NATURAL SCIENCES TRIPOS Part II

Saturday 1st June 2002 9.00 to 12.00

EXPERIMENTAL AND THEORETICAL PHYSICS (4)

Candidates offering the whole *of this paper should attempt the* whole *of Section A,* two *questions from Section B, and* two *questions from Section C.*

Candidates offering half *of this paper should attempt* three *questions from Section A,* one *question from Section B, and* one *question from Section C. They will be required to leave the examination after one and a half hours.*

Answers from Section A should be tied up in a single bundle, with the letter A written clearly on the cover sheet. Answers to each *question from Sections B and C should be tied up separately, with the number of the question written clearly on the cover sheet.*

Sections A and B each carry approximately a quarter of the the total marks. The approximate *number of marks allocated to each part of a question in Section C is indicated in the right margin. The paper contains 5 sides, and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

Answers should be concise, and relevant formulae may be assumed without proof. All questions carry an equal amount of credit.

1 Explain how a half-wave plate can be used to rotate the plane of polarisation of linearly polarised light through an angle θ.

2 Give a physical picture of the origin of the Lamb shift in the hydrogen atom.

3 It is very easy to blow a candle out at a distance of, say, 20 cm – but difficult to suck it out. Why?

4 Why is it difficult for a ship to exceed a speed of $\sim \sqrt{gl}$, where l is its waterline length?

5 Distinguish between Fresnel and Fraunhofer diffraction.

6 What is the moment of inertia of a solid cylindrical rod of length l, radius a and mass M about an axis through the centre of mass of the rod perpendicular to the axis of the rod?

SECTION B

Credit will be given for well-structured and clear explanations, including appropriate diagrams and formulae (detailed mathematical derivations are not required).

B7 Write an essay on laser cooling of atoms. Include a discussion of one model for cooling beyond the recoil limit and indicate how this has been exploited to obtain Bose–Einstein condensation.

B8 Write brief notes on **two** of the following:

(a) Rabi oscillations;

(b) squeezed light;

(c) the Lamb dip.

B9 Write an essay on Reynolds number. Include the arguments and estimates leading to the statement: "The large-scale properties of free turbulent flows are very nearly independent of the viscosity of the fluid".

B10 Write brief notes on **two** the following:

(a) the fluid dynamics of aircraft propeller operation;

(b) Bernoulli's theorem and when it is applicable;

(c) small amplitude gravity waves in water.

SECTION C

C11 An electromagnetic field is present in free space. Explain the physical significance of the Poynting vector

$$\boldsymbol{S} = \frac{1}{\mu_0}\boldsymbol{E} \times \boldsymbol{B}$$

and its relation to the electromagnetic energy density [3]

$$U = \frac{\epsilon_0}{2}|\boldsymbol{E}|^2 + \frac{1}{2\mu_0}|\boldsymbol{B}|^2.$$

Without detailed mathematical derivation, justify the expression for the angular momentum density of the field [3]

$$\boldsymbol{J} = \epsilon_0 \boldsymbol{r} \times (\boldsymbol{E} \times \boldsymbol{B}).$$

A plane electromagnetic wave propagates in free space along the z-direction so that its phase is given by $\Phi = kz - \omega t$. The electric and magnetic field components are

$$\boldsymbol{E} = E_0 (\cos\Phi, -\sin\Phi, 0); \qquad \boldsymbol{B} = \frac{E_0}{c}(\sin\Phi, \cos\Phi, 0)\ .$$

Describe the polarisation state of this wave and evaluate the Poynting vector and energy density. [4]

Suppose now that the wave has a lateral variation of amplitude described by a real function $E_0(x, y)$, where the variation of E_0 is very small on the scale of a wavelength. Show that there must now be a z-component of the field, given approximately by

$$E_z \approx -\frac{1}{k}\left(\sin\Phi\frac{\partial E_0}{\partial x} + \cos\Phi\frac{\partial E_0}{\partial y}\right)$$

and give the corresponding form for B_z. [5]

Evaluate the transverse components of $\boldsymbol{E} \times \boldsymbol{B}$ and hence show that the z-component of the angular momentum density is [5]

$$J_z(x, y) = -\frac{\epsilon_0 E_0}{ck}\left(x\frac{\partial E_0}{\partial x} + y\frac{\partial E_0}{\partial y}\right).$$

(TURN OVER for continuation of question C11

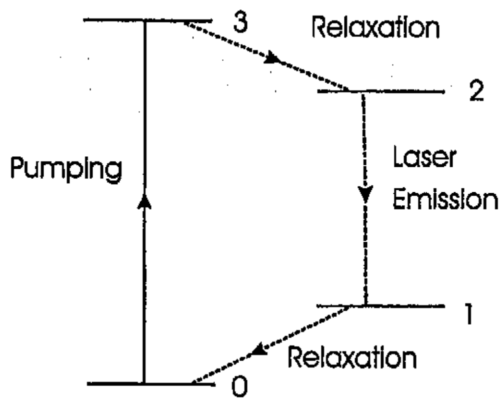
Show that, when integrated over the whole x–y plane, the angular momentum and energy satisfy the relation

$$\iint U \, dx \, dy = \omega \iint J_z \, dx \, dy$$

and comment on the significance of this result. [5]

C12 Explain the principles underlying the operation of lasers, illustrating your answer with a description of the Helium–Neon laser. [7]

The energy-level diagram for a particular 4-level laser is shown below.



If N_i is the number of atoms in level i, in what circumstances is it possible to approximate $N_1 \approx 0$, $N_3 \approx 0$, so that the total number of atoms $N_T = N_0 + N_2$? [3]

Setting $N_1 = N_3 = 0$ leads to simplified rate equations for the number of atoms N_2 in the upper level and the occupation number of photons q in the lasing transition

$$\begin{aligned} \frac{dN_2}{dt} &= W_P N_0 - qBN_2 - \frac{N_2}{\tau}, \\ \frac{dq}{dt} &= qBV_m N_2 - \frac{q}{\tau_c}. \end{aligned}$$

Define the quantities W_P, B, τ, τ_c and V_m and explain the meaning of all the terms in these equations. [4]

Describe the steady-state behaviour of the system as a function of the pump power. Find, in particular, the critical pump power W_{Pc} and population N_{2c} for the onset of laser action and show that the steady-state occupation number q_s in the lasing state is given by

$$q_s = \frac{N_{2c} V_m \tau_c}{\tau} \left(\frac{W_P}{W_{Pc}} - 1 \right).$$

Draw a graph of q_s and the steady-state population N_{2s} as W_P is varied from 0 to $2W_{Pc}$. [7]

To what extent is this model of a laser realistic? [4]

C13 A straight vortex in an incompressible liquid has a core of radius a which rotates as a solid body. Outside the core, the liquid has azimuthal velocity $v_\theta = K/(2\pi r)$, where r is the distance from the centre of the vortex and K is the vorticity. Show that, in cylindrical polar coordinates, [5]

$$\rho\frac{v_\theta^2}{r} = \frac{\partial P}{\partial r}$$

where ρ is the density and P the pressure.

Assuming the pressure in the stationary liquid is P_0, find the distribution $P = P(r)$ outside and inside the vortex core and hence determine the value of the pressure on the vortex core axis. [8]

How is the problem modified for a (compressible) isothermal ideal gas? Show that, in this case, outside the vortex core, [8]

$$P = P_0\,\mathrm{e}^{-r_0^2/r^2}.$$

Find the value of the length scale constant r_0. [4]

C14 The equation of motion of a viscous incompressible fluid is

$$\rho\dot{\boldsymbol{v}} + \rho(\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = \eta\nabla^2\boldsymbol{v} - \nabla P + \boldsymbol{f}.$$

Explain briefly the origin of each term. What are the boundary conditions on $\boldsymbol{v}$ for such a liquid at an interface with a solid substrate and at a free surface? [5]

Molten lava, flowing down a flat plane inclined at an angle α to the horizontal, can be treated as a very viscous fluid. For the purpose of this question, assume that no significant change in its properties occurs over the length of its flow. Calculate the effective viscosity of molten lava of density $\rho = 2500$ kg m^{-3}, given that a layer of thickness 3 m on a plane inclined at $\alpha = 5°$ has a surface velocity of 1 m s^{-1}. [8]

The lava flow meets the valley floor, which is a plane inclined by a smaller angle, $\beta = \alpha/2$. What value will the thickness reach some way down the valley? [7]

Estimate the characteristic time required for the steady-state flow of lava to become established after it starts flowing down the plane. [5]

END OF PAPER