

EXPERIMENTAL AND THEORETICAL PHYSICS (4)

*Candidates offering the **whole** of this paper should attempt to answer **all** the questions in Section A, **two** questions from Section B and **two** questions from Section C.*

*Candidates offering **half** of this paper should attempt **three** questions from Section A, **one** question from Section B and **one** question from Section C. They will be required to leave the examination after one and a half hours.*

*Answers from Section A should be tied up in a single bundle, with the letter A written clearly on the cover sheet. Answers to **each** question from Sections B and C should be tied up separately with the number of the question written clearly on the cover sheet.*

*Sections A and B will each carry approximately a quarter of the total marks. The **approximate** number of marks allocated to each part of a question in Section C is indicated in the right margin. The paper contains **5** sides and is accompanied by a book giving values of constants and containing mathematical formulae which you may quote without proof.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

SECTION A

Answers should be concise, and relevant formulae may be assumed without proof. All questions carry an equal amount of credit.

A1 What is the Lamb dip?

A2 A 633 nm laser beam carries an average power of 1 mW through an optical system comprising a sequence of mirrors and lenses. If the electrical breakdown strength of the air in which the components are located is 3 MV m^{-1} , is it possible for the laser to ionise the air at any point in the system? Justify your reasoning numerically.

A3 Why does a stream of water from a tap become narrower as it falls?

A4 Estimate the area of parachute canopy required to deliver a mass of 1000 kg at a vertical landing velocity of 3 m s^{-1} .
[The density of air is 1.3 kg m^{-3} .]

A5 A solid cylinder of length L and density ρ impacts a rigid surface end-on at velocity v . If the stress wave velocity in the cylinder is v_s , calculate the duration τ of the impact and hence the pressure P .

A6 In an experiment to detect a source of photons in the presence of a background, 11346 photons are observed in 1000 seconds. In a calibration, observing the background alone, 9872 photons are counted in the same time. Estimate the rate of emission from the source in counts per second and the likely accuracy of your estimate.

SECTION B

B7 Write brief notes on **two** of the following:

- (a) laser cavities and modes;
- (b) laser cooling of atoms;
- (c) ladder operators in the context of optical fields.

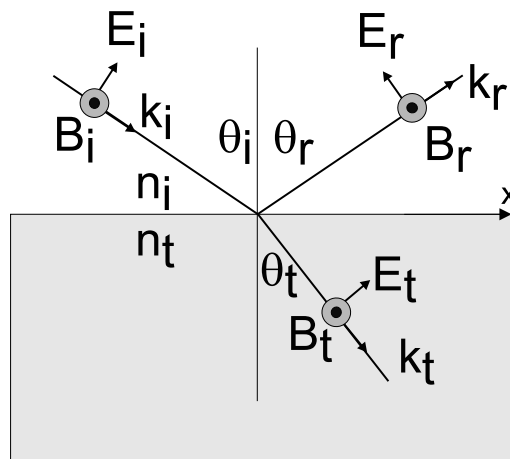
B8 Write an essay on the polarisation of optical fields. You should include the following topics: Jones vectors; Jones matrices; Stokes parameters; manipulating polarisations.

- B9 Write brief notes on **two** of the following:
- (a) the advection and diffusion of vorticity;
 - (b) bores and hydraulic jumps;
 - (c) boundary layer separation.

B10 Write an essay on wave generation by sources which move on the surface of fluids at constant velocity, with particular reference to ship waves in narrow canals and wave resistance.

SECTION C

C11 Discuss briefly the propagation of electromagnetic waves in a perfect dielectric medium having relative permittivity ϵ and relative permeability $\mu = 1$. Include a definition of the fields \mathbf{E} , \mathbf{H} , \mathbf{D} and \mathbf{B} , the wave impedance Z , the refractive index n and the wave velocity c/n , explaining how they are related to the permittivity ϵ . State the boundary conditions on the fields \mathbf{E} and \mathbf{D} at the boundary between media having different values of ϵ , and justify them briefly in terms of Maxwell's equations. [10]



The figure shows a linearly polarised plane wave, with electric field \mathbf{E}_i , incident on a boundary between two media having refractive indices n_i and n_t . The incident wave is scattered into a reflected wave \mathbf{E}_r and a transmitted wave \mathbf{E}_t . Assuming that the incident wave has the form

$$\mathbf{E}_i(\mathbf{r}, t) = \mathbf{E}_0 \exp(i(\mathbf{k} \cdot \mathbf{r} - \omega t))$$

write down an expression for the relation between \mathbf{E}_i , \mathbf{E}_r and \mathbf{E}_t at the boundary between the two dielectrics. Use your expression to prove Snell's laws: [7]

$$\theta_r = \theta_i ; \quad n_t \sin \theta_t = n_i \sin \theta_i$$

(TURN OVER for continuation of question C11

Show that the amplitude reflection coefficient is given by

$$\frac{|\mathbf{E}_r|}{|\mathbf{E}_i|} = \frac{(n_t \cos \theta_i - n_i \cos \theta_t)}{(n_t \cos \theta_i + n_i \cos \theta_t)} . \quad [6]$$

Explain why a pair of Polaroid sunglasses will help an observer see below the surface of a pond when the Sun is at an elevation of 37 degrees ($n=1.33$ for water). [2]

C12 An atomic discharge lamp is placed at the focal point of a lens. Assume that the lamp is a collection of atoms radiating with random relative phases. Show that the probability density $\text{Pr}(I)$ that the intensity at any point in the second focal plane is I is given by:

$$\text{Pr}(I) = \frac{1}{\langle I \rangle} \exp\left(\frac{-I}{\langle I \rangle}\right) .$$

[When adding contributions, it will be helpful to consider the field associated with each atom to be a random vector with Gaussian statistics in the complex plane.] [10]

The temporal intensity autocorrelation function $g(\tau)$ for light of time-dependent intensity $I(t)$ is defined by

$$g(\tau) = \frac{\langle I(t)I(t-\tau) \rangle}{\langle I(t) \rangle^2} .$$

By using the probability density function derived earlier, find the value of the autocorrelation function for $\tau = 0$.

Draw a diagram indicating roughly how you expect $g(\tau)$ to vary as a function of τ for the discharge lamp. Using a dotted line on the same graph, draw $g(\tau)$ as a function of τ when the discharge lamp is replaced by a laser having the same central wavelength. Explain, without detailed calculations, the form of your plots. [10]

Explain briefly how the form of $g(\tau)$ is related to photon bunching. [5]

C13 Explain the meaning of all the symbols and the significance of each of the terms of the Navier-Stokes equation describing the flow of an incompressible fluid

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla P + \eta \nabla^2 \mathbf{v} .$$

Write down also the incompressibility condition. [5]

The flow of incompressible, viscous liquid through a circular pipe of radius a and of total length L is generated by the pressure difference ΔP between its opening and its exit. Explain briefly the conditions under which flow is steady and laminar. For this case write down the non-zero terms in the Navier-Stokes equation for the longitudinal velocity v_z , using cylindrical polar coordinates. [5]

Hence, or otherwise, find and sketch the velocity profile $v_z(r)$ for the laminar flow in the main portion of the pipe. Show also that the volume flow rate Q is

given by

$$Q = \frac{\pi}{8\eta} \frac{\Delta P}{L} a^4 . \quad [5]$$

A machine for making sticks of bread forces dough of viscosity $\eta = 30 \text{ Pa s}$ through a pipe with $a = 4 \text{ mm}$ and $L = 10 \text{ cm}$. Taking the excess pressure as $\Delta P \approx 3 \times 10^5 \text{ Pa}$, estimate how long the device takes to discharge 25 ml of dough. Calculate also the maximum velocity in the flow. [5]

The machine forces the dough into the entrance of the pipe with a velocity which is independent of radius. Estimate approximately how far along the pipe the dough travels before steady-state laminar flow sets in. [5]

[Assume that the dough behaves as a Newtonian fluid under these conditions.]

C14 Under what conditions can the flow of fluids be described by a velocity potential that obeys Laplace's equation? Answer the following questions on the assumption that these conditions are fulfilled. [5]

Two long straight wires which lie parallel to the y -axis at $(x = 0, z = +d)$ and $(x = 0, z = -d)$ are surrounded by homogeneous fluid of density ρ which at large distances from the wires moves in the $+x$ direction with velocity V . The fluid circulation about the two wires is $+K$ and $-K$ respectively. Write down expressions for:

(a) the fluid velocity in the $z = 0$ plane;

(b) the pressure in the $z = 0$ plane,

as functions of x . [5]

By integrating the pressure over the $z = 0$ plane, obtain an expression for the vertical forces per unit length that must be applied to keep the wires in place. [5]

The lift force per unit length on the wings of an aircraft in steady flight at speed V through stationary air may be expressed as $\alpha\rho V^2$, where α depends on the design and configuration of the wings, so that the circulation about the wings is $K = \alpha V$. At take-off, however, when the same aircraft with its wings in the same configuration is moving at speed V relative to the ground, the lift force per unit length is $\alpha'\rho V^2$, where α' is less than α . Explain the distinction between α and α' and obtain an approximate expression for their ratio in terms of the height d of the wings above the ground. [10]

$$\left[\int_{-\infty}^{\infty} dx \frac{1}{x^2 + d^2} = \frac{\pi}{d} ; \quad \int_{-\infty}^{\infty} dx \frac{1}{(x^2 + d^2)^2} = \frac{\pi}{2d^3} . \right]$$