

NATURAL SCIENCES TRIPOS Part IB
NATURAL SCIENCES TRIPOS Part II (General)

Wednesday 29 May 1996 9 to 12

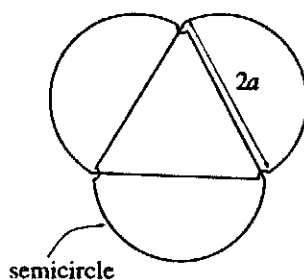
ADVANCED PHYSICS (1)

Attempt the whole of Section A, two questions from Section B and one question from each of Sections C and D.
Answers from each Section must be tied up in separate bundles, with the letter of the Section written clearly on each cover sheet.
Section A will carry approximately a quarter of the total marks.
The approximate number of marks allocated to each part of a question in Sections B, C and D is indicated in the right margin.

SECTION A

Answers should be concise, and relevant formulae may be assumed without proof.

A1 A uniform magnetic flux density B is perpendicular to the plane of the loop of wire illustrated. What is the magnetic flux linkage with the loop?



A2 The filaments of two incandescent light bulbs that are to be used at the same operating voltage are made from the same material. Estimate the ratios of the lengths and cross-sectional diameters of these filaments for bulbs rated at 60 and 100 W, if they give out light of the same colour?

A3 Explain why it is easier to balance vertically a long rod rather than a short rod on the end of a finger.

(TURN OVER)

A4 A spy satellite with a mirror of the same diameter as that of the Hubble Space Telescope (2.4 m) is placed in a geostationary orbit above the Earth. What is the smallest feature on the Earth's surface that can be resolved at optical wavelengths? [Radius of Earth = 6400 km.]

A5 A collimated beam of white light illuminates two closely-spaced parallel slits. The light transmitted on axis at a large distance is examined with a spectroscope. Describe the appearance of the spectrum when one of the slits is covered with a microscope slide 1 mm in thickness.

SECTION B

B6 Discuss **two** of the following: [19]

- the relation between the relative permittivity of a dielectric and the electrical properties of the individual molecules of which it is composed;
- Poynting's theorem and its physical interpretation;
- the uses and limitations of the magnetic scalar potential;
- the 'displacement current'.

B7 State the boundary conditions for D and E at the surface of a dielectric. [3]

When a dielectric sphere of radius a and relative permittivity ϵ is placed in a (previously) uniform electric field E_0 , the electrostatic potentials outside and inside the sphere are given by

$$\phi_{\text{out}} = E_0 r \cos \theta + \frac{Aa^3 E_0 \cos \theta}{r^2} \quad \text{and} \quad \phi_{\text{in}} = C E_0 r \cos \theta.$$

The symbols r and θ refer to spherical polar coordinates with origin at the centre of the sphere and $\theta = 0$ in the direction of E_0 .

Assuming these formulae, show that [8]

$$A = \frac{(1 - \epsilon)}{(2 + \epsilon)} \quad \text{and} \quad C = \frac{3}{(2 + \epsilon)}.$$

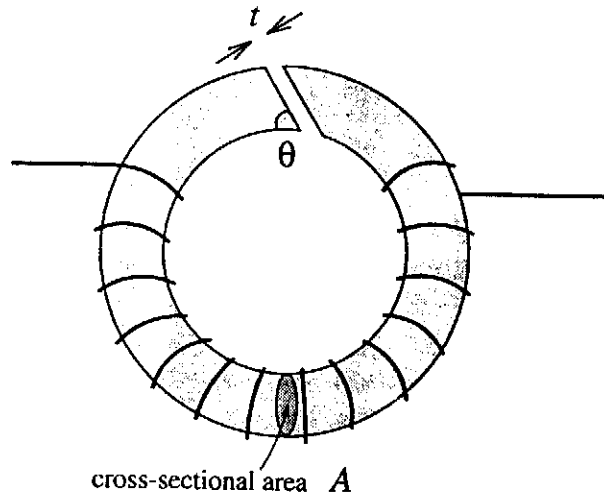
A large earthed sheet of metal lies perpendicular to a uniform field E_0 . When a hemisphere of radius a and relative permittivity ϵ is placed with its flat side against the sheet, how much additional charge appears in the metal sheet [8]

- under the hemisphere ($r < a$);
- beyond the hemisphere ($r > a$)?

B8 An iron torus of cross-sectional area A and relative permeability $\mu (\gg 1)$ is wound with N turns of insulated copper wire. Show that the self-inductance of the coil is [6]

$$L = \frac{N^2 A \mu \mu_0}{l},$$

where l is the length of the torus.



A slot of width t is cut in the torus at angle θ as shown in the diagram. Find (a) the self-inductance of the coil; [6]

(b) the magnetic field strength H in the slot when a current I flows in the wire. [7]

(Assume that $l \gg$ thickness of iron $\gg t$, but note that μt may be comparable with l .)

B9 A linearly polarised electromagnetic wave of frequency $\omega/(2\pi)$, with electric vector in the z direction, propagates in the x direction into an isotropic metal of conductivity σ . Show that [13]

$$E_z = E_{z0} \exp(px - i\omega t),$$

where $p \approx -(1 - i)/\delta$ and the skin depth $\delta = \sqrt{2/(\mu_0 \omega \sigma)}$, discussing in what circumstances the approximation is valid.

Show also that the amplitude reflection coefficient R for electromagnetic waves incident normally on the surface of the metal is given by [6]

$$\frac{1 - R}{1 + R} = \frac{(i + 1)\sigma\delta}{2} \sqrt{\frac{\mu_0}{\epsilon_0}} \quad r ?$$

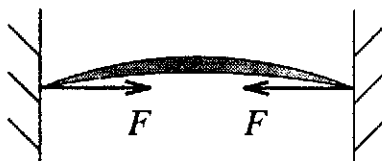
and comment on this result.

(TURN OVER

SECTION C

C10 Derive the relation between the moment of area and bending moment of a beam under stress, giving definitions of all the quantities involved. [9]

A light strut of length $2l$, which has a varying cross-sectional area but constant cross-sectional shape, is held under compression in the jaws of a vice, with the applied forces at each end horizontal, as shown below.



In this configuration it is found that the strut is bent into an arc of a circle. The radius of curvature of the bent strut is large compared with the length of the strut and the effects of gravity are negligible.

Prove that the cross-sectional area A of the strut at a point distance x from the centre of the strut is given by [10]

$$\left(\frac{A}{A_0}\right)^2 = 1 - \frac{x^2}{l^2},$$

where A_0 is the cross-sectional area at the centre of the strut.

C11 What Coriolis force acts on an object moving along a north-south line on the Earth's surface? [4]

A plane sets out to fly from London to the island of St. Helena in the South Atlantic at 800 km hr^{-1} . St. Helena is almost due south of London at latitude 16°S . Show that, to compensate for the Coriolis force, the pilot points the nose of the plane east of south at an angle ϕ , given by [6]

$$\phi = \sin^{-1} \left(\frac{2m\Omega v \sin \lambda}{T} \right),$$

where m, v and T are the mass, velocity and thrust of the plane, Ω is the Earth's angular velocity, and λ is the latitude. Assume that the plane is travelling in still air and that the drag is antiparallel to the plane's velocity.

At the equator the co-pilot takes over, keeps the velocity of the plane constant and remembers to correct for the Coriolis force, but gets the sign of the correction wrong. By considering the forces on the plane (the Coriolis force, C , the thrust, T , and the drag, D), by what distance will they miss the island of St. Helena? [9]

[Radius of Earth = 6400 km, mass of plane = $27 \times 10^4 \text{ kg}$. Thrust is 1/16 of the plane's weight.]

SECTION D

D12 Define *Fresnel zones*. Derive the relationship for the radius of the n th zone, [6]

$$r_n^2 \approx nd\lambda,$$

for illumination by a plane wavefront of wavelength λ observed at a distance d along the axis.

A zone plate of radius R consists of N zones, with all the odd zones blocked. Derive the principal and secondary focal lengths of the zone plate, and the relative intensities at these foci. [5]

By equating the incident power with the power in the principal focal spot, show that the approximate radius of the spot is [4]

$$\frac{R}{N\sqrt{2}}$$

If the odd zones are not blocked, but consist of glass of thickness such that the phase of the incident plane wave is changed by π , what is the width of the principal focal spot in this case? [4]

D13 Write an essay on optical microscopy. Include discussion of the resolution limitations and of the techniques of *dark field* and *phase contrast* microscopy. [19]

