NATURAL SCIENCES TRIPOS Part II

Saturday 3rd June 2000   9.00 to 12.00

EXPERIMENTAL AND THEORETICAL PHYSICS (4)

Candidates offering the **whole** *of this paper should attempt to answer* **all** *the questions in Section A,* **two** *questions from Section B, and* **two** *questions from Section C.*

Candidates offering **half** *of this paper should attempt to answer* **three** *questions from Section A,* **one** *question from Section B, and* **one** *question from Section C. They will be required to leave the examination room after one and a half hours.*

Answers from Section A should be tied up in a single bundle, with the letter A clearly written on the cover sheet. Answers to **each** *question from Sections B and C should be tied up separately with the number of the question written clearly on the cover sheet.*

Sections A and B will each carry approximately a quarter of the total marks. The **approximate** *number of marks allocated to each part of a question in Section C is indicated in the right margin. The paper contains 5 sides and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

SECTION A

Answers should be concise, and relevant formulae may be assumed without proof. All questions carry an equal amount of credit.

A1 Explain how aircraft can fly in their usual orientation, and suggest how some (high-powered) aircraft can fly upside-down.

A2 Describe Borda's mouthpiece, and explain the physics of fluid flow through one.

A3 Estimate the speed of the edge of the Moon's shadow at the Earth's equator during a total solar eclipse.

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A4 Why can we ignore the energy stored in the magnetic flux density produced by electrons in atoms, but not of the conduction electrons in a typical inductor?

A5 The reflecting mirrors of a laser cavity have an intensity reflectivity $R = 94\%$ and a separation of 10 cm. The laser power is switched off abruptly. Estimate the time required for the output intensity of the laser to decay by a factor of 2.

A6 A laser dye exhibits gain only for wavelengths in the range 600–700 nm and is used in a mode-locked laser. What is the shortest pulse that the laser could produce?

SECTION B

B7 Write brief notes on *two* of the following:

(a) laminar flows;

(b) surface waves;

(c) the measurement of fluid flow rate with Pitot tubes and Venturi meters.

B8 Write an essay on Reynolds, Mach and Euler numbers, explaining how they are useful to characterise fluid flow, including an explanation of dynamic similarity.

B9 Write brief notes on *two* of the following:

(a) the shapes of atomic spectral lines;

(b) laser cooling of atoms;

(c) the optical Bloch equations.

B10 Describe experimental techniques used in high resolution laser spectroscopy and discuss their application to the study of atomic transitions.

SECTION C

C11 The Navier–Stokes equation can be written as

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}\right) = -\boldsymbol{\nabla}p + \eta\boldsymbol{\nabla}^2\boldsymbol{u}.$$

Interpret the left hand side and each term of the right hand side of this equation. [6]

A solid spherical projectile of radius R and density ρ_s moves through a fluid of density ρ and viscosity η at a velocity v.

(a) For small velocities, taking the two terms on the left hand side of the Navier–Stokes equation to be of similar magnitude, and by estimating the magnitudes of the dominant terms of the Navier–Stokes equation, show that the magnitude of the drag force scales as $\eta v R$. You may neglect numerical factors, including π. [5]

(b) For large velocities, by comparing the dominant terms in the Navier–Stokes equation, show that the steady drag force scales as $\rho v^2 R^2$, independent of viscosity. Again, you may neglect numerical factors, including π. Interpret the work done by this drag force in terms of motion of the fluid. [7]

Estimate the characteristic velocity, v_c, that marks the boundary between the small-velocity and high-velocity regimes, and work out approximately how far the projectile will travel before stopping, if the initial velocity is $v_0 \gg v_c$. [7]

C12 Explain the origin of the Magnus effect, and discuss the magnetic analogue of a vortex in uniform flow. [6]

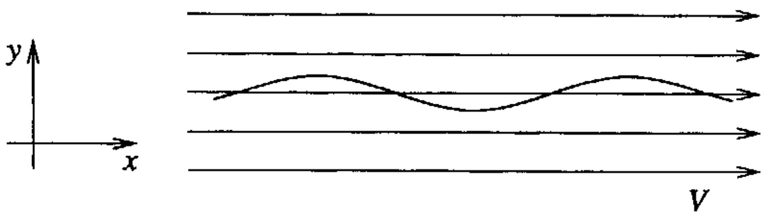
A straight vortex of length L, circulation K extends between two parallel plates, in a fluid of density ρ, with the vortex core perpendicular to the plates. By considering an idealised model for the vortex, with a core of radius a executing solid body rotation, show that the kinetic energy stored in the vortex can be written as [5]

$$\frac{\rho K^2 L}{4\pi}\left[\frac{1}{4} + \ln\left(\frac{R}{a}\right)\right],$$

explaining the significance of R. [2]

By considering the work done pulling the two plates apart slowly, derive the tension between the plates due to the vortex line. [6]

A long vortex of circulation $K\hat{\boldsymbol{x}}$ is held parallel to a uniform flow, with a velocity $\boldsymbol{V} = V\hat{\boldsymbol{x}}$. A small displacement in the position of the vortex core, of the form $\boldsymbol{r} = r\cos(kx)\hat{\boldsymbol{y}}$ is introduced, as shown in the figure below. What is the force on the vortex line? Describe qualitatively, using diagrams, the initial phase of the subsequent motion. [6]



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C13 Justify the representation of the polarisation state of light as a two component electric field vector with components E_x and E_y. [5]

A linear polariser has its transmission axes oriented at an angle θ with respect to the x axis. Show that the polarisation state transmitted by the polariser can be described by the components [5]

$$\begin{aligned} E'_x &= \cos^2\theta\, E_x + \cos\theta\sin\theta\, E_y, \\ E'_y &= \cos\theta\sin\theta\, E_x + \sin^2\theta\, E_y, \end{aligned}$$

where E_x, E_y describe the polarisation state incident on the polariser.

An electro-optic cell produces a voltage-dependent birefringence Δn. The cell is placed between two linear polarisers with their transmission axes oriented to be mutually perpendicular. One axis of the electro-optic cell is oriented at $\pi/4$ radians with respect to the transmission axis of the first polariser. By considering an appropriate matrix representation of each component or otherwise, show that the transmission of the composite device is given by [15]

$$T = \sin^2\left(\frac{\pi \Delta n d}{\lambda}\right),$$

where d is the thickness of the electro-optic cell, and λ is the wavelength of the light in vacuum.

C14 Explain how the spatial and temporal coherence of a light source determine the amplitude correlation between the fields measured at two different positions. [5]

The correlation between the intensity of an optical wavefield generated by a thermal source measured at two positions labelled 1, 2 is related to the corresponding amplitude correlation through the expression

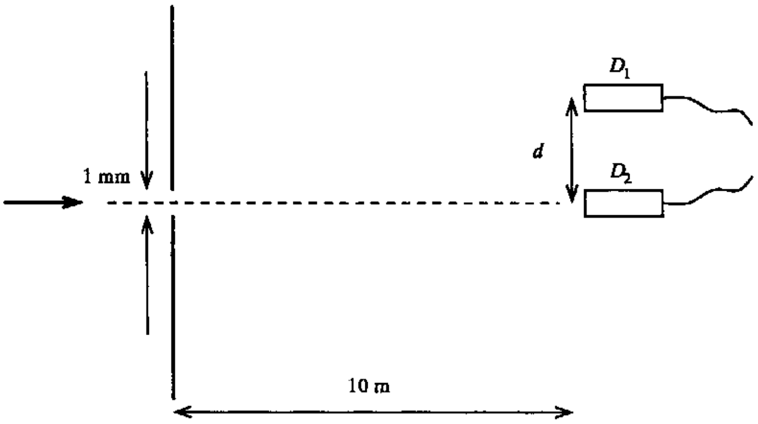
$$\langle I_1(\tau) I_2(0)\rangle = \langle I_1\rangle \langle I_2\rangle \left[1 + |\gamma_{12}(\tau)|^2\right],$$

where the coherence function $\gamma_{12}(\tau)$ is given by

$$\gamma_{12} = \frac{\langle E_1(\tau) E_2^*(0)\rangle}{\sqrt{\langle |E_1|^2\rangle \langle |E_2|^2\rangle}}$$

and all symbols have their usual meanings. Outline how this formula may be derived, explaining your assumptions. [10]

Monochromatic light of wavelength 500 nm is passed through a narrow slit of width 1 mm. The light intensity is measured 10 m distant from the slit using two identical detectors, one opposite the slit and a second separated from the first by a distance d, as shown in the diagram below.



Find the smallest value of d for which the instantaneous intensity correlation (i.e. for $\tau = 0$) between the two detected intensities is a minimum. [10]