(b) Show that the tangential stress τ_θ is related to the radial stress τ_r by the formula

$$\tau_\theta = \frac{d}{dr}(r\tau_r)\,.$$

(c) Show that the radial displacement satisfies the equation

$$r^2\frac{d^2R}{dr^2} + r\frac{dR}{dr} - R = 0.$$

(d) Hence, or otherwise, show that the radial pressure p is given by

$$p(r) \equiv -\tau_r = p_0 + (p_1 - p_0)\,\frac{a^2}{r^2}\frac{b^2 - r^2}{b^2 - a^2}.$$

What is the maximum absolute value of the tangential stress?

7 State the time-dependent Schrödinger equation for a particle of mass m moving parallel to x-axis. What are the boundary conditions on the wavefunction where the particle is incident upon a potential barrier of infinite height?

An infinite potential barrier is moving in the x-direction with velocity u. The potential is given by $V(x,t) = 0$ ($x < ut$); $V(x,t) = \infty$ ($x > ut$).

Consider a stream of particles undergoing reflection from this barrier. Verify that

$$\psi(x,t) = \exp(\mathrm{i}k_1x)\exp\left[-\mathrm{i}(\omega + uk_1)t\right] - \exp(\mathrm{i}k_2x)\exp\left[-\mathrm{i}(\omega + uk_2)t\right],$$

where ω is a constant, is a suitable wavefunction for this potential for $x < ut$, provided that k_1 and k_2 each satisfy

$$\frac{\hbar^2k_\alpha^2}{2m} = \hbar(\omega + uk_\alpha) \quad (\alpha = 1, 2).$$

Find expressions for the velocities (v_i, v_r) of the incident and reflected particles in terms of u and ω, and show that, provided $v_\mathrm{i} > u$, your result is in agreement with the equivalent result for a classical particle.

Discuss briefly what happens if $v_\mathrm{i} < u$.

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8 Explain the variational method for the estimation of the ground-state energy of a quantum-mechanical system.

In the centre-of-mass frame of the deuteron, the potential energy $V(r)$ can be approximated by

$$V(r) = -B\frac{\exp(-\lambda r)}{r},$$

where λ and B are constants and r is the separation between the proton and the neutron. Assume that the wave function of the system is spherically symmetric and of the form

$$\psi(r) = \exp(-\alpha\lambda r/2).$$

Find an expression for the binding energy of the deuteron in terms of α, λ and the reduced mass μ of the system.

Treating α as a parameter, show that the binding energy is minimised when α is a solution of the equation

$$\frac{\hbar^2\lambda}{2\mu}(1+\alpha)^3 = B\alpha(\alpha+3).$$

Given that the expectation value $\langle r\rangle$ of the neutron-proton separation in the ground state is equal to $3/(2\lambda)$, determine the value of α and hence find the numerical value of the binding energy.

[Assume: $m_p c^2 = m_n c^2 = 940\,\text{MeV}$; $\hbar^2\lambda^2 c^2 = 1.05\times10^4\,(\text{MeV})^2$.]

9 A system of volume V may be modelled as a gas having an energy density ϵ which is a function only of temperature T, where $\epsilon = 0$ at $T = 0$. Its pressure p is given by $p = \alpha\epsilon(T)$, where α is a constant. Show that the energy density takes the form

$$\epsilon(T) = AT^n$$

where A and n are constants, and determine the value of n.

Calculate:

(a) the entropy density;

(b) the heat capacity at constant volume;

(c) the heat capacity at constant pressure.

For what range of values of α is the system stable?