Part IB Physics B

Classical Dynamics and Fluids — Examples 1 — 2009

There will be about 2 questions for each lecture.

Problem grading:

(A) Problems that can be answered directly by quoting the lectured material or by straightforward calculation.

(B) Problems that require some algebraic formulation and manipulation as well as calculation.

(C) Problems which are either harder or longer than (B) problems. You should feel a sense of achievement in completing these.

Newtonian mechanics and the energy method

1. (A) A uniform solid cylinder is set spinning about its axis and is then gently placed, with the axis horizontal, on a rough horizontal table. What fraction of its initial kinetic energy is dissipated in sliding friction before the cylinder eventually rolls smoothly along the plane?

2. (B) A ladder of length l rests against a wall at and angle θ to the vertical. There is no friction between the top of the ladder and the wall and no friction between the bottom of the ladder and the ground.

   Write down the kinetic and potential energies of the ladder as a function of θ and $\dot{\theta}$.

   Using the energy method, or otherwise, derive the equation of motion of the ladder in terms of θ.

   [In order to work out the velocity of the centre of mass, write down expressions for the height and horizontal positions of the centre of mass as a function of θ and then differentiate with respect to time.]

3. (B) A small solid cylinder of radius r lies inside a large tube of internal radius R. Find the period of oscillation of the small cylinder about its equilibrium position.

   [Hint: first show that the angular velocity of the small tube ω is related to θ, the angle between the centre of mass of the solid cylinder, the centre of the large tube and the vertical by $\omega = \frac{R-r}{r}\dot{\theta}$.]

Rotating frames and fictitious forces

4. (A) Paraboloidal telescope mirrors can be made by 'spin casting', which involves rotating the molten glass and its container about a vertical axis as the glass solidifies. By considering the equilibrium of an element of the molten surface show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\omega^2}{g}x$$

where y is the height of the surface, x is the distance from the axis of rotation and ω is the angular velocity of rotation. For a mirror of focal length 2 m, what angular velocity is required?

[The equation of a parabola is $y = x^2/4f$, where f is the focal length.]

5. (B) A train at latitude λ in the northern hemisphere is moving due north with a speed v along a straight and level track. Which rail experiences the larger vertical force? Show that the ratio R of the vertical forces on the rails is given approximately by

$$R = 1 + \frac{8\Omega v h \sin\lambda}{ga}$$

where h is the height of the centre of gravity of the train above the rails which are at a distance a apart, g is the acceleration due to gravity, and Ω is the angular velocity of the Earth's rotation.

6. (B) A stone is dropped from a stationary helicopter 500 m above the ground at the equator. How far from the point vertically beneath the helicopter does it land and in what direction?

[You should try to solve this problem in two ways (a) by consideration of the angular momentum of the stone, (b) by invoking Coriolis force.]

7. (C) A weather map shows a shallow, stationary depression centred over a point on the Earth's surface at latitude 50° N. The isobars corresponding to pressures of 998, 1000 and 1002 mbar are concentric circles of radius 50, 200 and 350 km respectively and the temperature is 300 K. Estimate the wind speed on the middle of these isobars, neglecting the effects of friction between air and ground.

[1 mbar = 100 Pa. You should assume that the wind direction follows the isobars, i.e. rotating anticlockwise in circles around the depression as viewed from above.]

Orbits

8. (B) In a classical model of a multi-electron atom, electrons are assumed to move in a modified electrostatic potential $V(r)$, given by:

$$V(r) = -\frac{k}{r}\exp(-r/a)$$

where k and a are constants. Show that, in a such a potential, circular orbits are unstable unless

$$\frac{r}{a} < \tfrac{1}{2}\left(1+\sqrt{5}\right)$$

9. (A) A satellite travelling round the earth in a circular orbit with centre O experiences, at the point P, a sudden impulse which deflects it into a new orbit. Sketch the orbits to be expected if the impulse acts:

   (a) in the direction of motion of the satellite;

   (b) in the reverse direction;

   (c) outwards along the line OP.

   On each sketch you should show clearly, in relation to O and P, the points A and B at which the new orbit is furthest from, and nearest to O.

10. (B) Two stars of unequal mass are in circular orbit about one another. One day the more massive star suffers a spherically symmetrical loss of matter (it explodes). After explosion the masses of the stars are equal. Show that the binary system will be disrupted if $\alpha > 3$, where α is the ratio of the original masses.

11. (B) In an attempt to place a satellite into a geostationary orbit, the correct speed and radial distance are achieved but the direction of motion has an angular error θ in the plane of the orbit. Show that the maximum and minimum radii of the orbit are $a(1 \pm \sin\theta)$ where a is the radius of the geostationary orbit. By a short burst of its booster while at maximum radius, the satellite is able to put itself into a circular orbit. If the initial error θ is 1 minute of arc, calculate the period of this new orbit.

    Why was making this correction a very bad idea?

12. (B) A point mass m on the end of a light string of length l is free to swing as a conical pendulum. Show that, in terms of the (constant) angular momentum J_z of the bob about a vertical axis through the fixed point of support and the inclination θ of the string to this axis, the energy of the pendulum may be written as

$$E = V(\theta) + \tfrac{1}{2}ml^2\dot{\theta}^2$$

    where $V(\theta) = mgl(1 - \cos\theta) + J_z^2/(2ml^2\sin^2\theta)$ is the effective potential that determines motion in θ. By differentiating $V(\theta)$ twice with respect to θ, show

   (a) that the bob can move steadily round a circle, with $\theta = \theta_0$ and angular velocity Ω given by

$$\Omega^2 = \frac{g}{l\cos\theta}$$

   (b) that, if the pendulum is then given a little extra energy without changing its angular momentum, θ oscillates about θ_0 with angular frequency ω given by

$$\omega^2 = \Omega^2\left(1 + 3\cos^2\theta_0\right)$$

    Use these results to discuss the precession of almost circular orbits of a conical pendulum, assuming $\theta_0 \ll 1$.

    [For enthusiasts only: grade (C)]
    Now consider the conical pendulum in a frame of reference rotating about the vertical with angular velocity Ω. Write down the equations of motion, involving Coriolis force, which govern small displacements from its equilibrium at $\theta = \theta_0$, and hence verify the expression $\omega^2 = \Omega^2\,(1 + 3\cos^2\theta_0)$

13. (B) A spacecraft is in a circular orbit of radius $R = \alpha r$ around a moon, where r is the radius of the moon itself. A short 'burn' of the spacecraft's motor provides an impulse which halves its velocity without changing its direction, and this alters the orbit to one that just grazes the moon's surface. Deduce the value of α.

    If the same impulse had been applied in a radial direction, i.e. inwards towards the moon's centre, would the new orbit then have reached the moon?

    [Assume that the effect of the burn on the mass of the satellite is negligible. You did most of this question last year, but it's easier by Part IB methods.]

14. (C) α particles (atomic number 2) with energy E are scattered through an angle Φ by nuclei of atomic number Z. Assuming that the only interaction between the particles and the nuclei is the electrostatic force, show that the distance of closest approach is

$$\frac{Ze^2}{4\pi\epsilon_0 E}\left(1 + \mathrm{cosec}(\Phi/2)\right)$$

    α particles are scattered by lead ($Z = 82$) through an angle of 60°. As the energy of the α particles is increased the scattering is found to be in agreement with the Rutherford formula up to an energy E_0=25 MeV. Estimate an upper limit for the range of the nuclear force in lead.

Rigid Bodies

15. (A) A uniform disc of radius 0.1 m and mass 0.4 kg is rotating with angular velocity 1 rad s^{-1} about an axis at 45° to its plane through its centre of mass. What is (a) its angular momentum, and (b) its kinetic energy?

    [Assume the centre of mass is stationary.]

16. (B) A uniform rectangular tile drops without spinning until its corners reach positions $(0, 0, 0), (2a, 0, 0), (2a, 2b, 0), (0, 2b, 0)$, when it strikes the top of a vertical pole at a point very close to the $(0, 0, 0)$ corner. Just before impact the velocity of the tile was $(0, 0, -u)$. Assuming that the tile does not break, and that the impact is elastic (i.e. the kinetic energy of the tile is conserved), find immediately after impact

    (a) the velocity of its centre;

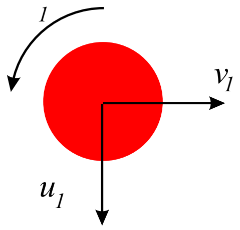
    (b) the angular momentum about its centre;

    (c) its angular velocity.

    Show that the velocity of the corner at $(0, 0, 0)$ becomes $(0, 0, +u)$ immediately after impact.

17. (B) Show that the moment of inertia of a uniform sphere of mass m and radius a about an axis through its centre is $\frac{2}{5}ma^2$.

A spherical 'super-ball' may be considered to be perfectly elastic, incompressible and rough, so that, when it collides with a surface, energy is conserved and the point of contact does not move. Such a super-ball is spinning about a horizontal axis at angular velocity Ω_1 as it falls towards a horizontal rough surface as shown below



When it hits the rough surface it is travelling with components of velocity u_1 normal to the surface and v_1 parallel to the surface and perpendicular to Ω_1. Find the linear and angular velocities u_2, v_2 and Ω_2 with which it rebounds.

[Neglect gravity.]

18. (B) A uniform disc of mass m and radius a is initially at rest. A small particle of the same mass m with an initial velocity u normal to the plane of the disc makes an elastic collision with the disc, striking it at a point midway between the centre and the rim. Find the velocity of the centre of mass of the disc and the angular momentum of the disc about its centre after the collision.

Suppose now that before the collision the disc was rotating about its own axis with angular velocity $\frac{2}{3}u/a$. Describe the motion after the collision as completely as you can, showing in particular that the disc returns to its initial orientation in the time taken for its centre to move through a distance $\pi a/\sqrt{2}$.

19. (B) A circular coin of radius a falls at speed u without rotating onto a smooth horizontal table. The perpendicular to its face makes an angle θ with the vertical. Determine the state of motion of the coin just after it strikes the table, assuming that the collision is elastic. Show that, when θ is small, the coin strikes the table a second time at an angle of $\frac{5}{11}\theta$.

20. (C) The moments of inertia of the Earth about polar and equatorial axes differ by 1 part in 300. Estimate the instantaneous rate of precession of the Earth's axis in mid-summer due to the couple exerted by the Sun alone, assuming the Earth's equatorial bulge to be concentrated in a ring round the equator. The Earth's axis is inclined at 23.5° to the normal to its orbit.

21. (B) A coin of radius a is spun on a perfectly rough table in such a way that its centre is stationary, while its axis precesses steadily about the vertical at a fixed inclination θ with angular velocity Ω. Show that $\omega_3 = 0$ and hence that

$$\Omega^2 = \frac{4g}{a \sin\theta}$$

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Show also that, if θ is a small angle, the head of the coin, viewed from above, appears to rotate with angular velocity $\Omega(1 - \cos\theta) \approx \sqrt{g\theta^3/a}$.

Quickies

1. Why does atmospheric friction make an artificial satellite speed up?
2. Why is the moon receding slowly from the earth?

Numerical Answers

Q1. 2/3.
Q4. 1.57 rad s^{-1}.
Q6. 24 cm to the East.
Q7. 28 km hr^{-1}.
Q11. 1 day + 38 seconds.
Q10. $\alpha = 7$.
Q14. 1.4×10^{-14} m.
Q15. $(0.7, 0, 1.4) \times 10^{-3}$ kg m^2 s^{-1} w.r.t. obvious axes; 0.75 mJ.
Q17. $u_2 = -u_1, v_2 = (3v_1 - 4a\Omega_1)/7, \Omega_2 = -(10v_1/a + 3\Omega_1)/7$.
Q20. 1.6×10^{-4} yr^{-1}.