NATURAL SCIENCES TRIPOS Part IB
NATURAL SCIENCES TRIPOS Part II (General)

Friday 2 June 1995 9 to 12

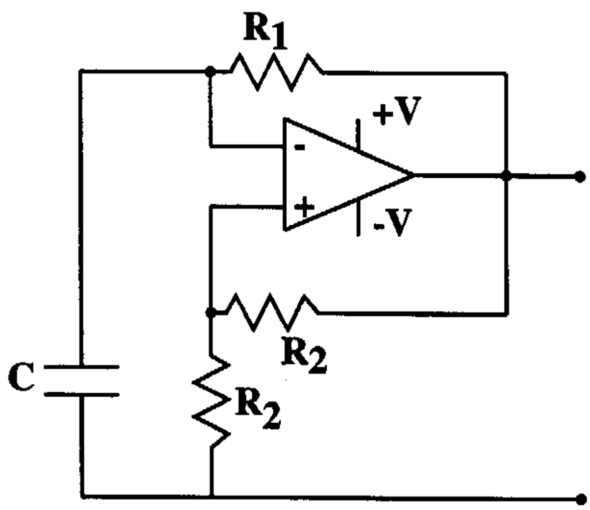
ADVANCED PHYSICS (3)

Attempt the **whole** *of Section A and* **three** *questions from Section B.*
Answers to Section A and **each question of Section B** *must be tied up separately, with the letter of the Section and the question number written clearly on* **each** *cover sheet.*
Section A will carry approximately a quarter of the total marks.
The **approximate** *number of marks allocated to each part of a question in Section B is indicated in the right margin.*

SECTION A

Answers should be concise, and relevant formulae may be assumed without proof.

1 The circuit below illustrates a type of oscillator. Discuss its mode of operation by describing the sequence of events following an assumed starting condition with the output saturated at $+V$ and the inverting input at $-V/2$, and sketch the variation of voltage with time at the inputs and output of the operational amplifier. What is the period of oscillation?



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2 Describe briefly the method you would use to measure **each** of the following:

(*a*) the resistivity of copper, assuming it is provided as foil, 0.1 mm in thickness [resistivity of copper $\simeq 10^{-8}\,\Omega\,\mathrm{m}$];

(*b*) the distribution of intensity with wavelength across the sodium spectral line doublet using a Fabry–Perot etalon;

(*c*) the force between two conducting plates, 10 mm square and separated by 1 mm, when charged to a potential difference of 1000 V.

3 Light from a distant source is focused on a detector which counts the number of photons arriving in time intervals Δt. The light source has a sinusoidal variation of period $4\Delta t$ superimposed on a steady mean intensity. Given the photon counts in 100 successive time intervals below, derive values for

(*a*) the mean intensity;

(*b*) the amplitude of the sinusoidal variation;

and give estimates of their uncertainties.

(1-20)	2	6	6	12	1	4	9	7	5	5	9	3	2	12	5	8	6	5	6	6
(21-40)	4	4	6	11	3	1	8	1	3	4	9	7	2	3	6	6	3	1	5	4
(41-60)	3	7	2	4	2	6	6	6	0	4	11	3	2	4	9	4	0	2	5	8
(61-80)	0	6	7	8	1	6	10	7	0	2	9	9	2	4	6	5	6	3	8	12
(81-100)	6	2	8	9	3	3	9	4	1	3	9	4	3	3	5	5	9	10	8	3

SECTION B

B4 Discuss the boundary conditions for the magnetic flux density **B** and magnetic field strength **H** at the surface of a magnetic material. [8]

A thin square sheet of a paramagnetic substance of side length l, magnetic susceptibility χ and density ρ, is suspended from the midpoint of one side in a uniform horizontal magnetic field of magnitude H. If the suspension exerts no significant couple on the sheet, what are the equilibrium orientations of the sheet, which one is stable and why? [7]

Show that the period of oscillation, T, of the sheet about the stable equilibrium position is given by [10]

$$T = \frac{\pi l}{\chi H}\left(\frac{\rho}{3\mu_0}\right)^{1/2}.$$

B5 Derive an expression for the time derivative of angular momentum with respect to a frame fixed in space in terms of the time derivative with respect to axes fixed in a rotating body. By this means derive the *Euler equations* for the motion of a rigid body not under the action of an external torque, i.e. [5] [5]

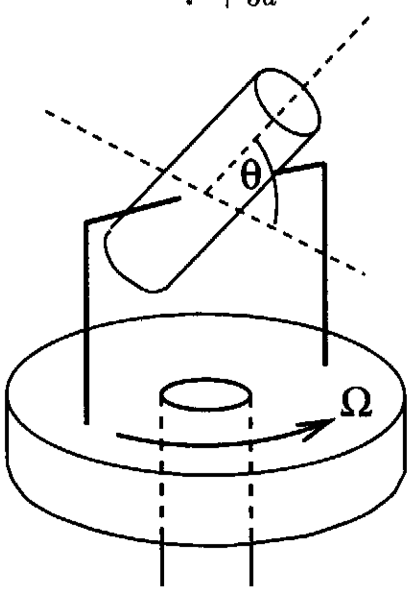
$$\begin{aligned} I_1\dot{\omega}_1 &= \omega_2\omega_3(I_2 - I_3), \\ I_2\dot{\omega}_2 &= \omega_3\omega_1(I_3 - I_1), \\ I_3\dot{\omega}_3 &= \omega_1\omega_2(I_1 - I_2). \end{aligned}$$

A uniform rod of length l, mass M, and with a circular cross-section of radius a, is mounted on a horizontal axle through its centre (see figure below). The rod moves in a plane perpendicular to the axle. Show that the moment of inertia of the rod about the axle is [6]

$$\frac{1}{12}M(l^2 + 3a^2).$$

The axle, which has frictionless bearings, is mounted on a platform that is rotating with angular velocity Ω. The axis of the platform passes through the centre of the rod. Using Euler's equation for motion about the horizontal axle, find a differential equation for the angle θ that the rod makes with the horizontal, as a function of time. Show that for small θ, and assuming $l^2 > 3a^2$, then the motion of the rod is simple harmonic, with angular frequency ω given by [9]

$$\omega^2 = \frac{\Omega^2\,(l^2 - 3a^2)}{l^2 + 3a^2}.$$



B6 Describe the *Abbe theory* of image formation, and explain its application to optical filtering. Include a diagram of the experimental setup required to filter images using plane wave illumination with monochromatic light and two lenses with the same focal length f. [7]

What are the effects of filtering an image using a mask that is: [6]

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(a) opaque within a circle centred on the optical axis, but transparent elsewhere?

(b) transparent within a circle centred on the optical axis, but opaque elsewhere?

A 'halftone' consists of dots or varying size arranged on a regular square grid. What mask could be used to filter a transparency of such a 'halftone' to remove the periodic dot pattern? [5]

An object is transparent with a constant refractive index, but has sinusoidal variations in thickness. The thickness varies as a function of x, giving variations in phase

$$\phi(x) = \phi_0 + \phi_a \sin(kx),$$

where ϕ_a is small. Show that if a mask as in (a) above is used, then the intensity variations of the filtered image are proportional to $(1 - \cos(2kx))$. Comment on the relative intensities corresponding to the thinnest and the thickest parts of the transparent object. [7]

B7 The wavefunction of a particle, of mass m and frequency ω in a three-dimensional harmonic oscillator potential of the form

$$V(x,y,z) = \frac{m\omega^2}{2}(x^2 + y^2 + z^2),$$

is given by

$$\psi(x,y,z) = \left(\frac{a}{\sqrt{\pi}}\right)^{3/2} A H_{n_x}(ax) H_{n_y}(ay) H_{n_z}(az) \exp\left(-\frac{a^2}{2}(x^2 + y^2 + z^2)\right),$$

where $A = (2^{n_x+n_y+n_z} n_x! n_y! n_z!)^{-1/2}$ and $a = \sqrt{m\omega/\hbar}$.

Sketch the form of this wavefunction in one dimension for the ground state and first two excited states. [6]

Sketch and describe the probability of finding a particle a distance x from the origin and compare with that expected classically. [6]

By calculating the expectation value of the energy for the ground state and the first excited state for a single particle in this potential well, show that they are consistent with energy levels given by [6]

$$E = \left(n_x + n_y + n_z + \frac{3}{2}\right)\hbar\omega.$$

Two identical spin-1/2 fermions are located in this potential with total energy $5\hbar\omega$. What is the probability of finding only one of the particles in the ground state with energy $\frac{3}{2}\hbar\omega$? [7]

[The Hermite polynomials for the ground state and first two excited states are: $H_0(\xi) = 1$, $H_1(\xi) = 2\xi$ and $H_2(\xi) = 4\xi^2 - 2$.]

B8 Show that the speed of propagation of transverse waves on a string of mass per unit length ρ under tension T is [5]

$$c = \sqrt{\frac{T}{\rho}}.$$

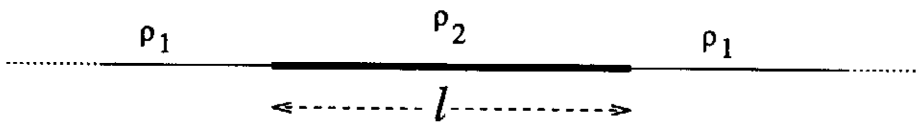
Define *characteristic impedance* for waves, and show that for transverse waves on a string it is [5]

$$Z = \sqrt{\rho T}.$$

A string of mass per unit length ρ_1 is joined to a string of mass per unit length ρ_2. Derive relationships for the amplitude *transmission* and *reflection* coefficients for a wave traveling along the first string towards the second. [5]

Sketch how these reflection and transmission coefficients vary with the ratio of the mass per unit length of the strings. Comment on the extreme cases, and the case where the reflection coefficient is zero. [6]

A long string of mass per unit length ρ_1 is joined to a string of mass per unit length ρ_2 of length l which is then joined to a long string of mass per unit length ρ_1 as shown below. Show that for waves incident from the left there is no reflected wave if l is a half-integer multiple of the wavelength of waves on the central string. [4]



B9 State and discuss Carnot's results concerning the efficiency of heat engines. [8]

By taking an ideal gas around a Carnot cycle, prove that the ideal gas temperature θ is proportional to the thermodynamic temperature T. [8]

Discuss a practical heat engine and analyse it in terms of a Carnot cycle. [9]

B10 Write an essay on the influence of defects and impurities on the strength and electrical properties of solids. [25]