

NATURAL SCIENCES TRIPOS Part II

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Saturday 29 May 1999 9 to 12

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EXPERIMENTAL AND THEORETICAL PHYSICS (4)

*Candidates offering the whole of this paper should attempt to answer all the questions in Section A, two questions from Section B and two questions from Section C.*

*Candidates offering half of this paper should attempt three questions from Section A, one question from Section B and one question from Section C. They will be required to leave the examination after one and a half hours.*

*Answers from Section A should be tied up in a single bundle, with the letter A written clearly on the cover sheet. Answers to each question from Sections B and C should be tied up separately with the number of the question written clearly on the cover sheet.*

*Sections A and B will each carry approximately a quarter of the total marks. The approximate number of marks allocated to each part of a question in Section C is indicated in the right margin. The paper contains 5 sides and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

SECTION A

*Answers should be concise, and relevant formulae may be assumed without proof. All questions carry an equal amount of credit.*

A1 Radiocarbon dating is to be used to estimate the age of the Turin shroud, using a sample containing 50 mg of carbon. For how long must the decays be counted if the sample is to be dated to  $\pm 250$  yr and its age is 2000 yr?

*[The half-life of  $^{14}\text{C}$  is 5730 yr and, when formed, organic matter has an activity of 15 decays/min per gram of carbon.]*

(TURN OVER

A2 It has been suggested that a superconducting solenoid can be used to store the energy produced by a power station overnight. If superconducting wire can be operated at a current density of up to  $3000 \text{ A (mm)}^{-2}$  in a field of up to 5 T, suggest suitable dimensions for a solenoid capable of storing up to  $2 \times 10^{13} \text{ J}$  of magnetic energy.

A3 Why is spontaneous emission important in a discharge lamp but not in a radio transmitter operating at 1 MHz?

A4 Estimate the time required for laser cooling of an atom.

A5 A diver is swimming in the deep open ocean on a day when there are swells of 1 m amplitude passing with a period of 2.5 s. How far below the surface must she swim so that the amplitude of motion is reduced to 1 cm?

A6 A trail of condensed vapour is sometimes observed to trail from the tip of an aeroplane wing. Suggest an explanation for this phenomenon.

## SECTION B

B7 Write brief notes on two of the following:

- (a) magnetic dipole transitions;
- (b) optical pumping;
- (c) two-photon transitions.

B8 Discuss the experimental evidence for the quantum-mechanical theory of light. Include in your answer a discussion of how the quantum-theoretical and classical descriptions of light can be reconciled.

B9 Write brief notes on two of the following:

- (a) the formation of meanders in river courses;
- (b) the Kelvin-Helmholtz instability;
- (c) the formation and propagation of vortex rings.

B10 Discuss, using examples, the advection and diffusion of vorticity in a viscous fluid.

## SECTION C

C11 The temporal intensity autocorrelation function  $g(T)$  for a light beam of time-dependent intensity  $I(t)$  is defined by:

$$g(T) = \frac{\langle I(t)I(t-T) \rangle}{\langle I(t) \rangle^2}.$$

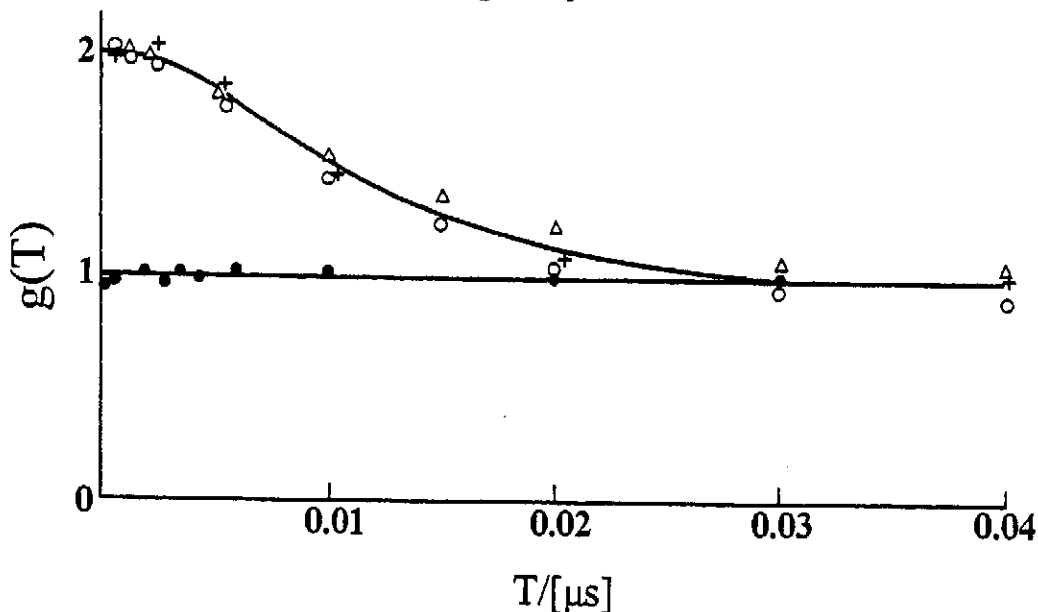
Explain why the intensity autocorrelation function is useful in characterising a light source. [3]

An atomic lamp emits monochromatic radiation and may be modelled as a finite set of independent identical oscillators which randomly emit radiation. By constructing the amplitude-phase diagram for such a set of oscillators, or otherwise, show that the probability density  $p(I)$  that the emitted intensity has the value  $I$  is given by:

$$p(I) = \frac{1}{\langle I \rangle} \exp\left(\frac{-I}{\langle I \rangle}\right) \quad [7]$$

Calculate  $g(T=0)$  for this model. [7]

Experimentally-measured intensity correlations of (a) a laser and (b) a discharge lamp, both operating on the same atomic transition, are shown in the figure below. Explain, without detailed calculation, how the properties of each source gives rise to the observed behaviour. Make an estimate of the linewidth of the atomic transition for the discharge lamp. [8]



[The filled circles refer to measured results for laser light and the remaining experimental points refer to the discharge lamp. The solid lines are the theoretically-predicted curves.]

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C12 Explain what is meant by (a) LS coupling and (b) jj coupling. Why are light atoms well described by the first case? [8]

In Mg there is a very strong single line in the absorption spectrum at 285.2 nm, which is the longest wavelength at which features appear in the spectrum. In the emission spectrum of Mg there is a well-defined series of triplets of which the lowest in energy are: 518.5 nm, 517.3 nm and 516.7 nm.

The ground state configuration of Mg is  $(1s)^2(2s)^2(2p)^6(3s)^2$ . What are the states which give rise to the above transitions? [7]

The corresponding wavelengths in  $K^{7+}$ , which has the same electronic configuration as Mg, are 51.9 nm in the absorption spectrum and 78.1 nm, 77.5 nm and 76.1 nm in the emission spectrum.

Comment in detail upon the similarities and differences between the spectra for Mg and  $K^{7+}$ . [10]

[In LS coupling the spin-orbit interaction energy has the form:

$$\xi(r) \langle \mathbf{L} \cdot \mathbf{S} \rangle = \xi(r) [J(J+1) - L(L+1) - S(S+1)] \hbar^2 ,$$

where  $\xi(r)$  depends on radial coordinates only.]

C13 In what circumstances can fluid flow be described by a velocity potential? [5]

A small bubble is expanding at a constant volume rate  $Q$  a distance  $d$  away from a solid plane in a fluid of density  $\rho$ . Show that the pressure distribution on the plane is given by

$$p(r) = p_0 - \frac{\rho Q^2 r^2}{8\pi^2 (d^2 + r^2)^3} , \quad [8]$$

where  $r$  is the radial distance from the point of symmetry on the plane, and  $p_0$  is the pressure at a large distance from the bubble.

Hence, or otherwise, determine the force on the bubble. [8]

Is the force repulsive or attractive? Explain why. [4]

C14 What is a hydraulic jump? [5]

A hydraulic jump is stationary in the laboratory frame. By considering the balance between the momentum flux across the jump and the pressure change, show that the relationship between the depths  $d, d'$ , on either side of the jump ( $d' > d$ ) and the incoming velocity  $u$  is given by

$$u^2 = \frac{gd'(d'+d)}{2d} ,$$

where  $g$  is the acceleration due to gravity. [8]

A stream of water 10 mm in diameter impinges vertically with a velocity of  $5 \text{ m s}^{-1}$  on a large horizontal tray. Show that the thickness of the sheet of water so formed falls off inversely with distance from the centre, so long as streamline flow is maintained.

[4]

Find the radius at which a hydraulic jump is observed, if the depth of the water at the edge of the tray is 10 mm.

[8]

*[You may assume that the tray is sufficiently large that, to sufficient accuracy, the post-jump flow has a uniform and constant depth of 10 mm. Ignore the effects of viscosity.]*

